

BASIC PRINCIPLE OF RIDING ON A SNAKEBOARD

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This paper describes the mathematical model of a derivative of a skateboard known as the snakeboard. Equations of motion of the model are derived and their analytical and numerical investigations are fulfilled assuming harmonic excitation for the angles of rotation by feet and a torso of the rider. The possibility of the forward motion for the snakeboard is analyzed.

KEY WORDS: snakeboard, dynamics, analysis of motion.

INTRODUCTION: The Snakeboard (Figure 1) is one of the modifications of a well-known skateboard. It allows the rider to propel himself forward without having to kick off the ground. The motion of the snakeboard becomes possible due to specific features of its construction and due to the special coordinated motions of the rider's feet and body. The first snakeboard appeared in 1989 and from this moment many fans among the amateurs of extreme sports it has found. Soon after the invention of a snakeboard the first attempts to describe the basic principles of motion have been made. The basic mathematical model for the snakeboard has been proposed by Lewis et al. (1994). In this paper we give the rigorous mathematical proof of the conditions for the forward motion of a snakeboard.

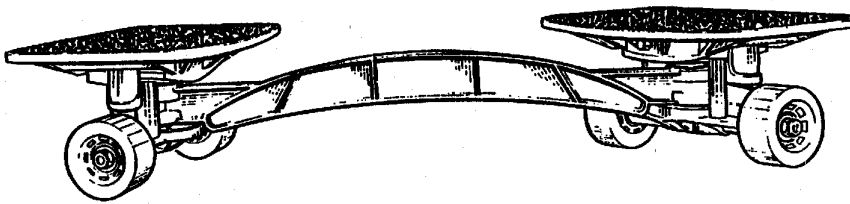


Figure 1: The Snakeboard. (Reproduced from Smith et al., (1991)).

The snakeboard consists of two wheel-based platforms upon which the rider is to place each of his feet. These platforms are connected by a rigid crossbar with hinges at each platform to allow rotation about the vertical axis. To propel the snakeboard the rider first turns both of his feet in. By moving his torso through an angle, the snakeboard moves through an arc defined by the wheel angles. The rider then turns both feet so that they point out and moves his torso in the opposite direction. By continuing this process the snakeboard may be propelled in the forward direction without the rider having to touch the ground.

METHODS: MATHEMATICAL MODEL AND EQUATIONS OF MOTION: The mathematical model of the snakeboard considered in this paper is represented in Figure 2. We assume that the snakeboard moves on the xy plane and let Oxy be the fixed coordinate system with origin at any point of this plane. Let x and y be the coordinates of the system centre of mass (point G) and θ is the angle between the central line of the snakeboard and the Ox -axis. In the basic model treated by Lewis et al. (1994) the platforms could rotate through the same angle in opposite directions with respect to a central line of the snakeboard (in other words, for the model described by Lewis et al. (1994) we have $\varphi_f = -\varphi_b = \varphi$, see Fig. 2). We suppose that platforms can rotate independently and their positions are defined by two independent variables φ_f and φ_b . The motion of the rider is modelled by a rotor. Its angle of rotation with respect to the crossbar is denoted by δ .

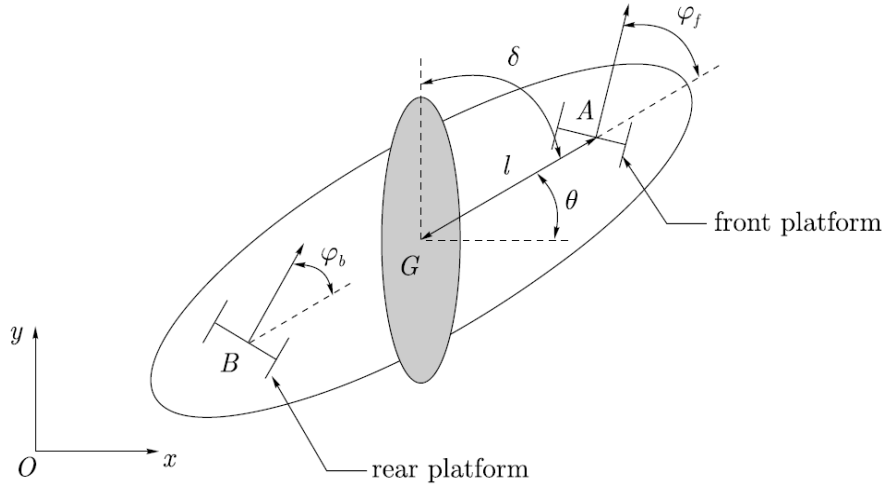


Figure 2: The Mathematical Model of the snakeboard. (Image modified from Lewis et al. (1994)).

