MATHEMATICAL MODELS IN SPORT BIOMECHANICS

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The relationship between sports and culture goes back to ancient times. The organization of the Olympia Games in ancient Greece was a cultural and ethical event that only a people of great intellectual refinement could have conceived.

Great artists of those times dedicated their work to sports.

They thus praised and transmitted to posterity the harmonious forms of the athletic body and the elegance of the athletic gesture, thereby highlighting the function of sports for physical improvement (Myron's disc thrower is an example of unsurpassed elegance (fig. 1)}. They highlighted also agonistic aspect by celebrating the winner; Pindar's odes are an outstanding example.

The maxim "mens sana in corpore sano" was philosophy's acknowledgement of the importance of physical exercise, i.e. of sports.

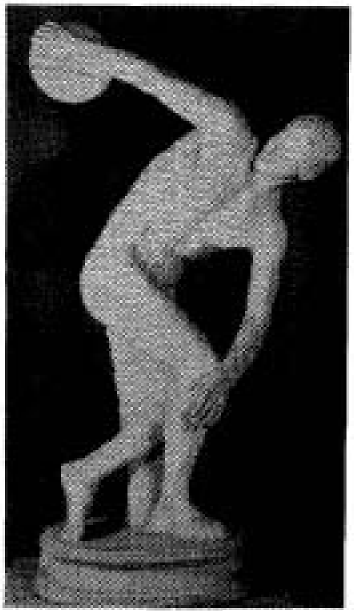


Fig. 1

Many centuries later, sports, as understood in our times, originated mainly in England and eventually became an important social and economic phenomenon.

The birth of modern sports can be set at the time of the first modern Olympic Games, held at Athens in 1896. From that time, the number of those involved in sports, in different ways, has been growing at a faster pace, so that sports has turned into a major social fact.

Athletic performance went from record to record.
For a long time progress was made on entirely empirical bases, by individual attempts and on the experience built up through the work of the trainers.

Leading athletes have been, are and always will be, persons of exceptional psycho-physical capabilities and probably also able instinctively to find the best way to move in our physical world to attain a certain result. In general, they did not know the physical laws of the environment in which they performed.

However, the idea of a correlation between the mechanical laws and physical exercise has always existed in a more or less latent way. This correlation is evident in certain sport disciplines originating from, and traditionally pursued in the Far East. As an example I mention judo, in which the athlete attempts to put to practical use certain principles of dynamics, the theory of which he almost certainly ignores, to turn his opponent's force to his own advantage.

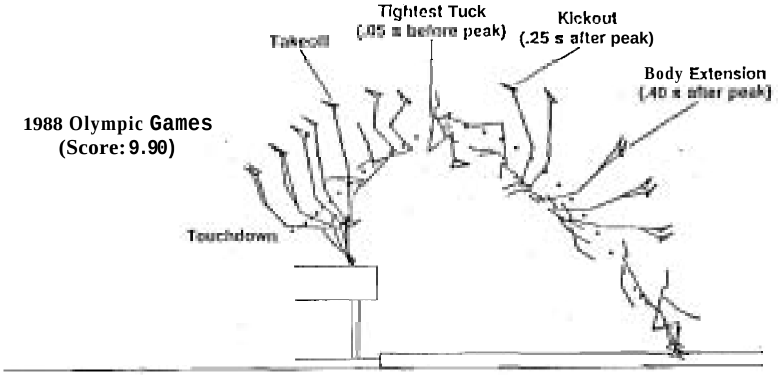
In a period of time difficult to pinpoint, roughly coinciding with the years of WWII or immediately thereafter, the idea of scientifically applying the principles of physics - chiefly of mechanics - to the study of sports took form.
It was thus possible, for one, to raise the high-jump record. All in all, sports has been relying on science in different ways to attain its aims.

Concurrently, growing interest was being shown by science for the study of the human body's motion: biomechanical research was being more and more pursued for various purposes but evidently found significant scope in its application to sports.

The introducting and widespread use of computers gave a strong impulse to research.

The computer can first be used in coordination with photographic shots of the moving athlete to obtain a schematic stick-type reproduction of the performance on the screen. This permits highlighting certain basic, especially meaningful elements.

