CUT-OFF FREQUENCY ESTIMATION OF KINEMATICAL DATA WITH DISCRETE FOURIER TRANSFORMATION AND REGRESSION ANALYSIS

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INTRODUCTION

This study addressed the problem of the differentiation process of noisy human motion data. The finite difference method was the first approach to replace the graphical method in obtaining smooth kinematic data and their derivatives. The finite difference method gives the derivatives estimates, smoothing the data at the same time, but Widule and Gossard, (1973) pointed out that the accelerations were too noisy and with high oscillations for direct application.

Winter, Sidwall, and Hobson, (1974) obtained a better approximation for the second derivative in a human locomotion after filtering their data with a low pass second order Butterworth filter. Pezzack, Norman, and Winters, (1977) compared the calculated accelerations from finite differences after digital filtering by a second order Butterworth filter, with accelerations from Chebyshev least squares polynomials followed by polynomial differentiation. Pezzack et al. (1977) demonstrated the higher quality of the Butterworth filter comparing the calculated data with analog real data obtained with an accelerometer. However the uncertainty of the cut-off frequency choice is a drawback when someone is dealing with digital filters or with frequency domain methods. The process of trial and error in order to find the right set of filter parameters could be tedious.

Cubic splines gave a better estimation of higher order derivatives than orthogonal polynomials when applied to force predictions (Zernicke, Caldwell, & Roberts, 1976).

Splines require a number of parameters to be estimated before an analysis and the work of Wood and Jennings (1979) was an attempt to automate the trial and error procedure. The work of Dierckx (1975) on b-splines and its application in biomechanics (Soudan & Dierckx, 1979) established the suitability of splines in human motion analysis. Another method that uses b-splines is the Generalized Cross Validation Criterion (GCV) (Utreras, 1981) that Woltring (1985) used to analyze Pezzack et al.'s (1977) data and Vaughans's (1982) data.

Fourier analysis has been employed to describe a measured motion and to examine the frequency domain of human movement., Cappozzo, Leo, and Pedotti, (1975) computed the harmonics of displacement data with different sampling intervals until the variance of the residuals approached a minimum difference and then calculate explicitly the velocities and accelerations from the Fourier coefficients. Jackson (1979) also estimated the cut-off frequency at the point where the variance of the residuals was minimum. Bar, Eden, Ishai, and Seliktar (1979) estimated the lowest cut-off frequency where the variance of the filtered signal from the original was the same with the known error amplitude.

The delineation of those spectrum frequencies that account for the majority of a signal variance has a crucial role in applied biomechanics. The purpose of this study was to propose a new smoothing technique for human motion data using a Discrete Fourier Transformation (DFT) that uses a statistical criterion for the estimation of the cut-off frequency. This smoothing method was constructed via a DFT of the displacement data combined with a regression analysis. DFT and regression method (DFTR) gives a meaningful way to identify this frequency through a statistical procedure, the regression's level, in contrast with the power spectrum graph (PSG) method that applies the "eye-balling" method. A secondary purpose was to compare this alternative method with three conditions representing characteristic human movements.

METHODS

DFTR transforms the data from the time domain to the frequency domain. The new constructed data formed an orthogonal basis of variables that they could enter a regression analysis for the evaluation of the statistically significant variables. Each variable represented a Fourier frequency, hence the valid frequencies and an optimum cut-off frequency could be estimated.

The DFTR method does not completely eliminate the trial and error procedure which is a characteristic of the most smoothing techniques, but it gives a fixed way to deal with it. After the regression chooses the valid frequencies for a certain ‡ level the researcher can decide if the smoothing is acceptable by examining the fitting of the data curve and the smoothness of the derivatives. A more strict ‡ level usually can improve the smoothness of the derivatives. Furthermore, the researcher can either force some of the frequencies out of the equation to increase the degree of smoothing or force some of the frequencies into the equation to increase the degree of fitting.

The maximum accepted frequency is considered the cut-off frequency.

