OPTIMIZATION OF DISCUS FLIGHT

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We use a 3-D model for men's and women's discus flight including initial discus flight path angle β_0, angle of attack α_0, pitch attitude δ_0 as well as release speed v_0 and initial spin rate p_0. We study in detail optimal release conditions depending on a constant wind velocity of v_0 =5 m/s blowing from different directions γ=0°, 10° up to 350°. Here γ =0°, 180°, 90°, and 270° correspond to tail wind, head wind, wind from the left, and wind from the right, respectively. The optimal wind for men is head wind from the right (γ= 220°). In this case optimal men's strategy at v_0=25 m/s is β_0 = 33°, α_0= 23°, δ_0= 30° with a range r=74.80 m. Optimal wind for women is wind exactly from the right (γ= 270°). The optimal women's strategy at v_0=24 m/s is β_0 = 41°, α_0= 32°, δ_0 = 30° with a range r=61.26 m. In all cases we assume an initial spin rate of p_0=50 rad/s. At the moment of release, the angle of attack α_0 of the discus symmetry plane should always be less than the flight path angle β_0. Also, we can show that a faster-spinning discus imparts greater gyroscopic stability and therefore achieves a better throw. We used evolutionary algorithms to perform the optimization.

KEY WORDS: Discus Flight, Release Conditions, Optimization, Evolutionary Algorithm.

INTRODUCTION: During the flight, drag and lift forces affect the discus. Due to lift, the discus often flies much further than the expected parabola distance. More than 10% of the throwing distance can be affected by aerodynamics, see Hildebrand (2001). Aerodynamic forces likewise generate a torque on the discus. Since the discus has an angular momentum at the moment of release, two classes of equations must be satisfied: First the equations of motion of the mass centre and secondly Euler´s gyroscopic equations. Recently, Hildebrand (2001), Hubbard (2002) and Hubbard and Cheng (2007) studied such a 3-D rotational model allowing the discus spin axis not to lie in a constant vertical flight plane.

The main purpose of this paper is to determine optimal release parameters depending on the wind direction. In all cases the modulus of wind velocity is assumed to be constant v_w = 5m/s. Our objective is the following. The system of equations of motion were established and numerically solved by the first author's work, see Hildebrand (2001). The throwing distance (range) is then a function of the release parameters which are initial speed v_0, flight angle β_0, wind velocity v_w, spin rate p_0, angle of attack α_0 (in our notation this is the angle between the discus plane and the ground) and pitch attitude δ_0 (which is the angle between the thrower's arm and the ground). We used drag and lift coefficients from Tutevich (1976). The moments of inertia for men's and women's discuses were determined experimentally with a torsion pendulum, see Sommerfeld (1950). These values are listed below in Table 1. They are quite similar to those obtained by Hubbard and Cheng (2007). Varying the release conditions (α_0, β_0, δ_0) as well as the wind direction γ we are able to determine the maximum range. This provides useful orientations for coaches and athletes. In the last part we study dependence of the optimal range r of both the wind direction γ and the initial spin rate p_0. We consider nominal release speed v_0 =25 m/s.

Table 1 Parameters for men's and women's discuses

discus	mass [kg]	diameter [cm]	moments of inertia	
			I_a [kgm²]	I_t [kgm²]
men's	2	22.1	0.0147	0.0085
women's	1	18.2	0.0044	0.0022

Here I_a denotes the moment corresponding to the discus axis while I_t is the moment with respect to an axis in the discus plane.

METHODS: The discus is balanced on its rotational axis. Therefore we have gyroscopic equations of angular velocities about discus axes as follows. If M_1, M_2, M_3 are the moments of aerodynamic force corresponding to three orthogonal axes, then, see Hildebrand (2001):

$$I_t \cdot \frac{d\omega_1}{dt} - (I_a - I_t)\omega_2\ \omega_3 = M_1, \quad I_t \cdot \frac{d\omega_2}{dt} - (I_a - I_t)\omega_3\ \omega_1 = M_2, \quad I_a \cdot \frac{d\omega_3}{dt} = M_3.$$

