DYNAMICS ANALYSIS OF PEDALING MOTION IN RACING CYCLE WITH COMPUTER SIMULATION

Akira Uesaki, Yoshiyuki Mochizuki[1], Tomoyuki Matsuo[2], Ken Hashizume[2], Koichi Omura[3], and Seiji Inokuchi[1]
Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka, Japan
[1]Laboratories of Image Information Science and Technology, Osaka, Osaka, Japan
[2]Faculty of Health and Sport Science, Osaka University, Toyonaka, Osaka, Japan
[3]Takarazuka University of Art and Design, Takarazuka, Hyogo, Japan

This paper reports the new method based on the computer simulation for the dynamics analysis of the pedaling motion in a racing cycle. At first, we describe three-dimensional mathematical models of lower limbs and the cycle, and then explain the formulation as the systems of Lagrange equations. Time-series angular displacements of each joint, the crank arm, and each pedal were obtained by capturing actual human pedaling motions. The 'ideal' pedal forces were computed by using the model of the cycle. The method for solving the 'inverse kinematics problem' is also proposed. As the results of the dynamic simulation, we obtained several dynamic properties of the three-dimensional pedaling motion. And the differences between the three-dimensional pedaling motion and the two-dimensional motion were also described.

KEY WORDS: three-dimensional mathematical model, Lagrange equations, inverse kinematics problem, Newton-Raphson method, inverse dynamics calculation

INTRODUCTION: Various approaches have been taken to analyse bicycle pedaling. Hull and Jorge (1985) constructed a mathematical model of a lower limb in pedaling, and obtained the joint torque data with the computer simulation. Gonzalez and Hull (1989) simulated the effect of five variables on the cost function derived from the joint moments developed during cycling. But these studies had two following assumptions: (1) the leg moved two-dimensionally during cycling, and (2) the foot segment of the leg was coincident with the pedal completely. In this paper, we proposed a new method based on the computer simulation for the dynamics analysis of the pedaling motion on lower limbs in a racing cycle. Compared with previous works, the features of our method can be described as follows: 1. A three-dimensional mathematical model of lower limbs to remove these unrealistic assumptions, 2. A mathematical model of a racing cycle that is constructed independently, 3. The 'ideal' pedal forces which are computed by using the model of the cycle, and 4. The method for solving the problem that the toe segment of the lower limb is apart from the pedal of the cycle. The purposes of this study were to investigate the differences between the three-dimensional pedaling motion and the two-dimensional motion when the ideal pedal forces were given, and to examine which motion was superior from the viewpoint of a burden on a rider.

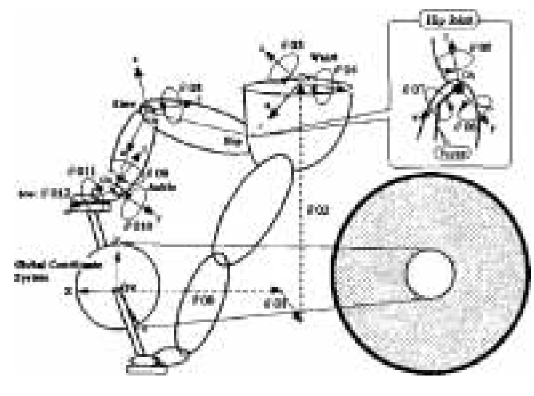


Figure 1- Mathematical model and location of hip joint

METHODS: Figure1 shows the three-dimensional mathematical model of the lower limbs. In our model, each leg consists of four segments. Especially, the feet are composed of two segments; the heel segment and the toe segment. The global coordinate system is located on the centre of the front gear of the cycle. It has five degrees of freedom for determining the locations of hip joints, and eight degrees of freedom in each leg. Especially, Femur is designed a structure with neck of femur. D-H representation (Denavit and Hartenberg, 1955) is applied to the local coordinate system, where the x-axis of it coincides with the longitudinal axis of the segment. A mathematical model of a racing cycle is constructed separately. In the same way, the global coordinate system is located on the front gear of the cycle, and the local coordinate systems are arranged on the crank and the pedals using D-H representation. We formulated these models as the systems of Lagrange equations. For example, Lagrange equations at hip joints of lower limbs and at the crank of the racing cycle can be represented by the following equations (1) and (2), respectively.

[illegible]

where $j = 0,1$ (0:right leg, 1:left leg)

$i = 0,\ldots,7$ (1)

[illegible]

+ (frictional force of air) + (frictional force of rolling) (2)

In these equations, j is the index for distinguishing between the right leg (j=0) and the left leg (j=1). θ_{ji} and θ_{cji} are the variables which correspond to the rotational angles at the i-th local coordinate system of lower limbs and of the racing cycle respectively. T_{ji} and T_{cji} are the matrices that transform the expressions on the i-th local coordinate system to the expressions on the global coordinate system. m_{ji} is the mass, J_{ji} is the matrix of inertia, r_{ji} is the positional vector of the centre of mass, and g is the vector of the gravitational acceleration. In equation (2), the last two terms of the equation express frictional force of air and of rolling respectively.