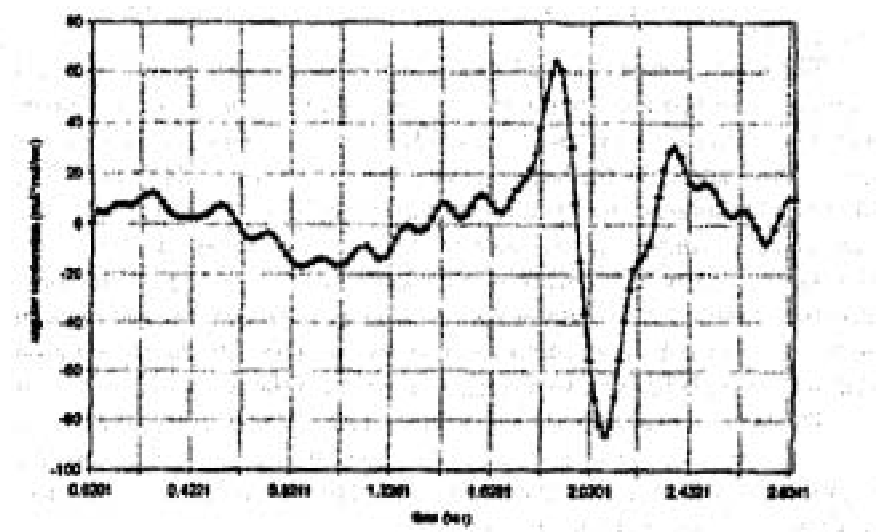
The PSG of the data represents the squares of the Fourier coefficients with respect to frequency. The flat part of the signal frequency spectrum is assumed to be connected with random error (Press, Teukolsky, Vetterling, & Flannery, 1992). In this case, the squared amplitudes of the high frequencies beyond the lowpass signal cut-off frequency should form a nearly straight line. The first estimation of the cut-off frequency would be exactly that point where the flat part of the graph begins. Even a crude approximation can give good results, when applied to data, as the obtained results differ from the optimal results by an amount that is second order in the precision to which the optimal cut-off frequency is determined (Press et al., 1992).

The matrix language of SPSSx was used to implement the DFT and regression method and the power spectrum graph method. Two routines written in fortran77 by Dierckx (1975) and Woltring (1985), were used for the b-splines and GCV methods respectively. The four methods a) DFTR, b) PSG, c) b-splines, and d) GCV were evaluated under four different data sets. The algorithms were evaluated with four applications: a) Pezzack et al.'s data (1977), b) data simulating a projectile motion, c) data simulating a counter motion, and d) data simulating a tracking motion. Pseudo-random error with normal distribution of 0 mean and 0.01 standard deviation (Lanshammar, 1982) was superimposed to the synthetic data.

The mean square error (MSE) and the mean signal to noise ratio (SNR) of the smoothed data were used as indexes of comparison among the algorithms for the three synthetic data sets. The results from Pezzack's et al. data (1977) were compared with
previously published results (Soudan & Dierckx, 1979; Wood & Jennings, 1979; Lanshammar, 1982) for the same data set.

RESULTS AND DISCUSSION

Pezzack et al.'s (1977) data by nature were good (Lanshammar, 1982) and all algorithms had reasonably fitting to the data. Comparison of the DFTR acceleration curve (see Figure 1) with the real acceleration curve (Pezzacket. al 1977; Lansharnmar, 1982) demonstrated the ability of the algorithm to preserve the peaks and shape of the curve. The oscillations between one and two seconds are larger than the fluctuations of the real curve, but the shape of the acceleration curve functions in the same fashion with the real curve for the rest of the sampling time, even at the boundary points.



DFTR and PSG had larger MSE than the b-splines and GCV methods for the counter and projectile motions. The b-splines and GCV methods estimated closer the real data and the real derivatives. The b-splines had the highest SNR (132.9) and it also had the smallest MSE (0.588) for the counter motion acceleration data; the GCV had the highest SNR (83.6) and

Figure 1. DFTR estimated acceleration data for Pezzack et al.'s data (1977).

it also had the smallest MSE (0.0002) for the projectile motion acceleration data, These results suggest that b-splines and GCV are suitable for projectile motions; whereas the DFTR and PSG should not be used with these type of motions. The frequency domain methods failed to give a good approximation due to the initial Fourier approximation that had large discrepancies at the boundary points and demonstrated oscillating Gibs phenomena.

