TOWARDS UNDERSTANDING HUMAN BALANCE – SIMULATING STICK BALANCING

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The purpose of this study was to develop a simulation model for stick balancing. Experimental result served as a guide for the developing progress. The progress started with a deterministic approach. We solved the Euler-Langrange equation and received the equation of motion. The controlling variable within this equation is the acceleration of the lower end of the stick. This parameter depends on the balancing strategy and ability of the human subject. We chose the van der Pol equation as an ansatz for describing it. A second attempt included the incorporation of a time delayed parameter. The third form included additional stochastic noise. We found close similarity between the measured and the calculated parameters tilt angle at reversal points, frequency expectation value, and others.

Keywords: balancing, movement pattern, simulation, stability, chaos theory, van der Pol equation

INTRODUCTION: Balance is an ability to maintain the center of gravity of a body within the base of support with minimal postural sway (Shumway-Cook, Anson et al. 1988). It is an essential feature for achievement in most human movement tasks. For the analysis of stick balancing we define two tilt angles. Those tilt angles and the respective angular velocity revealed to be connected with the acceleration of the lower end of the stick. These interrelations together with a characteristic sway frequency seemed to be the main ingredients for successfully balancing a stick. In this study we developed a simulation model based on this context. We draw a comparison between the result from the experiment and the simulation to check the degree of concordance.

METHODS: The movement situation is depicted in Figure 1. From the experiment we know there exist a moderate connection between the two tilt angles. However, for this study we decided to restrict the analysis to one plane of motion, the x-y-plane with x as the horizontal axis and y the vertical axis. The associated angle is β which is defined in equation (1.1).

$$\beta = \arctan\left(\frac{x_{up} - x}{y_{up} - y}\right) \quad \text{with} \quad x = x_{low} \quad \text{and} \quad y = y_{low} \tag{1.1}$$

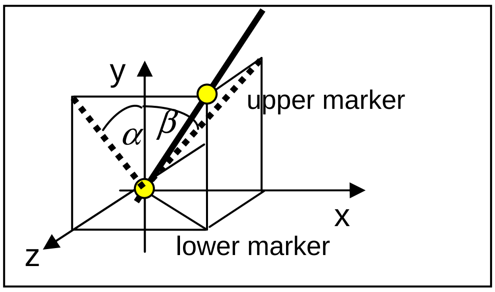

Figure 1: Stick with marker arrangement

We used the Hamilton principle respectively the Euler-Lagrange equation to derive the equation of motion. The stick was treated as a one-dimensional object of length 1 m with its mass evenly distributed along a strait line with the center of gravity symmetrically between the upper and lower ends. The Lagrange function is given in equation (1.2) with T the kinetic energy containing $T_v = \frac{1}{2} m \cdot v_{CoG}^2$ being the translation and $T_\omega = \frac{1}{2} \dot{\vec{\beta}}^T I \dot{\vec{\beta}}$ the rotation part. V stands for the potential energy. Here l is the stick length, m the stick mass, I the inertia tensor, $\vec{v}_{CoG}$ the velocity of the stick's center of

$$\begin{aligned} L &= T - V = T_v + T_\omega - V \\ &= \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2 + \dot{x} l \cos(\beta) \dot{\beta} - \dot{y} l \sin(\beta) \dot{\beta} + \frac{l^2}{4} \dot{\beta}^2 \right) + \frac{1}{24} m l^2 \dot{\beta}^2 - mg \frac{l}{2} \cos(\beta) \end{aligned} \tag{1.2}$$

gravity, $g = 9.81 \,{}^{m}\!/\!{}_{s^2}$ the gravitational acceleration, and β the tilt angle as defined above. The Euler-Lagrange equation (1.3)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\beta}}\right) - \frac{\partial L}{\partial \beta} = 0 \tag{1.3}$$

leads to the sought equation of motion (1.4). Here the mass of the stick canceled out. The remaining attribute of the stick is the length l. For practical reasons in an experiment with human subjects mass does play a role. If there is too much mass the subject cannot accelerate the finger holding the stick fast enough, if the stick is too light a subject will have problems to "feel" the movement. However, within a transition range mass does not greatly influence the outcome.

