OAR BLADE FORCE COEFFICIENTS AND A MATHEMATICAL MODEL OF ROWING

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The aim of this study was to validate the use of computational fluid dynamics (CFD) to determine oar blade force coefficients for use in a mathematical model of rowing mechanics to predict the performance of a boat. Experimental and CFD derived lift and drag force coefficients for a Macon oar blade were taken from previously published research. Each set of coefficients was used to drive a mathematical model of rowing, and predicted instantaneous and mean steady state boat velocity compared. Instantaneous boat velocity was similar throughout the stroke and mean boat velocity varied by only 1.33%. In conclusion, this investigation has demonstrated that lift and drag coefficients obtained by computational methods may be used successfully to predict boat behaviour in a mathematical model of rowing. The use of computational data closely matches model outputs derived from experimental data.

KEY WORDS: rowing, CFD, mathematical modelling, oar blade.

INTRODUCTION: In rowing, the boat is propelled through the water by musculoskeletal forces exerted by each rower on the boat. These are transferred to the water by the oars and oar blades. Due to these physical interactions, this system of rower-boat-oars-water lends itself to being modelled mathematically. Several complete mathematical models of rowing have been presented in the literature (Pope, 1973; Sanderson & Martindale, 1986; Millward, 1987; Brearley & de Mestre, 1996; Lazauskas, 1997; Brearley et al., 1998;Cabrera et al., 2006; Caplan & Gardner, 2007a). The oar blade motion is a key component of these models and how it is represented across the literature sees much variation.

Millward (1987), Brearley and de Mestre (1996) and Brearley et al. (1998) present models where the blade is assumed to remain fixed in the water during the drive phase of the stroke. Assuming that the blade remains fixed neglects the complex interactions between blade and water which produce lift and drag forces on the blade, affecting boat propulsion. Cabrera (2006) extended the work of Pope (1973), improving the model of the oar blade–water interaction, by defining the motion of the blade as moving backward through the water, with the water flow being perpendicular to the blade chord line. The force coefficients were taken from experimental data for a flat plate obtained from Hoerner (1965).

It has been shown, however, that the oar blade does not simply move backward during the stroke, generating only a drag force (Nolte, 1984). Instead the oar blade moves forward in the water during the first third of the drive phase, followed by a period where it slips backwards, and finally it moves forwards again as it approaches the boat (Figure 1). The angle of attack, α, between the line of the blade and its direction of motion varies transiently throughout the stroke and the path which it moves through. Additionally, the blade is known to act as a hydrofoil (Nolte, 1984; Dal Monte & Komor, 1989) and lift forces on the blade are produced as well as drag forces.

The importance of the fluid dynamic interaction between oar blade and water was considered by Caplan & Gardner (2007a) in their model. They used experimentally obtained force coefficients of lift and drag for a number of oar blade designs (Caplan & Gardner, 2007b) as variables in their model. This allowed for a more accurate simulation of the fluid dynamic behaviour of oar blades and therefore of propulsion to the boat, thus developing a more complete model of rowing.

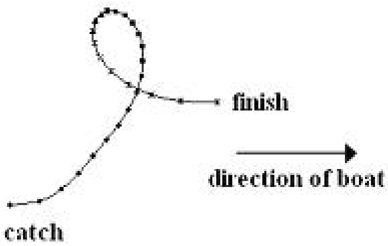

Figure 1: Path of oar blade during the drive phase of a stroke.

The fluid dynamic properties of rowing oar blades have recently been investigated using numerical methods, in particular through the use of computational fluid dynamics (CFD). Some preliminarily investigations have been carried out on flat plates by Leroyer et al. (2008) and Kinoshita et al. (2007). These studies have been able to provide additional details of the flow properties occurring around rowing oar blades, and Coppel et al. (2008) presented a CFD analysis of the Macon oar blade.

