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# Optimal Angle of Shotput and Long Jump

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In many materials, authors only list the optimal angles of delivery and take-off of world class athletes, and most textbooks quote the angles from these research sources. In fact, every athlete has his or her own optimal angle. If a general athlete acts as an elite athlete on an optimal angle, the result will fall short of his expectations. Both theoretical and experimental analysis have verified this viewpoint.

## 1. Optimal Delivery Angle of Put

We shall assume the motion of the put to be a parabolic motion; we shall also assume a delivery height  $h=1.8\text{m}$ . We obtain

$$\sin\theta = v / \sqrt{2(v^2 + gh)} \quad (1)$$

$v$  is velocity of delivery,  $g$  is gravity acceleration, and  $h$  is drop in height.

**TABLE 1**  
Optimal Angle and Distance in Different Velocities of Delivery

v(m/s)	8	12	16	20	30
$\theta$	38.7°	41.9°	43.1°	43.8°	44.4°
Smax(m)	8.14	16.4	27.8	42.6	93.4

According to Table 1, the higher the velocity of delivery, the closer the optimal angle is to 45. When the distance exceeds 20 meters, the variance of the optimal angle is quite small, so most students in middle school and college should not use the optimal angle range from 41 to 43 as recommended by textbooks. The diagram is known to all: the real angle and optimal angle can be obtained on the spot by means of a stopwatch and a tape measure.

## 2 Optimal Take-off Angle of Long Jump

The approach run velocity in the long jump is higher than that of the take-off velocity. We shall view the track of the mass center of the human body as a parabola; we shall assume that  $w$  is the velocity of the approach run, that  $u$  is the velocity of the take-off, ignore the difference in height, and the optimal take-off push angle will be

$$\cos \alpha = \sqrt{(w^2 + 8u^2 - w) / 4u} \quad (2)$$

We shall assume  $w/u = k$ , by means of a velocity triangle. The optimal take-off angle can be written as

$$\operatorname{tg} \theta = (2k\sqrt{k^2 + 8 - 2k^2 + 8})^{1/2} / (\sqrt{k^2 + 8 + 3k}) \quad (3)$$

Formula (3) indicates that there is no optimal take-off angle that can suit everybody. The optimal take-off angle is only related to the ratio between the velocity of the approach run and the velocity of the take-off velocity. Table 2 is the result of this evaluation

**TABLE 2:**  
Optimal Take-off Angles for different Velocity Ratios

k	0	0.6	0.8	1.0	1.5	2.0	3.0
$\theta$	45°	34.9'	32.1°	30.0°	25.2'	21.5°	16.3°

**TABLE 3:**

The Variance of a Group of Elite Athletes' Optimal Take-off Angles

No.		1	2	3	4	5
M	v	8.93	8.85	8.50	8.95	8.05
	$\theta'$	21°00"	21°09"	23°16"	18°41"	18°05"
	$\alpha$	80°	80°	81°	76°	75°
E	w	7.84	7.71	7.27	7.79	6.98
	u	3	3.24	3.39	3.00	2.59
	k	2.42	2.38	2.14	2.57	2.70
	$\theta$	19°00"	19°00"	20°54"	18°09"	17°02"
	$\theta' - \theta$	2°00"	2°09"	2°22"	0°32"	0°53"

$\alpha$  – angle of take-off

M – measurements

E – evaluations

Table 3, the measurements are quoted from the work of Spanochonak. At the time, he didn't know of our evaluations. The variance of take-off angle between measurements and evaluations is so small that we **cannot** think of it as chance. The results elucidate that formula (3) is usable.