$$\ddot{\beta} = \frac{3}{2l}\left((g + \ddot{y})\sin(\beta) - \ddot{x}\cos(\beta)\right) \tag{1.4}$$

As a simplification, from here onwards $\ddot{y}$ is suppressed and we are left with the angular acceleration in the form of equation (1.5).

$$\ddot{\beta} = \frac{3}{2l}\left(g\sin(\beta) - \ddot{x}\cos(\beta)\right) \tag{1.5}$$

We are simulating balancing with the vertical coordinate of finger being constant. The angular acceleration $\ddot{\beta}$ is depending on g, β, and eminently on $\ddot{x}$. g is constant, β we are going to calculate, but the most important parameter is the acceleration $\ddot{x}$. This is the acceleration of the finger and as such exclusively depending on a subject's performance. We are developing an iterative solution (1.6) by writing down the first three terms of a Taylor series.

$$\beta(t+\delta t) = \beta(t) + \dot{\beta}(t)\delta t + \frac{1}{2}\ddot{\beta}(t)\delta t^2 = \beta(t) + \left(\int_0^t \ddot{\beta}(t')dt' + \dot{\beta}_0\right)\cdot\delta t + \frac{1}{2}\ddot{\beta}(t)\delta t^2 \tag{1.6}$$

Here $\dot{\beta}_0$ is the angular velocity at $t = 0$ and δt the step length of the iteration. $\ddot{x}$ necessarily must allow for a stable result. It must also take into account the empirical relationship between β, $\dot{\beta}$, and $\ddot{x}$. Such an ansatz we found in the van der Pol equation (Goldobin, Rosenblum et al. 2008), which we write here in the form of equation (1.7).

$$\ddot{x} = \mu_1 \cdot (\beta_b - \beta)\cdot\dot{\beta} - \mu_2 \cdot \beta \tag{1.7}$$

Here μ_1, μ_2 and β_b are constants. Putting (1.7) into (1.5) leads to a deterministic equation. In a next attempt (1.8) time delay is included through the substitution $\dot{\beta}(t) \to \dot{\beta}(t-\tau)$.

$$\ddot{x} = \mu \cdot (\beta_b - \beta)\cdot\dot{\beta}(t-\tau) - \nu \cdot \beta \tag{1.8}$$

The solution number three (1.9) introduces stochastic components.

$$\ddot{x} = \xi(\mu_1,\sigma_1)\cdot(\beta_b - \beta)\cdot\dot{\beta}(t-\tau) - \xi(\mu_2,\sigma_2)\cdot\beta \tag{1.9}$$

$\xi(\mu_1,\sigma_1)$ respectively $\xi(\mu_2,\sigma_2)$ represent normal distributed stochastic noise with the first parameter being the mean and the second the square root of the variance. One of our test parameters is the Frequency expectation value. It is defined as

$$\langle\nu\rangle = \int_0^{\nu_s/2} \nu\cdot\left|F(\nu)\right|\cdot d\nu \Bigg/ \int_0^{\nu_s/2}\left|F(\nu)\right|\cdot d\nu \tag{1.10}$$

Here ν is the frequency, $F(\nu)$ the Fourier transform of β, and ν_s the sampling frequency.

RESULTS: All iterations were done with $\delta t = 0.005$ seconds. For the deterministic ansatz (1.5) the iteration returns, as expected, a stable solution Figure 2. There is a transition time, when the movement amplitude changes from the initial value of 1° at an angular velocity of zero as well as an initial acceleration of zero towards a stable situation. Afterwards, the solution is stable at a period two with a frequency of 0.61 Hz and absolute amplitude of 1.99 °.

