INVERSE KINEMATICS IN THE SUCCESS OF THE THROW IN BASKETBALL GAME

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The paper analyses the problematic of the human arm's movement in order to get a successful direct throw in basketball. For the human arm we analyzed the mechanic model of the double pendulum, formed by the arm and the forearm, with the hand being immobilized by the forearm. The classic equations of throwing the ball (dependent on the initial speed and throwing angle vector) have been correlated with the corresponding equations of the arms' movement. After imposing the mathematical conditions necessary for the ball to get into the basket, meaning certain inequality-type restrictions, we determined the kinematic dependents between the human arms' movement and the trajectory of the ball. The numeric analysis for several initial values of the throw allowed the assessment, through validation or non-validation, of the kinematic conditions for the success of the throw, and also several discussions regarding the influences of various kinematic parameters upon the throw.

KEY WORDS: geometrical conditions, mathematic model, trajectory, upper limb

INTRODUCTION: The ball trajectory during a throw depends upon the initial throwing conditions (the throwing angle, the initial speed vector, the geometrical coordinates of the throwing point) and on the type of movement (uniform, diversified etc.), representing a classic problem of the mechanics. Nevertheless, the success of the throw in basketball depends not only on the initial throwing conditions and on the type of movement, but also on the final coordinates of the ball and the incidence angle towards the basket. Schroder and Bauer (1996) treat, among others, the aspects related to finalizing the throw in basketball, putting into evidence the geometrical parameters for the success of the throw. Budescu and Iacob (2005) present, in the chapter called "Kinematics", the classic equations of throwing a body, with applications in sports, and with an emphasis on the kinematic dependents of the body trajectory. Budescu et al. (2005) talk about certain issues regarding the dependence between the arm's movement and the kinematic conditions for the success of the throw in basketball. We mention the fact the arm is considered here as a simple pendulum kinematic model.

In this paper, our aim was to analyze the mathematical dependence between the flexion-extension movement of the human arm, seen as a double pendulum, when throwing the basket ball and the success of the direct free throw towards the basket. The main objectives of the study were: emphasizing the geometrical parameters of the flexion-extension movement of the human arm, which have a great influence upon the success of the direct free throw in basketball; determining the kinematic restrictions which may ensure the success of a throw, as well as correlating these restrictions with the arm's movement parameters; mathematically emphasizing the relation between the movement of the arm (double pendulum type) and that of the ball during the direct free throw in basketball.

The purpose of the study was to offer the theoretic support for specialists in biomechanics for the individualized calculus of the optimal angular amplitudes for the flexion-extension of the arm and of the forearm during the direct free throw at the basket, which may ensure the success of the throw in basketball.

METHODS: The study has used the analytical method in order to study the throw in the basketball game. There is a schematic presentation of the basketball player and of the ball trajectory in Figure 1.

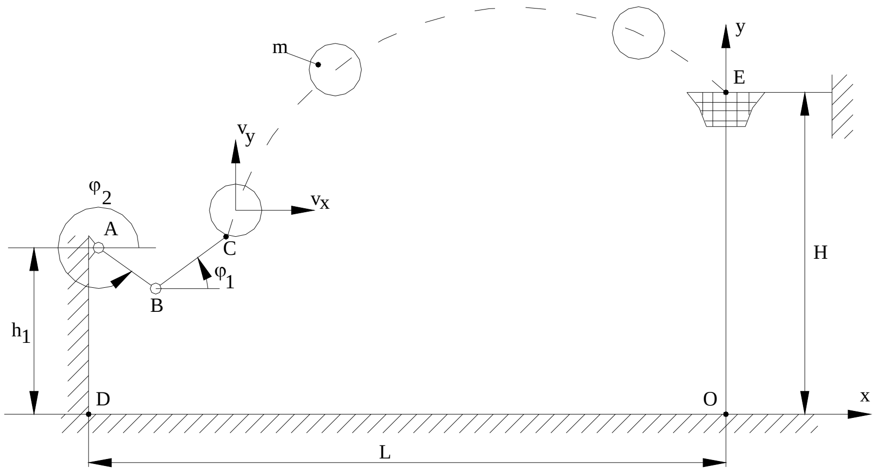

Fig. 1 Schematic of basketball player and ball trajectory

In Figure 1 letters represent as follows: h_1 is the height of the basketball player, ABC is the player's arm, modeled as a double pendulum, L is the distance from which the ball is thrown, m is the ball, v_y and v_x are the initial vertical and horizontal speeds of throwing the ball, H is the height of the basket.

