BRACHISTOCHRONE (I.E.,SHORTEST TIME) IN SKIING DESCENTS

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ABSTRACT

Time needed to traverse on skis a straight line between two points A and B marked on the ski slope was compared to to the time needed to carve a ski turn which followed a cycloid curve connecting the A and B points.

Descents over the longer path outlined by the cycloid curve were faster than the descents over straight line traverses in 14 out of 18 tests. In each of the runs which procedured shorter time on the traverses considerable skidding of the skis (instead of the desired carving) was observed.

The Brachistochrone prediced by the calculus of variations indeed yield a path which the skier descended in a minimum time.

The purpose of this study was to compare the time needed to traverse on skis a straight line between points A and B to the time needed to carve a ski turn which followed a curve connecting the A and B points. (Fig.1)

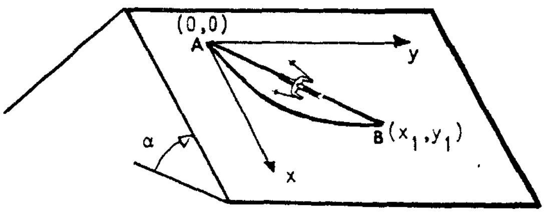


Figure 1: Slope in the experiment (not to scale) Y axis is horizontal and the X axis is along the fall-line.

Our intuition suggests that the fastest line connecting two points on traverse is a straight line. For example Dan Bean (national coach) in an article, describing "Fundamentals of Giant Slalom" in American Ski Coach, November 1987, p.12 states:
"There is no argument against the fact that the fastest way to get from point A to point B is a straight line." Numerous ski coaches make similar statements describing descents at an angle to the fall line.

However, variational calculus shows that the brachistochrone for a passage of a particle sliding without friction from A to B under the force of gravity is not a stright line, but a cycloid curve. (Fig.2)

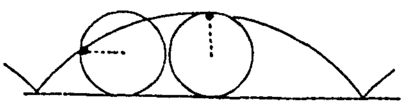


Figure 2: Common Cycloid Curve a path traced out by a point on a wheel that rolls along a straight line.

In fact the theory of the calculus of variations had its beginning in the solution of the brachistochrone problem. It was first solved (theoretically) by a Swiss scientist, Johan Bernoulli, already in 1696, (and perhaps was re-discovered by the Swiss National Team of today? Smile!)

To the best of my knowledge the first controlled experiments, adressing the brachistochrone problem in real skiing situation, were performed in 1988 by our research team.

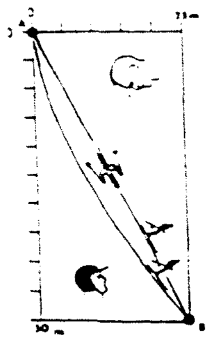
SLOPE AND SUBJECTS

The experiment was performed on a 30-40 degree slope at Squaw Valley(April 13, 1988). The slope was salted. Timing equipment was made by Southern Cross Computers. The straight traverses and cycloid curves were marked with the colour dye. Actual skiing on the straight lines and on the curves was executed by three experienced skiers. All subjects used slalom-type skis.Steve, 154 lb, 70 kg skis 200 cm. Alenka, 114 lb, 52 kg skis 195 cm. Sean, 194 lb, skis 88 kg 205 cm. The start was accomplished by using a gravity start (from a stand-still) procedure.

THE RESULTS

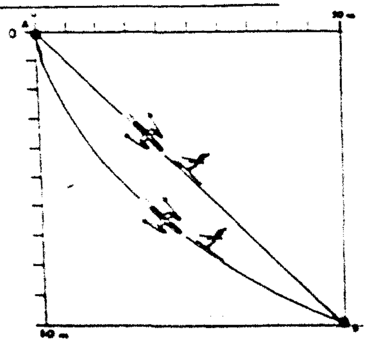
Test I

	Skier	Time in seconds Straight Line	Curve Line
Run#1	Steve	5.22	5.06
	Sean	5.08	5.00
	Alenka	5.21	(+) 5.23
Run#2	Steve	5.05	4.91
	Sean	5.07	5.05
	Alenka	5.26	5.19
Run#3	Steve	4.98	(+) 5.13
	Sean	5.09	4.73
	Alenka	5.25	5.19

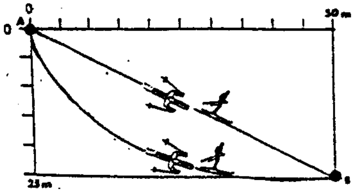


Test II

	Skier	Straight Line	Curve Line
Run#1	Steve	6.28	(+) 6.39
	Sean	6.41	6.22
	Alenka	6.70	6.20
Run#2	Steve	6.13	6.03
	Sean	6.38	6.07
	Alenka	6.36	5.98



Test III



PAIR #3

Run#1*	Steve	5.83	(+)5.84
	Sean	5.91	5.67
	Alenka	6.14	5.60

Notes: All times in seconds. * Because of rain and strong wind there was only one run in Test III.