The motion of the centre of mass is described by gravity, drag and lift forces as well as speed $\mathbf{v}$ and wind velocity $\mathbf{v}_w$. Let $\mathbf{w} = \mathbf{v}_w - \mathbf{v}$. The aerodynamic forces are determined by $\mathbf{F}_D = \frac{\rho}{2} A w^2 c_D \cdot \frac{\mathbf{w}}{w}$ and $\mathbf{F}_L = \frac{\rho}{2} A w^2 c_L \cdot \frac{\mathbf{w} \times (\mathbf{n} \times \mathbf{w})}{w^2}$. Here A denotes the area of the discus, ρ is the atmospheric density and w is the absolute value of $\mathbf{w}$. Note that drag and lift coefficients c_D and c_L depend on the angle between discus axis and direction of $\mathbf{w}$. These equations were numerically solved for a given set of nine initial parameters. These nine parameters form an individual in the evolutionary algorithm, see Weicker & Weicker (2003). At most four parameters are changed simultaneously in one evolution step. Usually wind direction and initial spin are fixed. A population is formed out of 50 individuals. We perform 20 mutations and 20 crosses in one evolution step and use a quadratic selection algorithm to choose the fittest 50 individuals for the next generation. Fitness function is the range. In general we let β_0 vary freely. It turns out that in all cases the optimal initial flight path angle β_0 is between 28° and 45°. We assume the following restrictions on α_0, β_0 and δ_0 to be satisfied, $0° \le \beta_0 - \alpha_0 \le 10°$ and $0° \le \delta_0 \le 30°$ which is justified by the abilities of any thrower. In all cases wind velocity is v_w = 5 m/s.

RESULTS: All tables are computed for right throwers. For left throwers take the values at γ'= 360°-γ. Figure 1 shows the optimal release parameters (α_0, δ_0, β_0) for a men's discus with initial spin rate p_0 = 50 rad/s and v_0= 25 m/s depending on wind direction. It turns out that optimal wind is head wind from the right (northeast); tail wind is worst. The maximal range in these cases differs by 10 m (r_1=74.60 m and r_2= 65.30 m). In all cases we obtain that the angle of flight direction (solid gray line) is about 10° bigger than the angle between discus and the ground (angle of attack, dashed line). For the sake of clarity, the pitch attitude δ_0 is drawn with a negative sign. It turns out that the range behaves quite sensitive to δ_0. In particular, changing δ_0 at γ=220° (northeast) from 0° to 30°, the range increases from 72.50 m to 74.60 m.

Figure 2 shows optimal release parameters (α_0, δ_0, β_0) for a women's discus with initial spin rate of p_0 = 50 rad/s and v_0= 25 m/s depending on wind direction. It turns out that optimal wind comes from the right (east, γ=270°); wind from the left is the worst. It is seen that the maximal range in these cases differs by 7 m (r_1=66.12 m and r_2= 58.82 m).