The aim of this investigation was to use Caplan & Gardner's (2007a) mathematical model of rowing with both the experimentally derived values of lift and drag coefficients from Caplan & Gardner (2007b) and the CFD predicted results from Coppel et al. (2008). A series of simulations were carried out using the experimental and predicted values, and comparison was made between the two simulations.

METHODS: The mathematical model of rowing used in this investigation (Caplan & Gardner, 2007a) defines the rowing system using Newton's second law ($\sum F = ma$) where the rowing propulsive forces must be applied to the boat to overcome the various sources of resistance. The water resistance is determined experimentally from Wellicome (1967) for an eight, and the air drag is defined from experiments on a seated man by Hoerner (1965) multiplied by the number of rowers. The crew were modelled as a single mass moving back and forth with half of a simple harmonic motion within the boat. The equation of motion for the rowing model can be written as:

$$P - D = m\frac{dv_{shell}}{dt} + M\left(\frac{dv_{crew}}{dt} + \frac{dv_{shell}}{dt}\right) \quad (1)$$

where, m is the mass of the shell, oars and coxswain, M is the combined mass of the crew, v_{shell} is the absolute velocity of the shell, v_{crew} is the velocity of the crew relative to the boat, P is the propulsive force provided by the rowers and D is the drag force, which is proportional to the velocity of the boat. The rowing model is driven by the change in the angular velocity of the oar shaft about the oarlock caused by the rower pulling the oar handle, and also by the motion of the rower.

The model was built in Simulink (Matlab, Mathworks, USA) and solved using the in-built Runge-Kutta variable rate solver with a maximum time-step of 0.005 s. The model produced a continuous output of all variables with time. Boat velocity was the integral of the modelled boat acceleration. This allowed the performance of the rowing boat to be monitored in terms of attained boat velocity as a function of a given stroke.

A heavyweight men's eight with a combined mass of 740 kg and with a stroke rate of 30 strokes per minute was used. Their drive time was assumed to be 0.92 s and recovery time 1.08 s, giving a total stroke time of 2 s. The Macon oar blade, which is typically used for beginners, and has a projected surface area 0.108 m^2 was used in the simulations and is shown in Figure 2.

In order to determine the lift and drag forces generated through the stroke the lift and drag coefficients of the Macon oar blade as a function of angle of attack were predefined. Here both the values obtained from the experiments of Caplan & Gardner (2007b) and those obtained from the CFD investigation of Coppel et al. (2008) were used, and the outputs compared. Although in rowing the blade moves continuously through a path from catch to finish (as shown in Figure 1), in the experimental and computational work it was only

possible to sample a discrete number of angles of attack. The investigated angles were 0^0, 20^0, 45^0, 90^0, 115^0, 135^0, 160^0, 180^0 and this represented the complete range of angles of attack that the blade goes through during the drive phase of the stroke. A cubic-spline interpolation was used to interpolate the data between each angle of attack. This gave a continuous reading of the values of lift and drag coefficient during the model simulation.
The simulation was started from the catch position and ran for a duration of 20 s to provide data for 10 strokes. A steady state velocity had been achieved by 5 strokes. The last stroke was then isolated and used as comparison between the experimental and CFD results.

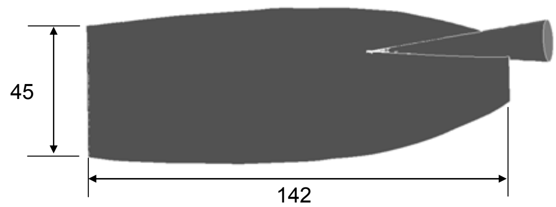

Figure 2: Geometry of Macon blade (dimensions defined in mm).

RESULTS AND DISCUSSION: Table 1 presents the absolute values of lift and drag coefficient for the Macon oar blade at the 9 angles of attack sampled. The values for the CFD predicted and the measured experimental values are given. These coefficients were used as predefined variables in the rowing model (Caplan & Gardner, 2007a).