CONCLUDING REMARCS

When the slope was prepared for the experiment numerous bystanders said: "It will never work!" - fortunately, it did. Specifically the experiment was succesful in 14 out of 18 runs. Most of the time the cycloid curve produced shorter times than the straight line. In four runs the skier was faster on the straight line than on the curve. In each of the unsuccesful runs both the skier and and the observers noted considerable skidding of the skis instead of carving (for carving-skidding line see Fig. 3).

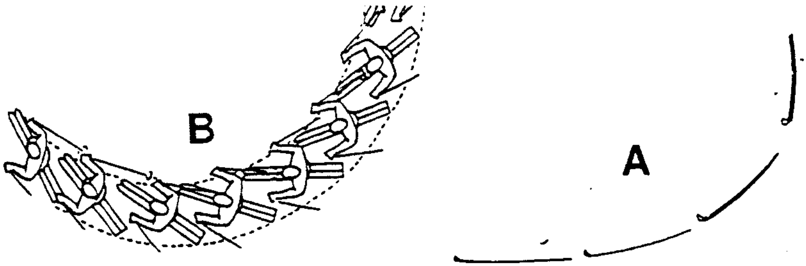


Figure 3: For the Brachistochrone to be effective a carved track, i.e., a track with minimum skidding is needed as in Fig.3, A. Skidded track which followed the cycloid curve as in Fig.3, B produced slower times than a straight line of traversing descent.

In this brief report many facts had to be left out, and without a doubt many more remain to be discovered. If in the future the whole truth emerges, will it be an extension or a contradiction of the story I told? - A Zen master could not dream up a finer conundrum!

THE FORMULAS

The slope is a plane which makes an angle(α) to the horizontal plane. The Y-AXIS is horizontal and the X-AXIS is along the fall line. The equation of the cycloid curved from origin (0,0) to the will be:

$$X = \frac{1}{4G\,C\,\sin\alpha}(1 - \cos U)$$

$$Y = \frac{1}{4G\,C\,\sin\alpha}(U - \sin U)$$

When G is the acceleration of gravity, C is a constant, and U is a parameter in the interval $0 < U < U_1$, C and U are calculated from the equations:

$$X_1 = X(U_1)$$
$$Y_1 = Y(U_1)$$

From the formulas one can deduce that the form of the curve does not depend on the slope angle but the time of the passage does. Friction is neglected in the presented calculations.

However, modern ski bases are evidently so slick (coefficient of friction as little as 0.02) that they reduce friction enough to bear out Bernoulli's Theory.

THE CURVES

The curves were calculated and graphed at the University of Uppsala, Institute of Theoretical Physics, Uppsala, Sweden and presented in Figure 4.

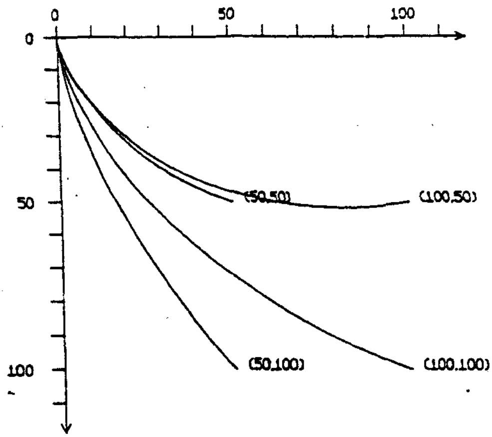


Figure 4: Minimum time curves used in the experiment (meter units).

DISCUSSION

Casual analysis may consider the advantage of the curvilinear path which at the outset of the descent is closer to the fall-line (than the straight traverse line), affords more acceleration and therefore more velocity to start with. However, this observation must be weighted against the advantage of the shortest distance of the straight traverse path possesses. At this point intuition can no longer be helpful and there remains only the mathematical method.

The mathematical solution to the brachistochrone is well known. It was published by the two Bernoulli brothers, Newton, and Leibniz, although the techniques at arriving at the solution were quite different. Today virtually every text of classical dynamics or variational calculus deals with the brachistonechrone problem in theory, but application of this theory to sports is rare.

The empirical results obtained in this research suggest a practical advice for competitive skiing: the shortest line of descent between the gates may not be the fastest.

THE RESEARCH TEAM

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Alenka Vrecek, Sean Walkerly, Steve Gould, and David Leer, all experienced instuctors coaches and racers.