# A SIMULATION STUDY OF THE INTERNAL TWISTING TORQUE IN THE FOUETTÉ TURN


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The purpose of this study was to investigate the effects of the magnitude of the twisting torque for one revolution of a Fouetté turn. Simulations were performed using a simple model comprising the supporting leg and the remainder of the body. It is shown that when the dancer turns more than one revolution with a small twisting torque, the turn will be decelerated and will finally stop. A large twisting torque is required at the start of each turn in order to increase the angular momentum which will subsequently decrease during the turn due to friction.



**KEY WORDS:** turn, simulation, angular momentum.


**INTRODUCTION:** Skilled ballet dancers can continuously perform repeated Fouetté turns (Figure 1) for more than 30 revolutions, starting from one or two revolutions of the pirouetté which is started with both feet in contact with the floor. Friction during slipping reduces the initial angular momentum at the beginning of each turn. The dancers have to regain the angular momentum for the next revolution from the swing of the free leg and the arms enabled by a large frictional torque ($T_F$) exerted on the supporting foot (Laws, 1984, Imura et al., 2008). This is achieved using a torque T to produce the twisting motion of the free leg, upper body and arms relative to the supporting leg. The net external rotational impulse of $T_F$ should be zero or positive after one revolution in order to maintain or increase the angular momentum. The behaviour of $T_F$ depends on the magnitude of T relative to the limiting frictional torque which is the product of the friction coefficient, the normal ground reaction force and the radius of the foot contact. Thus, the magnitude of T regulates the continuity and speed of the turn under a certain friction coefficient between shoes and floor.

The purpose of this study was to investigate the effects of the magnitude of T on one revolution of Fouetté turn using a simple model comprising the supporting leg and the remainder of the body.

**Figure 1: Sequential view of one revolution of Fouetté turn. Each picture is shown every 10% time of one revolution. The left most picture shows the configuration of the body at the middle of full foot contact (pictures from Imura et. al, 2008).**

**METHODS: Model:** One typical dancer's body (mass 50 kg) was used for a model comprising two cylinders (Figure 2): the supporting leg (L) and the remainder of the body (B). The twisting torque T which rotates B relative to L, the radius (r) of the foot contact area, the moments of inertia ($I_B$ and $I_L$) of the bodies B and L and the normal ground reaction force (N) were specified using monotonic quintic functions based on experimental data (Imura et al., 2008) as shown in Figure 3. The averaged N through the rotation was one body weight. The coefficient of friction (μ) was estimated to be 0.2 from experimental data (Imura et al., 2008).

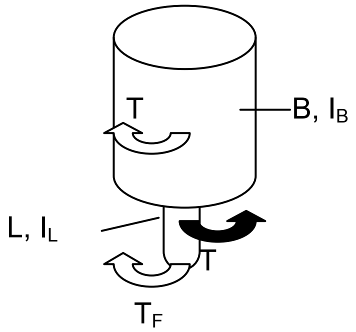


**Figure 2: The model comprised the supporting leg L and the remainder of the body B. Initial torque directions are shown.**

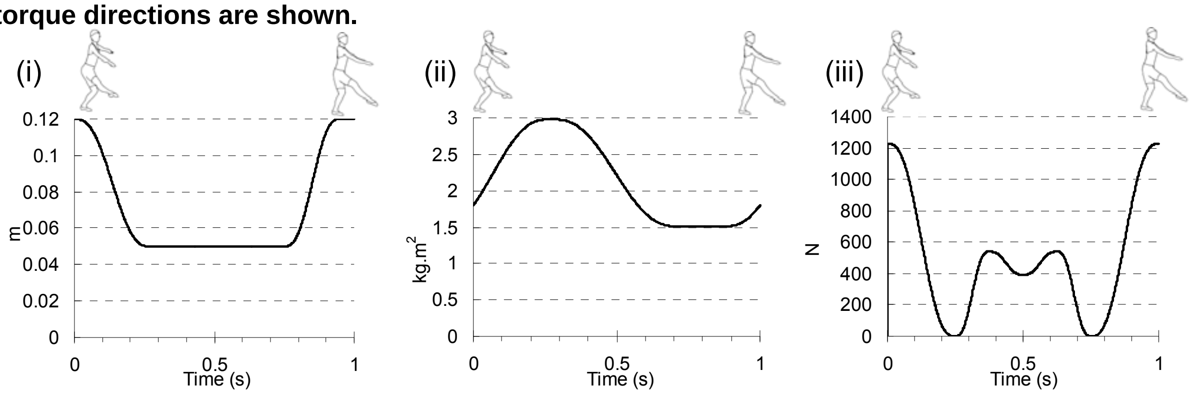


**Figure 3: Time histories of: (i) Radius of the foot contact area (ii) moment inertia of body B (iii) normal ground reaction force.**