All methods had a good fit with the real tracking motion data, except the GCV, which demonstrated some discrepancies near local maxima and minima. DFTR had the best approximation as it had the highest SNR(37.9) and the smallest acceleration MSE (0.004). The second best approximation according to the SNR index of table 1 was that of b-splines. The tracking simulated data were periodic and thus DFTR and PSG have an advantage over techniques with polynomial formulation. The same reasoning is applied to the projectile and to the counter motion, where b-splines and GCV have a priori advantage due to their polynomial nature. The gain for the DFT is like an ideal filter (see Figure 2), with no attenuations and a sharp signal decline near the estimated 6 Hz
cut-off frequency.

The results suggest that DFTR is suitable for smoothing periodic motions, but further research should point to short term Fourier transform and wavelets for non-periodic motions. DFTR can be used and as a method to just estimate the cut-off frequency of a signa, 1 a requirement for other smoothing techniques.

Table 1.
Descriptive statistics and fit indexes of the error approximation of the four algorithms for the tracking motion.

	Statistics				
Methods	Mean	S.D.	Max	Min	MSE
	Displacement (m)				
Real err.	0.000497	0.01007	0.0257	-0.0287	0.000101
Fourier	0.000492	0.01007	0.0255	-0.0285	0.000101
DFT Reg.	0.000498	0.00041	0.0013	-0.0001	0.000000
5 Har.	0.000494	0.00097	0.0023	-0.0012	0.000001
B-spline	0.000491	0.00149	0.0117	-0.0015	0.000002
GCV	0.000498	0.01126	0.0163	-0.0247	0.000126
	Velocity (m/sec)				
DFT Reg.	-0.000003	0.00529	0.0083	-0.0083	0.000027
5 Har.	-0.000004	0.02474	0.0429	-0.0478	0.000609
B-spline	0.012305	0.07548	0.6329	-0.1803	0.005819
GCV	0.005410	0.24675	0.6610	-0.6128	0.060610
	Acceleration (m/sec2)				
DFT Reg.	-0.000010	0.63462	0.9701	-0.895	0.004007
5 Har.	-0.000657	7.13228	13.3212	-12.332	0.506151
B-spline	-6.074135	22.20557	6.3333	-140.508	5.275172
GCV	12.994500	36.94028	78.5671	-38.237	15.266185

	Fit Indexes		
Methods	SmNS	NSR	SNR
	Displacement		
Real error	5100.51653029823	25.50258265141	0.03921171489
Fourier	40.48959326904	0.20244796635	4.93954085118
DFT Reg.	5.27770481463	0.02638852407	37.89526072881
5 Har.	6.88238463811	0.03441192319	29.05969522430
B-spline	6.43043575622	0.03215217878	31.10209130176
GCV	68.23443470833	0.34117217354	2.93107139899

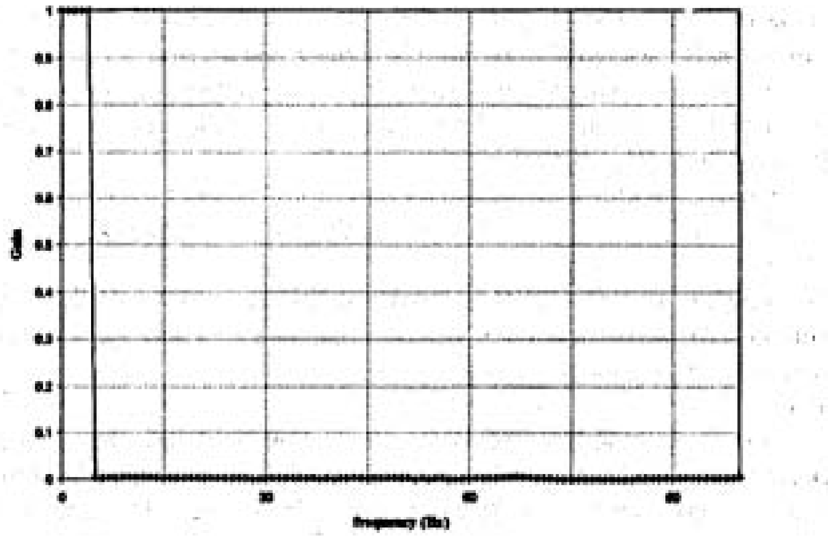


Figure 2. DFTR gain characteristics for the tracking data.

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