Further we describe the motion of platforms using new variables ψ_1 and ψ_2 connected with variables φ_f and φ_b by relations:

$$\psi_1 = \varphi_f - \varphi_b, \quad \psi_2 = \varphi_f + \varphi_b.$$

If $\varphi_f \neq \varphi_b \neq 0$ then there is a point on the line passing through a crossbar which has a zero lateral velocity and hence only the velocity along the crossbar. We denote this velocity by V . Control of the Snakeboard is realized by rotations of the platforms through φ_f and φ_b and by rotation of the rotor through δ . Therefore we assume that the variables δ , ψ_1 and ψ_2 are known functions of time t i.e.

$$\delta = \delta(t), \quad \psi_1 = \psi_1(t), \quad \psi_2 = \psi_2(t).$$

These variables are the controlled variables in this problem.

Parameters for the problem are:

m_b : the mass of the crossbar;

m_r : the mass of the rotor;

m_p : the mass of every platform (we assume the platforms are identical);

$m = m_b + m_r + 2m_p$: the total mass of the system;

J_b : the moment of inertia of the crossbar;

J_r : the moment of inertia of the rotor;

J_p : the moment of inertia of every platform;

l : the length from the board's center of mass to the location of the wheels;

$$k^2 = \frac{J_b + J_r + 2J_p + 2m\ell^2}{m\ell^2}, \quad d_1 = \frac{J_r}{m\ell}, \quad d_2 = \frac{J_p}{m\ell}.$$

Equations of motion of the considered model of a snakeboard have the form (Kuleshov, 2007):

$$\dot{x} = V \cos \theta - \frac{V \sin \psi_2 \sin \theta}{\cos \psi_1 + \cos \psi_2}, \quad \dot{y} = V \sin \theta + \frac{V \sin \psi_2 \cos \theta}{\cos \psi_1 + \cos \psi_2}, \quad \dot{\theta} = \frac{V \sin \psi_1}{(\cos \psi_1 + \cos \psi_2) \ell}, \quad (1)$$

$$P_1(t) \dot{V} + P_2(t) V = Q(t), \quad (2)$$

$$P_1(t) = 1 + \frac{k^2 \sin^2 \psi_1}{(\cos \psi_1 + \cos \psi_2)^2} + \frac{\sin^2 \psi_2}{(\cos \psi_1 + \cos \psi_2)^2}, \quad Q(t) = -\frac{(d_1 \ddot{\delta} + d_2 \ddot{\psi}_2) \sin \psi_1}{\cos \psi_1 + \cos \psi_2},$$

$$P_2(t) = \frac{(\dot{\psi}_2 \sin \psi_2 \cos \psi_2 + k^2 \dot{\psi}_1 \sin \psi_1 \cos \psi_1)}{(\cos \psi_1 + \cos \psi_2)^2} + \frac{(\dot{\psi}_1 \sin \psi_1 + \dot{\psi}_2 \sin \psi_2)(k^2 \sin^2 \psi_1 + \sin^2 \psi_2)}{(\cos \psi_1 + \cos \psi_2)^3}.$$

Equation (2) determines the dependence of the velocity V on the controlled variables $\delta(t)$, $\psi_1(t)$ and $\psi_2(t)$. Suppose that $V(0) = V_0 = 0$. Then the solution of equation (2) can be written as follows:

$$V(t) = - \frac{(\cos \psi_1(t) + \cos \psi_2(t))}{\sqrt{k^2 \sin^2 \psi_1(t) + (\cos \psi_1(t) + \cos \psi_2(t))^2 + \sin^2 \psi_2(t)}} \times \int_0^t \frac{\sin \psi_1(\tau) (d_1 \ddot{\delta}(\tau) + d_2 \ddot{\psi}_2(\tau)) d\tau}{\sqrt{k^2 \sin^2 \psi_1(\tau) + (\cos \psi_1(\tau) + \cos \psi_2(\tau))^2 + \sin^2 \psi_2(\tau)}}. \quad (3)$$

Having the expression for $V(t)$ we obtain from the third equation of the system (1)

$$\theta(t) = \theta(0) + \int_0^t \frac{V(\tau)}{\ell} \frac{\sin \psi_1(\tau)}{(\cos \psi_1(\tau) + \cos \psi_2(\tau))} d\tau. \quad (4)$$

Using this formula we can obtain from the first two equations of the system (1):

$$\begin{aligned} x(t) &= x(0) + \int_0^t V(\tau) \left(\cos \theta(\tau) - \frac{\sin \psi_2(\tau) \sin \theta(\tau)}{\cos \psi_1(\tau) + \cos \psi_2(\tau)} \right) d\tau, \\ y(t) &= y(0) + \int_0^t V(\tau) \left(\sin \theta(\tau) + \frac{\sin \psi_2(\tau) \cos \theta(\tau)}{\cos \psi_1(\tau) + \cos \psi_2(\tau)} \right) d\tau \end{aligned} \quad (5)$$

Thus, the problem of a Snakeboard dynamics at arbitrary controlled variables $\delta(t)$, $\psi_1(t)$ and $\psi_2(t)$ is completely solved in terms of integrals (3)-(5). However the calculation of these integrals for given controlled variables and the analysis of the exact solution is a rather complicated problem. Below we assume the harmonic excitation for the controlled variables.