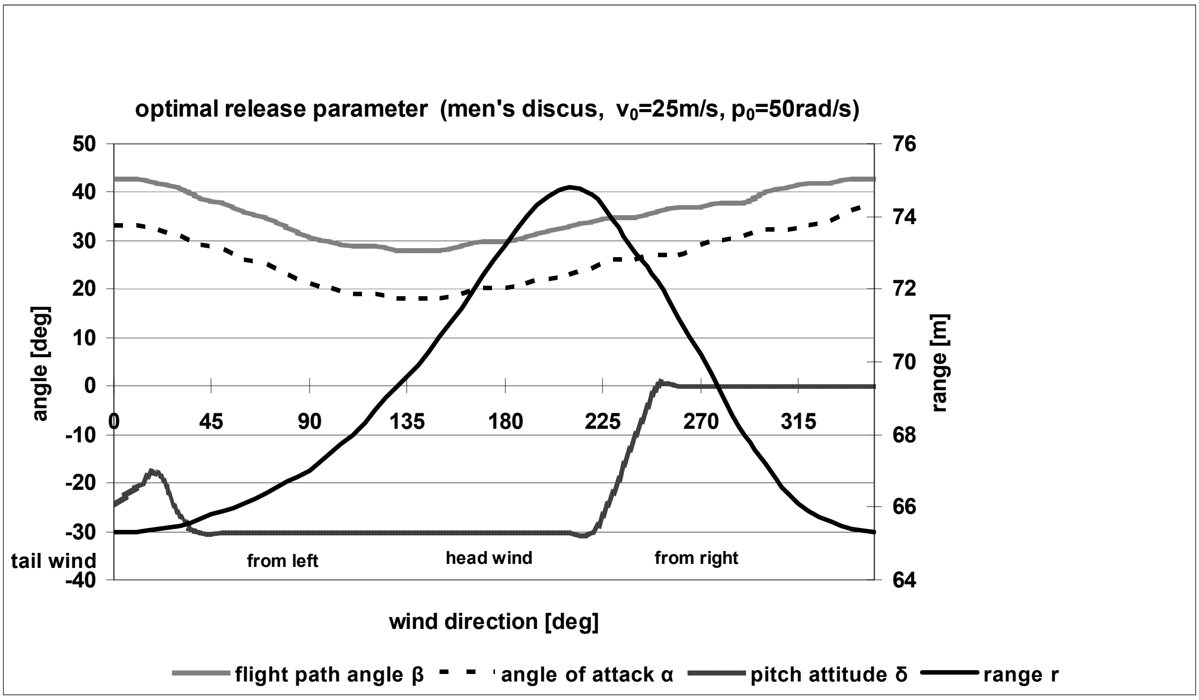


Figure 1: Optimal release parameter and maximal range depending on wind direction (men's discus)

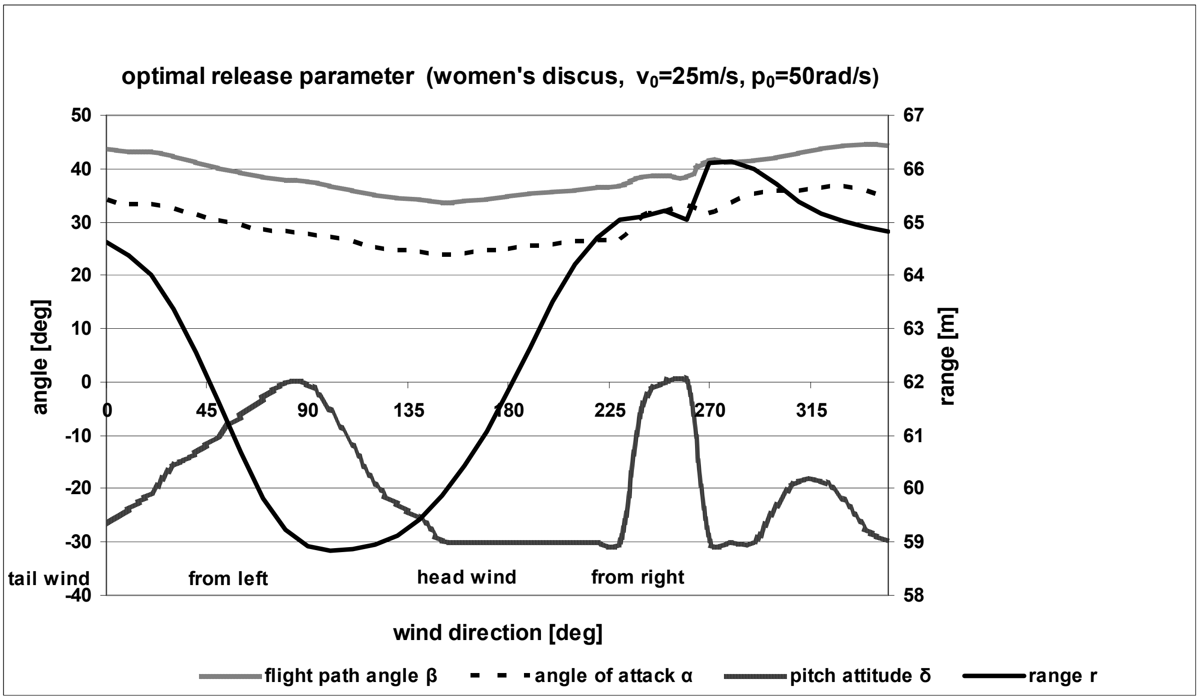
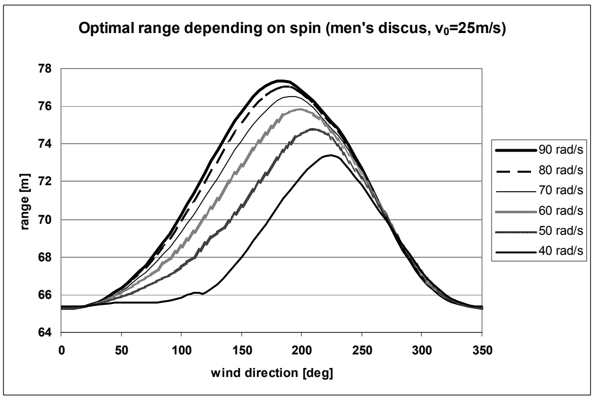
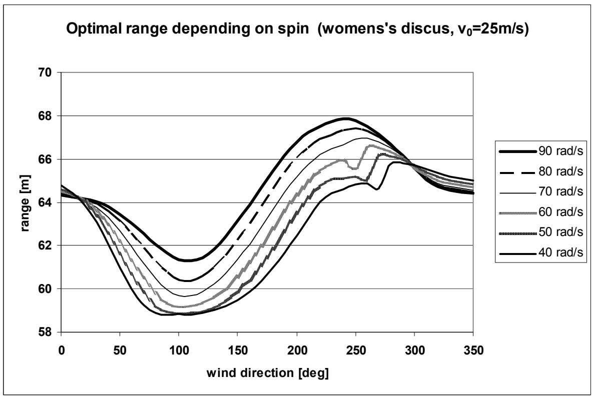


