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## INTRODUCTION AND GENERALITIES

Modern athletics and gymnastics are characterized by more and more complex exercises whose mechanics is very often out of any intuitive understanding. Consequently, big problems may arise planning, teaching and coaching new exercises.

The aerial phase, as an example, which can be relatively long, is sometimes problematic to explain because the athletes do not act against any easily recognizable external force, as the reaction of the ground or that of the equipment. Moreover, the aerial phase of some exercises is generally composed of different sub phases; while learning they are experimented separately by athlete who rebuilds the whole exercise afterwards; therefore a difficult coaching item concerns, for example, how to explain the variations of the movements required to obtain the same sub-phases once they are linked in the full exercise, since the initial conditions are in fact different.

Another coaching problem come from the remark of the practical advantages connected with the presentation of new original figures at the main international meeting, such as olympic games: this drives the technician to design new exercises that, because of their difficulties can generally be tested only by elite athletes, that obviously should not put their safety on risk.

This kind of reasons gave recently impulse to the development of sport biomechanics computer simulation systems, especially oriented to teaching and coaching.

In the last decade a lot work has been done in developing systems for the solution of the inverse dynamic in human motion: it concerns the calculation of inertial forces and couples acting on the body segments and the forces and couples transmitted by the joints, only in few cases the force required to some muscles is also evaluated. This kind of calculations require the knowledge of the athlete position during the whole action which can be obtained by analyzing and digitizing, automatically manually, the images recorded during the exercise.

An exiguous number of researches, on the contrary, deal with the direct dynamic problem whose solution is the basis for computer simulation of the exercises: concisely, it concerns the calculation of the body trajectory and orientation knowing the external forces acting on it and the relative movement of the body segments (or the forces exerted by the muscles). The system of programs that has been developed in our Department and is now under test, belong to the latter mentioned researches and is oriented to sport teaching and coaching.

## THEORETICAL BACKGROUND

The mathematical approach used is based on matrix method recently

developed (Casolo, Legnani 1991). It can be briefly summarized as follow. The method is a generalization of the homogeneous matrix approach, and it makes a coherent use of new 4x4 matrices in order to obtain a kinematic and dynamic model of rigid bodies chains.

According to our notation the kinematic behavior of each rigid body can be described by the following matrices:

$M$  - Position matrix [ the usual transformation matrix). Describing position and orientation of a body.

$W$  - Velocity matrix. It describes both the angular and the linear velocity of a body.

$H$  - Acceleration matrix which contains the angular and linear acceleration of a body.

All these matrices generally have three subscripts  $(ijk)$  in order to specify the bodies to which the contents is relative and the frame assumed as reference. For instance  $W_{ijk}$  is the projection onto the frame  $(k)$  of the relative speed of body  $j$  with respect to body  $i$ .

Velocity and acceleration matrices, because of their structure, are related to the time derivative of the position matrices by simple relations:

$$W = M' M \quad H = M'' M^{-1}$$

The usual relative kinematic theorems can be simply written in matrix notation in order to find the "absolute motion" of bodies (eg.  $i$  and  $k$ ) as resultant of the "drag motion" (between  $i$  and  $j$ ) and the "relative motion" (between  $j$  and  $k$ ). The matrix form of the position and the velocity composition in space and the Coriolis' theorem can be simply written as it follows.

$$M_{ik} = M_{ij} M_{jk} \quad W_{ik(r)} = W_{ij(r)} + W_{jkr} \quad (1)$$

$$H_{ik(r)} = H_{ij(r)} + H_{jkr} + 2 W_{ij(r)} W_{jkr}$$

where  $r$  can be any reference frame.

For the dynamics we introduce two other matrices that are:

$\Phi$  - Action matrix. It describes the system of forces and torques applied to the body.

$J$  - Pseudo-inertial matrix. It describes the mass distribution of a body.

$\Phi_{kst}$  is, as an example, the matrix describing in frame  $(s)$  the forces and torques applied to the body  $k$ .

It is then possible to express, as an example, the action  $\Phi$  that must be applied to a rigid body in order to force an absolute acceleration  $H$  on it as it follows:

$$\Phi_k = (H_{Ok} J_k) - (H_{Ok} J_k)^t$$

where  $t$  is for the transposed matrix and  $o$  is the label of the inertial frame.

This method face coherently, with the same matrix approach to whole question of mathematical analysis and synthesis of 3D human motion: from the position analysis to the direct and inverse dynamics. It allows to simplify the notation and the number of equations generally implied in the dynamics of multi-chains of rigid bodies. Moreover, a library of routines(Legnani et al. 1990) appositely written for solving all the operations involving the described matrices help in writing efficient, concise but understandable programs.

## APPLICATIONS

The set of programs for the simulation of sport exercises developed in our Department in connection with the University of Brescia consists in a set of operating modules each performing a particular task.

- SPACE-LIB is above mentioned library of routines.