The position and velocity equations for point C are the following:

$$x_C = -L + AB \cdot \cos(\varphi_2) + BC \cdot \cos(\varphi_1) \tag{1}$$

$$y_C = h_1 + AB \cdot \sin(\varphi_2) + BC \cdot \sin(\varphi_1) \tag{2}$$

$$\dot{x}_C = -\omega_2 \cdot AB \cdot \sin(\varphi_2) - \omega_1 \cdot BC \cdot \sin(\varphi_1) \tag{3}$$

$$\dot{y}_C = \omega_2 \cdot AB \cdot \cos(\varphi_2) + \omega_1 \cdot BC \cdot \cos(\varphi_1) \tag{4}$$

The equations (1) to (4) represent the initial conditions for the ball in its fly toward the basket.

Thus:

$$v_x = \dot{x}_C = -\omega_2 \cdot AB \cdot \sin(\varphi_2) - \omega_1 \cdot BC \cdot \sin(\varphi_1) \tag{5}$$

$$v_y = \dot{y}_C = \omega_2 \cdot AB \cdot \cos(\varphi_2) + \omega_1 \cdot BC \cdot \cos(\varphi_1) \tag{6}$$

The throwing angle is: $\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)$.

It is well known that the trajectory of the ball is a parabola. In the reference system xOy shown in Figure 1, the equation of the ball trajectory is:

$$y = A \cdot x^2 + B \cdot x + C \tag{7}$$

where:

$$A = -\frac{g}{2 \cdot v_x^2};\; B = \frac{g \cdot C_3}{v_x^2} + \frac{C_1}{v_x};\; C = C_2 - \frac{g \cdot C_3^2}{2 \cdot v_x^2} - \frac{C_1 \cdot C_3}{v_x} \tag{8}$$

In relations (8), C_1, C_2 and C_3 are the following:

$$C_1 = v_y = \omega_2 \cdot AB \cdot \cos(\varphi_2) + \omega_1 \cdot BC \cdot \cos(\varphi_1) \tag{9}$$

$$C_2 = y_C = h_1 + AB \cdot \sin(\varphi_2) + BC \cdot \sin(\varphi_1) \tag{10}$$

$$C_3 = x_C = -L + AB \cdot \cos(\varphi_2) + BC \cdot \cos(\varphi_1) \tag{11}$$

The top of the ball trajectory (the parabola) is, in our case, the point U of the coordinates x_u and y_u.

$$x_u = -\frac{B}{2 \cdot A} \text{ and } y_u = -\frac{B^2 - 4 \cdot A \cdot C}{4 \cdot A} \tag{12}$$

In Figure 2 there is shown the ball when it reaches the basket. The segment D_p represents the D_m diameter of the ball, projected along the D_c diameter of the basket:

$$D_p = \frac{D_m}{\sin \alpha_E} \tag{13}$$

where α_E is the falling angle of the ball through the basket, at point E (see Figure 1).

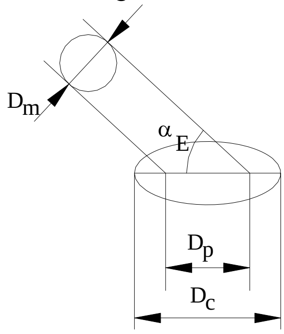

Fig. 2 Schematic showing ball entry to basket

Several conditions have to be met in order for the ball to pass through the basket. These conditions are as follows:

$$D_p < D_c \tag{14}$$

$$-\frac{D_c - D_p}{2} < x_E < \frac{D_c - D_p}{2} \tag{15}$$

$$x_u < x_E \text{ and } y_u > y_E \tag{16}$$

where: $$x_E = 0 \text{ and } y_E = H \tag{17}$$

From relations (14), (15) and (17) we can see that conditions (14) and (15) are equivalent.
From condition (16), $y_u > y_E = H$, and relations (8), (9), (10), (11) and (12) we may conclude that:

$$v_y^2 + 2 \cdot g \cdot y_c > 2 \cdot H \cdot g \;\Rightarrow\; v_y > \sqrt{2 \cdot g \cdot (H - y_c)} \tag{18}$$