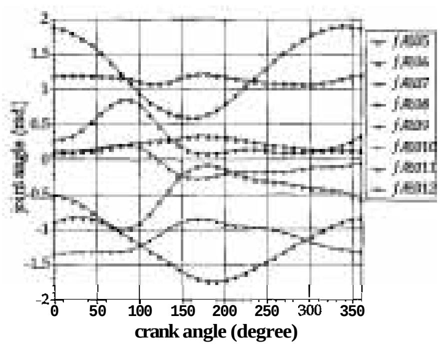
At first, angular displacements of each joint, the crank arm and each pedal were obtained for real movements measured in this study (see below). After that, noise reduction for each angular displacement was carried out with DFT (discrete Fourier transform) and IDFT (inverse DFT). Errors in measurement and the noise reduction yield the possibility that the toe segment separates from the pedal. We, therefore, have formulated the problem as the 'inverse kinematics problem', and solved it with Newton-Raphson method (Stoer J. & Bulirsch R. 1993). It can be represented by equation 3.

$$\Theta_{i+1} = \Theta_i + k\mathbf{J}^{-1}(\mathbf{r} - \mathbf{r}_i)$$
$$\mathbf{r}_i = \mathbf{f}(\Theta_i) \qquad (3)$$

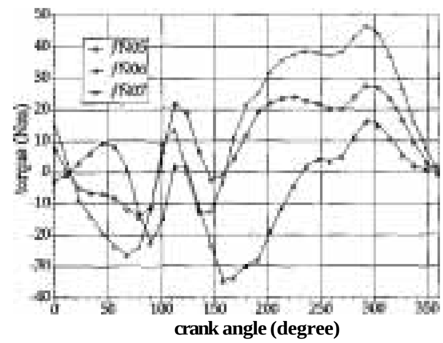
In equation 3, r and r_i is the vector which expresses locations and postures of the pedal and of the toe segment at a time respectively. θ_i is the vector of the joint angles at a time in i-th iteration, J is the Jacobian matrix, k is the constant value, and f is the function which represents the relation between the joint angular vector and the locations and postures of the toe segment. After solving the problem at each time, angular displacements of all joints are modified. With the angular displacements of the cycle, the inverse dynamics calculation for the cycle model was executed, and we transformed the obtained torques about the crank spindle and the pedal spindles to the pedal forces using the algorithm which is based on Moore-Penrose's generalised inverse matrix (Uesaki, Mochizuki, Omura, & Inokuchi, 1998). The computed pedal forces are the ideal forces in the sense that the sum of squares of them is minimised, and the consequent forces for lifting the pedal in one leg are sometimes nearly equal to the consequent forces for pushing the pedal in the other leg. Next, we executed the inverse dynamics calculation only for the human model with joint angular displacements and pedal forces.

Three subjects (subject A, B and C) with twelve markers were required to perform pedaling on a roller in forward velocity of 13.9 m/s with their own cycles, and were digitally filmed using four infrared cameras (MacReflex NP, 120Hz, Qualisys, Sweden). During pedaling, the mechanical load (RDA Interrim Trainer, Minoura, Japan) which corresponded to the frictional force of air was given on the racing cycle. The dynamics analysis simulation for lower limbs was executed with angular displacements of all joints and of the cycle. All physical parameters of lower limbs for the dynamic simulation were calculated using their anthropometric data measured prior to the pedaling experiments.

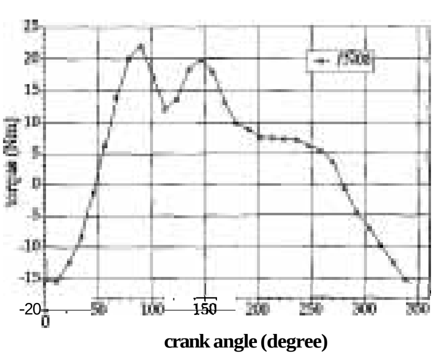
RESULTS AND DISCUSSION: Modified joint angular displacements of the right leg in subject A are shown in Graph 1. 0° indicates that the crank arm is vertical. Sinusoidal changes were seen in θ_{07} and θ_{08}. These results showed that the flexion/extension angles at the hip joint and the knee joint changed smoothly to a crank arm revolution. From the result of θ_{05}, the hip joint rotated to the direction of the abduction in the range of 0°-120° and 180° to 290°, and to the direction of the adduction in the range of 120°-180° and 290°-360°. The profile of θ_{06} indicated that the hip joint rotated to the direction of the external rotation in the range of 0°-180°, and to the direction of the internal rotation in the range of 180"-360". The angular changes of 0.25 radian were seen in both angles. It suggests that the three-dimensional mathematical model of lower limbs should be used. Concerning θ_{09} and θ_{011}, unexpected large changes at the ankle joint were observed because the constructed model