Figure 2: Optimal release parameter and maximal range depending on wind direction (women's discus)

In Figures 3 and 4 we show the optimal range for men's and women's discus depending on the initial spin rate and the wind direction. In general the best throw is obtained at the highest spin rate. The higher the spin the better is head wind. Head wind (γ=180°) is optimal for a men's discus with spin 90 rad/s, the corresponding range is r=77.36 m. Head wind from the right (Northeast, γ=230°) is optimal for women's discus with spin 90 rad/s, the corresponding range is r= 67.85 m. As the spin rate decreases, the optimal wind direction turns from north to east and the optimal range decreases, too. The worst wind for men is tail wind (γ=0°) while worst wind for women comes from the left (γ=110°).





Figures 3 and 4: Optimal range depending on spin and wind direction

DISCUSSION: We have computed optimal release parameters (flight path angle, angle of attack, pitch attitude) for women's and men's discus depending on wind direction (v_w = 5 m/s) and initial spin rate. The release velocity was 25 m/s. Due to different mass and moments of inertia, we obtained different pictures for optimal range and optimal wind direction. However, the optimal flight path angles and angles of attack behave quite similar for both men and women. In contrast to the work of Hubbard & Cheng (2007) we discussed not only head and tail wind but all other wind directions, too. Similarly to Hubbard & Cheng (2007) our evolutionary algorithm shows that optimal release conditions must be wobble-free.

It was shown for a men's discus that optimal wind direction changes from head wind (γ=180°) to wind from northeast (γ=230°) when the initial spin rate decreases from 90 rad/s to 40 rad/s. In case of a women's discus the optimal wind direction changes from γ=240° (eastnortheast) to γ=290° (eastsoutheast) when the initial spin rate decreases from 90 rad/s to 40 rad/s.

CONCLUSION: The optimal release parameters from Figures 1 and 2 can immediately be applied by coaches and athletes. Knowing the wind direction the athletes can adjust flight path angle, angle of attack and pitch attitude. Other tables with different release velocity and different wind velocities had been computed. To get a more realistic view and to reduce the amount of computer time it is necessary to get smaller parameter spaces.

We used evolutionary algorithms to optimize release parameters. It turns out that this method is much more efficient up to factor ten compared with standard search algorithm.

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