- ANTHROPM computes the geometrical parameters and build the pseudo-inertia matrix  $J$  for all the body segments. This program is based on regression equations obtained by Mc Convill et al (1980) from a large number of anthropometric measurements, and compute the few data lacking from this work by mean of standard geometry and medium density (Hanavan 1964, Dempster 1955) of the segment involved. In order to speed the preliminary tests and to allow the simulations also when few data on the subject are available, or, on the contrary, to obtain a precise characterization of the subject, this module accepts from a minimum of two (height and weight) to a maximum of fifty anthropometric parameters.

- LOM helps to build the relative "low of motion" of the joints or, in other words, the function  $\varphi = f(t)$  for all the D. of F. of the model that one want to ask the athlete to move during the simulated exercise. For each time interval considered it allow to chose the acceleration path.

It is also possible to use l.o.m. directly obtained by data recorded during real exercises.

- DY\_MAN is the main module for the solution of the direct dynamic problem: from the knowledge of the relative motion of body segments and the external forces applied, it calculate the athlete body trajectory and orientation during the hipotetical exercise.

From the mathematical point of view this problem will be solved by integrating a set of six order equations which relate the acceleration of the trunk (that we consider reference body ) with its position, its velocity and with limbs movement; the integration initial condition are known. The relative motion of the segments is described by means of the  $M$   $W$  and  $H$ , thus the determination of trunk trajectory and orientation is reduced to the evaluation of the position matrix of the trunk at each step of time.

Labeling with  $o$  the fixed frame and  $\square$  the trunk frame,  $M_{01}$ ,  $W_{01}$  and  $H_{01}$  are the absolute position, velocity and acceleration matrices of the trunk. Remembering that  $H_{01} = W'_{01} W_{01}^2$  (Casolo, Legnani 91), since at each instant the position and the velocity (but not the acceleration) of the trunk are known it is possible calculate immediately  $M_{01}$  and  $W_{01}$ , while  $W'_{01}$  is the only unknown and will be evaluated taking into account the athlete's movements.

All the absolute matrices of the rigid bodies forming the athlete's model can be calculated starting from the initial conditions and the l.of m. by applying the (1) to the four chains forming the whole body and here also the only unknown component results to be  $W'$ . At the end of formal manipulation it is than possible to express the dynamic equilibrium in function of the unknown component of trunk acceleration:

$$W' = f(\phi^*, J^*) = f(M_{1j}, W_{1j}, J_{1j}) \quad (2)$$

where  $\phi^*$  depends of the velocity and position of the body and  $M$ ,  $W$ ,  $J$  depends on the libs movement. The formal expression(2) can be seen as a six

order differential equation that can be integrated numerically in order to find, the trunk position and orientation at each step.

From the practical point of view in order to simulate an aerial exercise Dy\_man require: a file containing the initial conditions (initial position and initial velocity of all the body segments), a file containing the law of motions of the joints that will be moved **and** obviously the inertial and geometric parameters of the body as calculated by module Anthropom.

GRPHMAN is a graphic 3D post processor for Dy\_man output. It uses geometrical parameters produced by Antropos and represent the athlete while is performing as he will be seen by one ore more cameras placed anywhere in the space. The hidden lines feature, demonstrated to be very important in order to well understand the exercise.

## CONCLUSIONS

The recently developed matrix approach to the dynamic of rigid bodies chains, simplifies the set up of the simulation systems devoted to sport exercises. Preliminary tests demonstrate the efficiency of the programs and their adequacy for planning and checking new techniques or new possible exercises without exposing the athlete to any risk. The simplicity of the input module and the clear output make the system suitable for sport didactics in order to show the effect of certain movements on whole body motion.

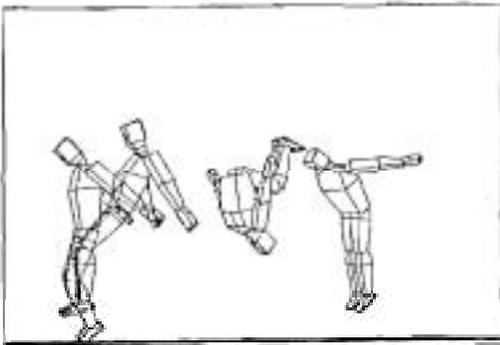
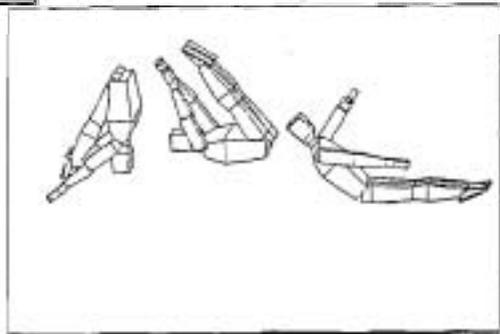


Fig.1 - gymnastic exercises  
simulation: graphic  
output from GRPHMAN



## REFERENCES

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