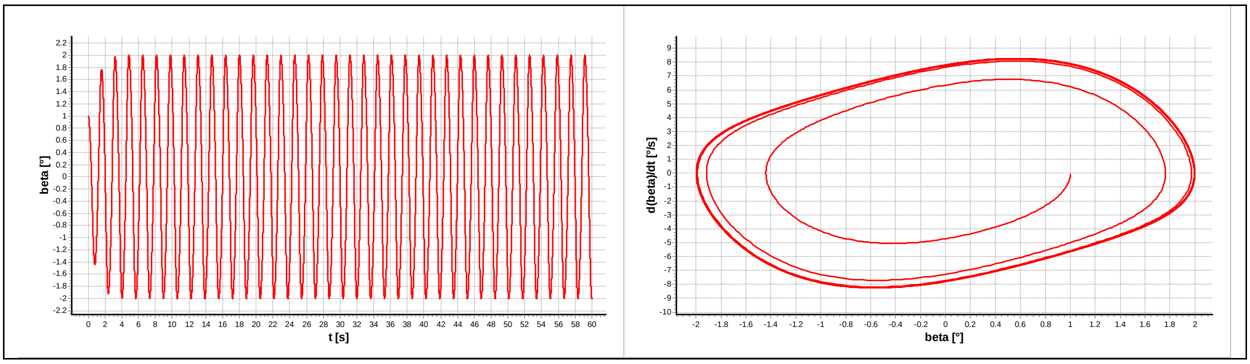

Figure 2: Space-time (left) and phase-space (right) diagrams of a deterministic configuration with $\mu_1 = 1$, $\mu_2 = 10$, and $\beta_b = 1$

We delayed the angular velocity term of (1.7) to arrive at (1.8). $\dot{\beta}(t-\tau)$ is delayed with $\tau = 0.15s$. Again, after a transition time, we got a stable two period (Figure 3) with a slightly higher amplitude of 2.09 ° at a frequency of 0.84 Hz.

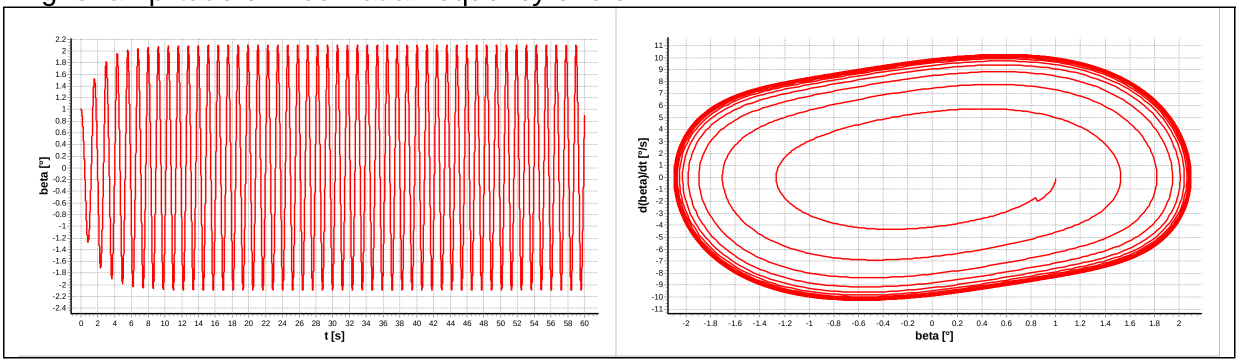

Figure 3: Space-time (left) and phase-space (right) diagrams of a deterministic configuration with time delay $\tau = 0.15$ sec

In the next step we altered (1.8) by adding normal distributed stochastic noise and arrived at equation (1.9). $\xi(\mu,\sigma)$ represents the normal distributed noise. The first parameter in the bracket stands for the mean (as above $\mu_1 = 1$ and $\mu_2 = 10$), the second for the standard deviation. Figure 4 shows a simulation with $\sigma_1 = 1$ and $\sigma_2 = 10$.