From condition (16), $x_u < x_E$, it results that $x_u = -\frac{B}{2 \cdot A} < x_E = 0$. But from relation (8) it can be observed that $A < 0$, thus B has to be $B < 0$. From relation (8), (11) and Figure 1, we can conclude that:

$$B = \frac{g \cdot x_c + v_x \cdot v_y}{v_x^2} < 0 \text{ , hence: } g \cdot x_c + v_x \cdot v_y < 0 \;\Rightarrow\; v_x < -\frac{g \cdot x_c}{v_y} \tag{19}$$

By analyzing relations (18), (19) and Figure 1, it can be observed that x_c and y_c are the horizontal and vertical distances from which the ball is thrown toward the basket. In comparison to the dimensions of the basketball ground, the lengths of the player's arm and forearm may be neglected. Thus, the possibility of the ball to fall into the basket is more likely influenced by the initial speeds v_x and v_y, than by the initial position s x_c and y_c. Therefore, y_c could be approximated with the height h_1 of the player, and x_c could be approximated with the distance L (see Figure 1).
The falling angle α_E of the ball through the basket, at point E (see Figures 1 and 2), must then be $0 < \alpha_E < \frac{\pi}{2}$ and may be calculated as follows:

$$\tan(\alpha_E) = \left| 2 \cdot A \cdot x_E + B \right| \tag{20}$$

but $$x_E = 0 \text{ and } \tan(\alpha_E) = \left| B \right| \tag{21}$$

By taking into account relations (8), (9), (11), (13), (14), and (21), it yields:

$$\left| \frac{g \cdot x_c + v_x \cdot v_y}{v_x^2} \right| > \tan\left[\sin^{-1}\left(\frac{D_m}{D_c}\right) \right] \tag{22}$$

Thus, the velocities v_x and v_y have to meet the conditions (18), (19) and (22), in order for the ball to pass through the basket.
From relations (5) and (6), the angular velocities of the arm and forearm of the player are the following:

$$\omega_1 = \frac{v_y \cdot \sin\varphi_2 + v_x \cdot \cos\varphi_2}{BC \cdot \sin(\varphi_2 - \varphi_1)} \tag{23}$$

$$\omega_2 = -\frac{v_x}{AB \cdot \sin\varphi_2} - \frac{\omega_1 \cdot BC \cdot \sin\varphi_1}{AB \cdot \sin\varphi_2} \tag{24}$$

RESULTS: In Table 1 we present several cases of throws obtained for certain initial throwing values, with the following input data: H=3.05 (m), L=6.25 (m), h1=1.8 (m), D_c=0.45 (m), D_m=0.24 (m), g=9.81 (m/s^2) and the initial ball velocities given in Table 1.

Table 1. The assessment of the mathematical conditions for the throw

Crt. no.	v_y (ms^{-1})	Cond. (18)	v_x ($m.s^{-1}$)	Cond. (19)	α_E (°)	α (°)	Cond. (14) $D_p < D_c$
1	6.00	true	3	true	78.261	63.43	0.24<0.45 true
2	6.50	true	4	true	65.624	58.39	0.26<0.45 true
3	7.00	true	5	true	46.465	54.46	0.33<0.45 true
4	7.50	true	6	true	24.376	51.34	0.58<0.45 false
5	8.00	true	7	true	6.1877	48.81	2.22<0.45 false
6	8.50	true	8	false	5.9653	46.73	2.30<0.45 false

The initial throwing speeds of the ball proposed in Table 1 are realistic, having similar magnitude with the speeds experimentally determined in paper (Budescu et al., 2005).

DISCUSSION: From Table 1 we can see that condition (19) is not fulfilled for the last throw, and that condition (14) is not fulfilled for the three last throws. Also, as the throwing angle decreases, the condition (14) is not fulfilled. The theoretical calculus concluded that:

- if the ball is thrown at an angle smaller than 40° or bigger than 50°, the conditions (14)-(17) are no longer fulfilled and the ball does not pass through the basket;
- condition (15) is the most restrictive one, so that, during the basketball practices, we should teach the player how to throw towards a fixed point;
- an acceptable throwing distance, taking into account the results of the study, is equal or smaller than 6.25 [m];
- the chance for a successful throw increases if the player jumps before throwing with a speed as big as possible.

CONCLUSIONS: The paper may be useful to sportsmen, trainers and researchers in biomechanics in sports, to determine the optimum angles of arms and throwing speed of the ball. The angles may be personalized for the sportsman's anthropometrical dimensions, to ensure the success of throwing at the basket. After getting to know these angles, we may adjust the corresponding mobile orthoses for the arm, used during trainings, especially for practicing the throw at the basket.

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