**Simulations:** $T_F = T$ when $T \le \mu \cdot N \cdot r$ and $T_F = \mu N r$ once L slipped. The angular accelerations of the body B and leg L ($\ddot{\phi}_B$ and $\ddot{\phi}_L$, respectively) were determined from the equations: $\ddot{\phi}_B = T / I_B$ and $\ddot{\phi}_L = (T_F - T) / I_L$, from which the angles and angular velocities of the bodies ($\phi_B, \dot{\phi}_B$ and $\phi_L$, $\dot{\phi}_L$) were calculated using stepwise integration. One revolution was simulated from midstance when the supporting foot fully contacted the floor. The initial value of T was varied from 6 to 30 Nm which was the limiting frictional torque and the initial angular velocity of B was varied from 6.0 to 8.0 radians/s. A search was made for turns which satisfied the following conditions: (1) The leg L turns approximately 6.28 radians. (2) The peak angle between B and L is less than 1.57 radians. (3) The relative angle between B and L at the end of the turn is within 0.1 rad of the value at the beginning of the turn. An empirical process was used in which simulations were run individually by hand and input was varied.

**RESULTS:** Two simulations were found satisfying the required conditions:(a) one with large initial torque and (b) one with small initial torque. Figures 4 and 5 show representative data for (a) and (b). In (a) the initial angular velocity of B and the magnitude of T were 6.2 radians/s and 28 Nm. In (b) corresponding values were 6.7 radians/s and 15 Nm. In both cases, T had to become negative before limiting friction became zero in order to accelerate the leg L in the direction of the turn. Twisting torque T during slipping was not different between (a) and (b) in order to rotate one revolution. In (b) the rotational impulse of $T_F$ was negative at the end of the turn while in (a) the impulse was positive and close to zero(Figure 6). The whole body rotated one revolution in each turn but the angular velocity of body B decreased in (b) while it remained the same in (a).

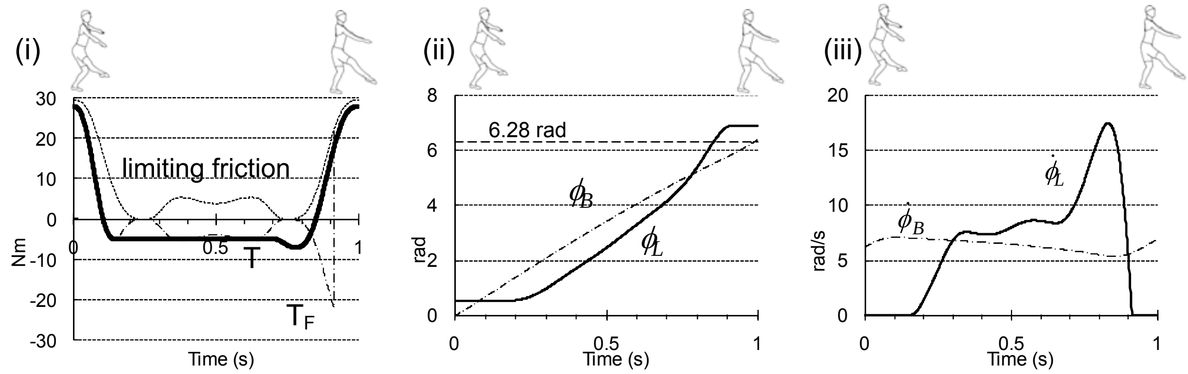


**Figure 4: Kinematics and kinetics of the larger torque turn (a): (i) T, $T_F$ and limiting $T_F$ (ii) $\phi_B$ and $\phi_L$ (iii) $\dot{\phi}_B$ and $\dot{\phi}_L$. $\dot{\phi}_B$ is the same at the start and end of the simulation.**

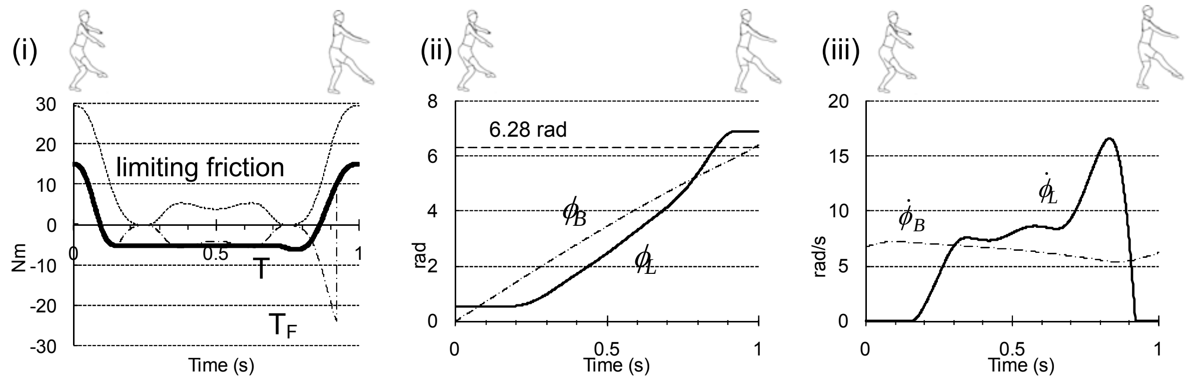


**Figure 5: Kinematics and kinetics of the small torque turn (b): (i) T, $T_F$ and limiting $T_F$ (ii) $\phi_B$ and $\phi_L$ (iii) $\dot{\phi}_B$ and $\dot{\phi}_L$. $\dot{\phi}_B$ is smaller at t=1 compared to t=0.**