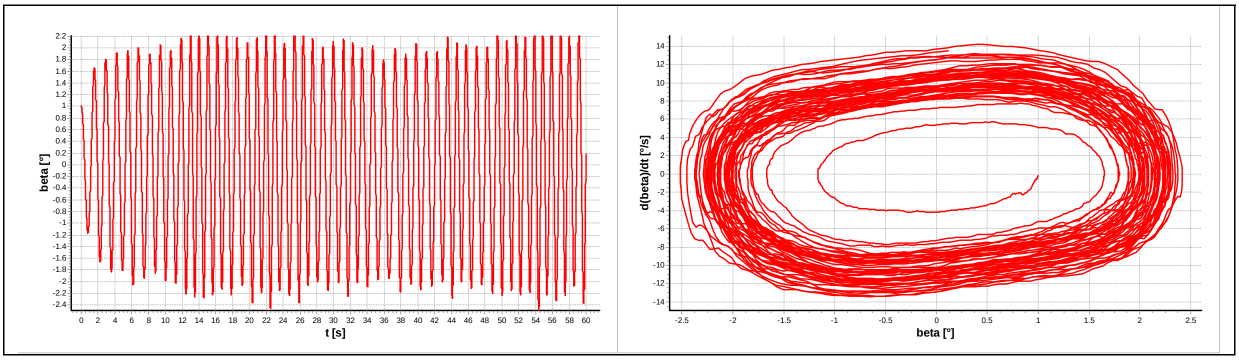

Figure 4: Space-time (left) and phase-space (right) diagrams of a deterministic configuration with time delay $\tau = 0.15$ sec and normal distributed noise

We still receive a fairly stable result. The maximal and minimal value of the amplitude within a cycle is not constant anymore, but still with just moderate variations. The mean frequency again slightly increases to about 0.88 Hz with a variation in the order of ± 0.1 Hz. The variables for a comparison between measurement and simulation are given in Table **1**.

Table 1: Simulation-measurement comparison of four different parameters

Parameter / Simulation	Deterministic	Delayed	Stochastic	Measurement
$\lvert\bar{\beta}\rvert$ [°] for the reversal points	1.99	2.09	2.16±0.13	2.38±1.41
Correlation coefficient β □ $\ddot{x}$	0.94	0.95	0.75	0.53±0.09
Time shift to r being maximal [s]	0.41	0.30	0.28	0.13±0.03
Frequency expectation value [Hz]	9.78	6.06	3.81	7.82±3.32

DISCUSSION: The mean tilt angle $\lvert\bar{\beta}\rvert$ for the reversal points is in the right range around 2 °. However, the variation in the simulation results is much smaller than those of the measurements. This hints toward an additional source of uncertainty that is not represented by the equations used in this simulation. The other parameters derived in the simulation, correlation β □ $\ddot{x}$, time shift, and frequency expectation value are close to those of the measurement. Still, deviations do occur. We varied the time delay to react towards an increased tilt angle β respectively tilt angle's angular velocity $\dot{\beta}$. The solution is fairly stable against time delay with regard to the angular velocity $\dot{\beta}$. Even a time delay of $\tau = 0.2\ \mathrm{sec}$ still results in a stable solution. On the other hand, a small time delay of $\tau = 0.025\ \mathrm{sec}$ with regard to the tilt angle β results in the stick falling.

CONCLUSION: The presented work is a first step towards a proper simulation. The equations allow quantifying the effects within the arrangement. We are able to figure out the influence of the deterministic part compared with the stochastic components. All these findings give confidence for an enhanced model and possibility to apply analogous simulations to other human balancing tasks such as walking.

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All numerical analyses within this study and also the generation of diagrams were done using the software StatFree (Vieten 2006). It is freely available on the internet at www.uni-konstanz.de/FuF/SportWiss/vieten/Software/.