Table 1: Comparison of measured (Caplan & Gardner, 2007b) and predicted (CFD) lift (CL) and drag (CD) force coefficient values.

Angle (deg)	Experimental		CFD model	
	CL	CD	CL	CD
0	0.02	-0.03	0.00	0.01
20	0.70	0.22	0.64	0.40
45	1.20	1.16	0.95	1.31
70	0.66	1.72	0.53	1.80
90	0.04	1.85	0.04	1.97
115	-0.77	1.74	-0.74	1.76
135	-1.19	1.25	-1.17	1.43
160	-0.80	0.03	-0.38	0.49
180	-0.14	0.04	0.01	0.01

The experimental and CFD model values presented show reasonable correlation with one another, except for some significant variations, for example at 160^o. However, it is their influence on boat performance as determined by the output from the rowing model that was considered in this investigation. The boat velocity variation over the same stroke using the experimental values and then the CFD predicted values is shown in Figure 3. There is very little difference in the two boat velocity predictions, with the CFD and experimental coefficients resulting in good agreement in the corresponding boat velocity profiles. In the early part of the stroke, during the initial part of the drive phase, $0<t<0.7$ the curves of boat velocity coalesce, although deviate away from each other as the minimum boat velocity is approached. Towards the end of the drive phase and during recovery $0.8<t<2.0$ there is a very slight difference between the boat velocity curves.
It was also possible to predict the mean boat velocity over this stroke. The mean boat velocity using the experimental values of force coefficients was 5.25 ms^{-1} and using the CFD predicted values it was 5.32 ms^{-1}. This produced a percentage error of 1.33% between the measured and predicted values. The error could be reduced further by improvements in the CFD predictions of the lift and drag coefficients. This could be achieved by addressing some of the assumptions made in the CFD model, such as employing a free surface condition, rather than an open symmetry boundary, at the top of the flume.

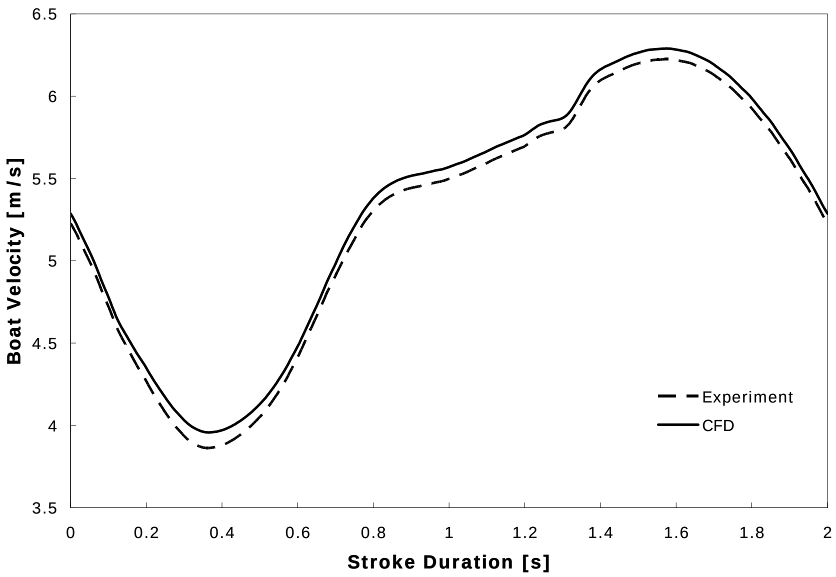

Figure 3: Variation in boat velocity during a rowing stroke. Output when experimental values and CFD predicted values of force coefficient are used.

CONCLUSION: It has been shown that a CFD simulation of the flow around the oar blades can be used in conjunction with a mathematical model of rowing to provide predictions of the motion of a boat in rowing. The results show very good agreement (within 1.33%) with previously published experimental data of boat performance (Caplan & Gardner, 2007b).

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