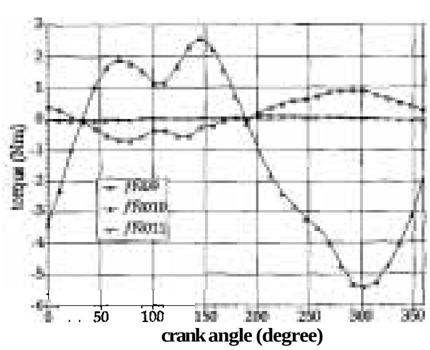
Graph 1- Joint Angular Displacements in Subject A



Graph 2- Hip Joint Torques in Subject A



Graph 3. Knee Joint Torque in Subject A



Graph 4- Ankle Joint Torques in Subject A

had only one degree of freedom at the toe joint. Since the foot segment of the leg, however, has the small inertia tensor and mass, it can be considered that there is little effect in the dynamic simulation.
The joint torques at the hip joint, the knee joint, and the ankle joint of the right leg in subject A are shown in Graph 2 to 4. From the results of τ_{05} and τ_{06}, we could recognise that the unexpected large torques were generated in spite of the small angular changes in θ_{05} and θ_{06}. Calculating the total amount of the hip joint torques at each time, the large torque was observed in the range of 180°-300°. The rider, therefore, generated the large power at the hip joint when he pulled up his leg and the pedal against the gravitational force. The knee joint torque qualitatively coincided with Hull's results (1985). Because the knee joint torque changed from the extension torque to the flexion torque at about 45°, and inversely at about 270°, it was considered that the knee joint torque mainly contributed not to the vertical movements of the leg and the crank arm, but to the horizontal movements. In our previous study, we constructed the two-dimensional mathematical model of lower limbs that had the same physical parameters, and carried out the inverse dynamics calculation for the two-dimensional model (Uesaki, Mochizuki, Omura, & Inokuchi, 1999). From the results under the same velocity condition (13.9 m/s), the knee joint torque in the two-dimensional model had the positive peak value of 26.0 Nm at about 150°, and was always larger than the knee joint torque obtained by the three-dimensional model in the range of 100° to 360". These results indicated that the knee joint torque might be influenced by the small movements at the hip joint. From Graph 4, τ_{09} and τ_{011} were very small. It was found the shape of τ_{010} was similar to the shape of τ_{08}, but the most important difference between the shapes of τ_{010} and τ_{08} was that τ_{010} shifted to the negative torque suddenly after it reached the positive peak at 145".

CONCLUSION: As the results of the dynamic simulation for bicycle pedaling, we have obtained several dynamic properties that have been unknown. At first, it was demonstrated that the small movements at the hip joint affected the hip joint torques remarkably. Secondly, the knee joint torque in the three-dimensional model was smaller than that in the two-dimensional model. We recommend that the riders should not perform pedaling two-dimensionally, because the abduction/adduction and the external/internal rotation at the hip joint may reduce a burden imposed on the knee joint. Additionally, they need to train their muscles for the abduction/adduction and the external/internal rotation at the hip joint, since the unexpected large hip joint torques are produced.
In the future works, pedaling motions in many riders and some other conditions need to be analysed by the same method, and we want to investigate what the 'ideal' pedaling motion is. The obtained information may be useful for improvement in skill of riders and in their training plan.

REFERENCES:
Denavit, J., & Hartenberg, R. (1955). A kinematic notation for lower-pair mechanism based on matrices. *ASME* Journal of Applied Mechanics, **22,** 215-221.
Gonzalez, H., & Hull, M.L. (1989). Multivariable optimization of cycling biomechanics. Journal of Biomechanics, 22, 1151-1161.
Hull, M.L., & Jorge, M. (1985). A method for biomechanical analysis of bicycle pedalling. Journal of Biomechanics, **18,** 631-644.
Stoer J., & Bulirsch R. (1993). *Introduction* to numerical analysis. Springer-Verlag.
Uesaki, A., Mochizuki, Y., Omura, K., & Inokuchi, S. (1998). Computer simulation for analyzing pedaling force in a competitive bicycle. Proceedings of the 42 Annual Conference of the Institute of System, Control and Information Engineers. 61-62. (in Japanese)
Uesaki, A., Mochizuki, Y., Omura, K., & Inokuchi, S. (1999). Computer simulation for dynamics analysis of pedaling motion on lower limbs in a racing cycle. The 48th Japan National Congress for Theoretical & Applied Mechanics. 181-182. (in Japanese)