I would like to mention as an example a very recent study of Yoshiaki Takei of the Department of Physical Education, Northern Illinois University, consisting in an analysis of photographs of horse-vaulting by 51 participants to the 1988 Seoul Olympics.
For each athlete, a figure like fig. 2 was drawn to represent the vaulting schematically as mentioned before.
It can be seen that the model is plane and the figure is of the stick type.
The purpose of this study is to correlate the success of the performance as evaluated by the judges, with certain medical parameters, specifically, the trajectory of the center of gravity. This trajectory is displayed on the monitor, and the athlete can evaluate for example the correlation of a more or less long or high trajectory with the final score.



Selected positions and the trajectory of the center of gravity of the two highest-scored handspring and salto forward tucked vaults performed at the 1988 Olympic Games (Y. Takei IJSB Vol.8, n. 2)

Fig. 2

The computer also made it possible to work out mathematical models of the human body by representing it as a manikin consisting of a number of articulated segments, i.e., a kinematic chain, which schematically represents the human body.

Certain anthropometric data are needed to construct the model: geometrical dimensions (lengths, widths, diameters, etc.) and inertial dimensions (masses, etc). More precisely, the data that are absolutely necessary are the dimensions of the body segments (chest, limbs, etc.), the position of the center of gravity and the mass of each segment, mass distribution (i.e., the moment of inertia) within each segment.

There are various kinds of mathematical models, chiefly as concerns mass distribution in connection to type of application.
A widely used model is Hanavan's: he schematically represented each segment with simple geometrical patterns of homogeneous material (fig. 3).

A first use of mathematical models is the following.
A series of photographs of the athlete performing an exercise is first taken as seen before in the case of horse vaulting.
The athlete is represented in the computer by a mathematical model of the type described above. The computer thus has the athlete's anthropometric data available.
From the photographs, the distance covered, acceleration and speed of each segment are calculated and the inertia forces acting on the athlete's body deducted.

Then, we write the dynamic equilibrium equations to calculate the interactions (forces and moments) between adjacent segments at the articulations. This corresponds to solving what is known as the "inverse" dynamic problem.

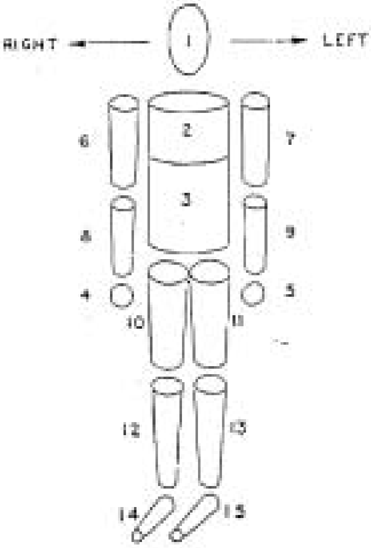


Fig. 3

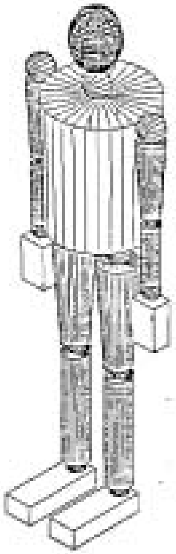


Fig. 4

The forces and moments acting on the articulations are exerted by the athlete's muscles, bones and ligaments.
Therefore, it should be possible to evaluate the action of each of them in this way. The problem is complicated by the fact that the number of unknowns exceeds the number of equations and its solution calls for some additional assumptions.
Several attempts have been made to this end, but the question is still wide open.

It should be stated planar models and plane motion are mostly used. Consequently computer displays stick figures as seen before.
Another program solves the direct dynamic problem, by obtaining the law of motion from known forces. Programs of this kind are seldom used and generally assume motion as plane.
I shall now report on a three dimensional model, developed in our Department.
I should first say that the mathematical model on which we report was obtained by entering the subprogram anthropos.
The input of anthropos are the anthropometric data of the subject that we know (like height, weight etc.). The output are all the correspondent data of the body (length and mass of segments, position of all centers of gravity, etc.), calculated on statistical criteria.

The joints and segments are shown in fig. 4; it can be seen that all joints have three degrees of freedom with the exception of the elbow and knee, which have one degree of freedom only.

To make the meaning of the program and of its results clear, we shall first show a very simple application with these examples.