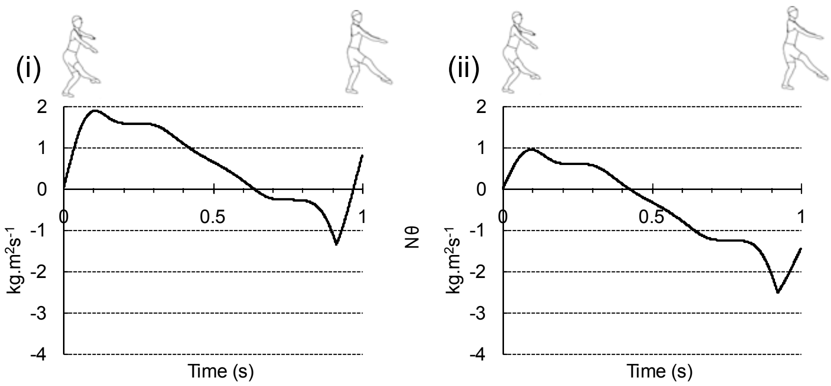


**Figure 6: The rotational impulse of the frictional torque: (i) larger torque turn (a), (ii) smaller torque turn (b).**

**DISCUSSION:** To start to slip in the direction of the turn, T has to be negative and have magnitude greater than the limiting friction ($\mu N r$). This explains the mechanism to generate the angular momentum for the next revolution: $T_F$ is exerted on the supporting foot and gives the rotational impulse in the direction of the turn when the dancer swings the free leg before the limiting friction becomes small. If T is still positive when it exceeds limiting friction the supporting leg of the dancer will slip in the opposite direction to the turn.

The difference between the large initial T and small initial T caused the difference in the angular momentum of the remainder of the body at the end of the simulation. T was changed at the same timings in (a) and (b), the rotational impulse of $T_F$ was positive in (a) and negative in (b) though B and L rotated one revolution in each case while the angular velocity of B decreased in (b). Before slipping T should be less than limiting friction, and $T_F$ is equal to

T. Thus, the rotational impulse in (b) did not increase as much as that in (a) before slipping. Because the decrease in rotational impulse of $T_F$ during slipping was the same in (a) and (b), the rotational impulse over one turn depends on the value of T at the start and end of the simulation. Thus, the Fouetté turn will continue but will be reduced in speed because the rotational impulse cannot be recovered with little torque as in (b). In such a case, the dancer has to swing the leg quicker and with more force from the muscles in the next turn to regain the angular momentum, recognizing that there is insufficient angular momentum to complete the turn.

To start the Fouetté turn, the dancer begins to turn around the supporting leg by exerting force on the floor with the free leg. This will produce more angular momentum than in the following turn because the moment arm between both feet is larger than the radius of the supporting foot (Laws, 1978). Thus the initial angular velocity of B reflects the angular momentum the dancer already has before the following Fouetté turn. For one revolution, the initial velocity was larger in (b) than in (a). This means the dancer adjusts the magnitude of T for one revolution according to the initial angular momentum. However, the magnitude of T for the swing is practically limited because the dancer has to face the front after one revolution. The dancer has to sense the angular momentum gained by the initial movement of the turn and exert the appropriate T for the following turns.

The time course of the angular velocities will be changed according to the input pattern of T. The restriction of the angle difference between B and L restricts the magnitude of T both initially and during slipping. More intricate input of T would provide another way of producing one revolution of the Fouetté turn, reflecting more complicated movement of the free leg. However, the magnitude of T for the swing regulates the speed and the continuity of the Fouetté turn.

More T will be required when the friction coefficient is larger for continuing the turn because the decrease in the angular momentum during slipping will be greater. However, the dancer will slip in the opposite direction if T exdeeds limiting friction. There is a limit as to how much friction can be accommodated since there is a limit to the amount of relative rotation that can occur between B and L.

**CONCLUSION:** To perform a number of consecutive Fouetté turns, it is important to maintain the angular momentum and swing the free leg with appropriate torque just sufficient to recover the angular momentum lost by friction. Hence the Fouetté turn starting with some pirouetté turns is difficult because the dancer has to regulate the magnitude of T for the swing just after the pirouetté and generate additional angular momentum. In a practical situation, the dancer pays attention to the friction of the floor, sensing the appropriate magnitude of the required twisting torque when performing turns. The dancer might be able to estimate the magnitude from the amount of rotation of the upper body, regulating the swing of the free leg and arms.

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