RESULTS: Observations of actual snakeboard riders suggest that sinusoidal inputs provide a good starting point for our investigations:

$$\delta = a_0 \sin(\omega_0 t), \quad \psi_1 = a_1 \varepsilon \sin(\omega_1 t), \quad \psi_2 = a_2 \varepsilon \sin(\omega_2 t).$$

Here ε is a parameter. We assume that ε is sufficiently small such that for the angles ψ_1 and ψ_2 the following approximate formulae

$$\sin \psi_1 \approx \psi_1, \quad \sin \psi_2 \approx \psi_2, \quad \cos \psi_1 \approx 1 - \frac{\psi_1^2}{2}, \quad \cos \psi_2 \approx 1 - \frac{\psi_2^2}{2}$$

are valid. In other words, we will neglect the terms of order higher than the second on the parameter ε . This assumption is completely justified by the snakeboard construction.

The snakeboard is assumed to have its initial condition at the origin in the space state, i.e.

$$x(0) = 0, \quad y(0) = 0, \quad \theta(0) = 0.$$

Taking into account all these assumptions we have the following simplified formula for the velocity $V(t)$:

$$V(t) \approx \frac{d_1 \varepsilon a_0 a_1 \omega_0^2}{2} \int_0^t \sin(\omega_0 \tau) \sin(\omega_1 \tau) d\tau + \frac{d_2 \varepsilon^2 a_1 a_2 \omega_2^2}{2} \int_0^t \sin(\omega_1 \tau) \sin(\omega_2 \tau) d\tau.$$

This integral will be a periodic function except the case

$$\omega_0 = \omega_1 = \omega_2 = \omega.$$

In this case the velocity $V(t)$ is a linear function of time:

$$V(t) = \frac{(d_1 a_0 + d_2 a_2 \varepsilon) a_1 \omega \varepsilon}{4} (\omega t - \sin(\omega t) \cos(\omega t)).$$

Thus we can formulate the main principle of a Snakeboard dynamics: to propel the snakeboard forward the rider should rotate his torso and his feet so that the frequencies of rotation by a torso and feet would be equal.

From the third of equations (1) an approximate formula for $\theta(t)$ has the form:

$$\theta(t) = \frac{d_1 a_0 a_1^2 \varepsilon^2}{8\ell} \left(\sin(\omega t) - \omega t \cos(\omega t) - \frac{\sin^3(\omega t)}{3} \right).$$

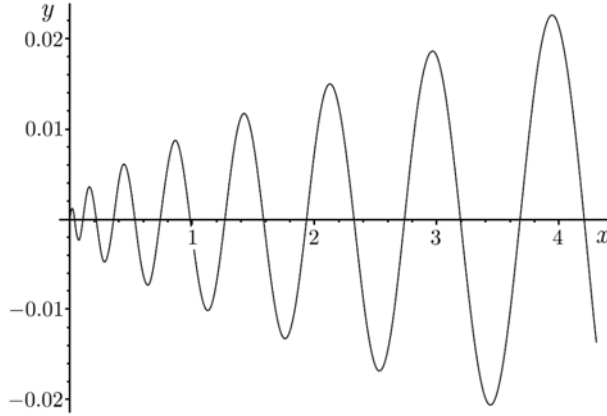


Figure 3: Trajectory of the snakeboard's center of mass.

Obviously, the function $\theta(t)$ as well as $V(t)$ is a linearly growing function of time. Therefore in a time interval of order $1/\varepsilon$ the angle θ remains proportional to ε . Thus in this time interval we can consider θ as a small angle. For small values of θ we have for $x(t)$ and $y(t)$:

$$x(t) = \frac{(d_1 a_0 + d_2 a_2 \varepsilon) a_1 \varepsilon}{8} (\omega^2 t^2 - \sin^2(\omega t)),$$

$$y(t) = \frac{d_1 a_0 a_1 a_2 \varepsilon^2}{4} \left(\sin(\omega t) - \omega t \cos(\omega t) - \frac{\sin^3(\omega t)}{3} \right).$$

Fig. 3 shows the trajectory of the snakeboard's center of mass in the considered case.

CONCLUSION: In this paper we found the condition for the forward motion of a snakeboard. We have proved that the rider propels the snakeboard forward using the 1:1 resonance between two frequencies: the frequency of rotation by a torso and the frequency of rotation by feet. These two frequencies should be equal.

Other snakeboard gaits have been investigated numerically in (Lewis et al., 1994) and analytically in (Kuleshov, 2007). It can be proved that every snakeboard gait can be achieved by a corresponding choice of resonant condition between the frequencies of rotation by a torso and feet. These results will be helpful for all people who makes the first steps in a snakeboard riding.

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