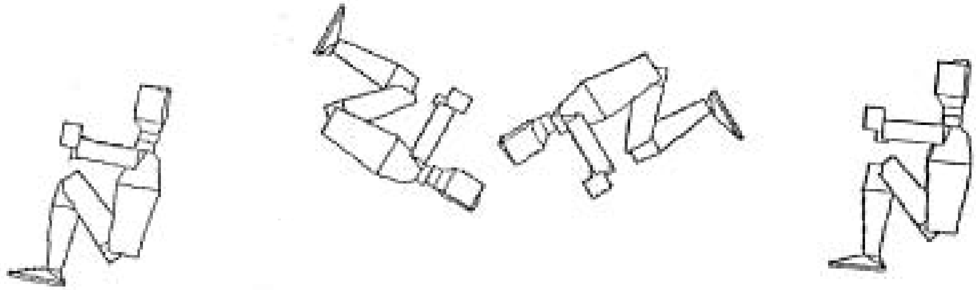


Fig. 5

The fig. 5 shows the evolutions of an athlete who takes a free movement in space in the phoetal position. His motion with respect to the center of gravity is uniform and continues unchanged with time through the entire movement, according to the law of inertia. The final configuration of the body is the same of starting.

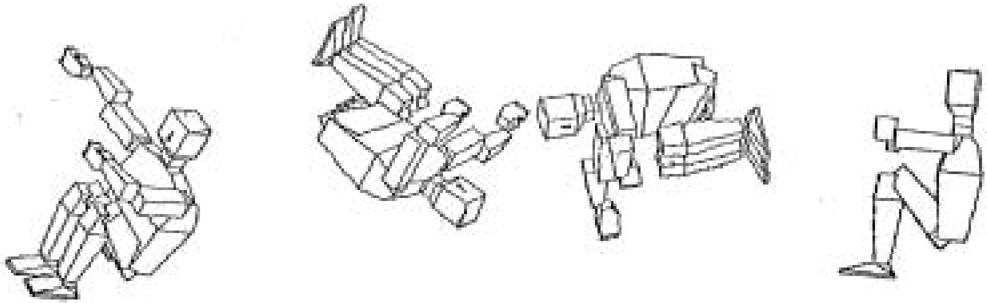


Fig. 6

The fig. 6 shows the same athlete: initially, he moves according to the same pattern and in the same posture as before; at a certain momento however, as specified in the program, he extends his right arm. .It can be clearly seen that the body's motion is affected by the protruding arm.
In final position the body is turned from the initial position.
In a similar way, any movement of any segment can be simulated and its effects displayed on the screen. Evidently, several segments can move simultaneously.

The fig. 7 simulates a dive and shows how the diver can perform evoiution during the fall by suitably moving his arms and legs.

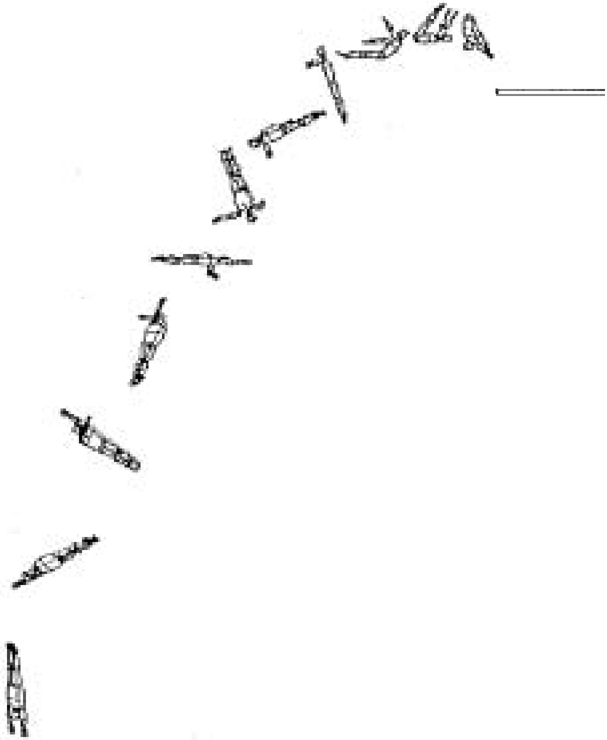


Fig. 7

As you can see, in these figures the body is not shown as a stick figure abefore, but its various segments are tridimensional; this has no mathematical or inertial significance and is only meant to facilitate the interpretation of the figures and motion.

Programs of this kind might be quite useful for athletes; if this approach is pursued, one might expect that in the near future athletes will be able to study their exercises on the basis of their anthropometric parameters sitting in front of a screen.