

A.GUSKOV, Moscow Technical University, Moscow, RF  
 A.VOROBIEV, Russian Athletic Federation, Moscow, RF  
 G.ARIEL, Ariel Life Systems, Inc., La Jolla, CA, USA

A mathematical model of a long jumper flight dynamics is proposed. The model incorporates the factor of the full aerodynamic force exerted during the flight phase under varying conditions. The full aerodynamic force is applied to the body at some angle to the vector of air speed and can be represented as vector sum of drag and lift forces. The magnitudes of the drag and lift components, as well as their role in the length of the jump are affected by a number of parameters, such as flight kinematics, wind velocity, mean altitude, etc., which vary from athlete to athlete, between attempts and competition grounds. Assumption is made that athlete's jumping technique is independent of ground level and wind velocity.

Methods

Analysis the video recordings of the jumps of the leading long jumpers were processed using Ariel Performance Analysis System (APAS) to obtain kinematic pattern of the each attempt.

Effective distance of the jump  $l_s$  is determined by horizontal distance between the CM (body center of mass) and foot mark at the take-off  $l$ , CM horizontal flight distance  $l_o$  and horizontal dis-

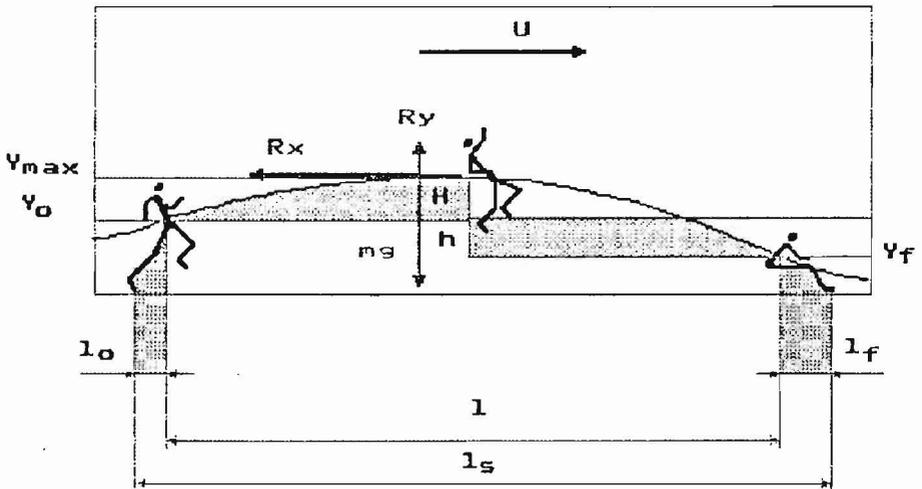


Fig.1 Evaluation of the long jump.

tance between the CM and closest to the board body segment mark during landing  $lf$  (fig.1).

$$ls = l + lo + lf \quad (1)$$

Effective distance of the jump in vacuum might be determined as:

$$ls_v = lo + lf + 2\dot{x}_0 \sqrt{\eta} (1 + \sqrt{1 + 1/\eta}) \quad (2)$$

were  $\eta = H/h$ ;  $H$ ,  $h$  correspondently CM height increasing and decreasing. Evident following inequality:

$$ls_v > ls \quad (3)$$

To determine flight distance  $l$  in atmosphere conditions, athlete's movement was considered as a single point movement with known mass  $m$ , which coincided with CM position, and was affected by aerodynamic force  $R$  and gravity force  $P=mg$  (fig.1). Aerodynamic force was approximated by the quadratic function for speeds below the speed of sound.

The corresponding equations of motion are:

$$\begin{cases} m\ddot{X} = -\rho A C_x / 2 (\dot{X} - U)^2 \sqrt{1 + (\dot{Y} / (\dot{X} - U))^2} (1 + C_y / C_x \dot{Y} / (\dot{X} - U)) \\ m\ddot{Y} = -\rho A C_x / 2 (\dot{X} - U)^2 \sqrt{1 + (\dot{Y} / (\dot{X} - U))^2} (\dot{Y} / (\dot{X} - U) - C_y / C_x) - mg \end{cases} \quad (4)$$

were  $U$  - wind velocity,  $\rho$  - air density:

$$\rho = \rho_n \exp(-a/a^*) \quad (5)$$

$\rho_n$  - air density on the elevation level  $a = 0$ ,  $a^* = 7160m$ ,  $C_x, C_y$  - drag and lift quotients, which depend upon body form and  $A$  - which is the body's characteristic cross sectional area.

Taking into account that in our case  $C_y \ll C_x$  and  $|\dot{Y}| < |\dot{X} - U|$  from system (4) effective length  $ls$  might be obtained:

$$ls = lo + lf + h \left[ \frac{U tf}{h} + \frac{1}{\kappa} \ln(1 + (1 - U/\dot{x}_0) \frac{\dot{x}_0 tf}{h} \kappa) \right] \quad (6)$$

were  $tf$  - flight time, which was determined using following equation:

$$tf = \tau_f \sqrt{h/2g}; \quad \tau_f = 2\sqrt{\eta} (1 + \sqrt{1 + 1/\eta}) \quad (7)$$

and  $\kappa$  - dimensionless drag quotient, defined as:

$$\kappa = \rho C_x A / 2m \quad (8)$$

With known effective distance  $ls$  equation (6) might be transformed for  $\kappa$  quotient definition. Following parameters should be considered:

$$\lambda = l/h; \quad \eta = H/h; \quad \Psi = U/\sqrt{2gh}; \quad \Phi = \dot{x}_0/\sqrt{2gh}; \quad \sigma = a/a^* \quad (9)$$

Then Eq.6 can be overwritten as:

$$\lambda - \Psi\tau f = 1/\kappa \ln(1+(\Phi-\Psi)\kappa\tau f) \quad (10)$$

Parameters  $\lambda, \Psi, \eta, \Phi$  obtained from the results of the long jump kinematic analysis and wind velocity value. Then utilizing one of the numeric methods dimensionless drag quotient  $\kappa$  can be calculated.

To obtain comparable data the normalized dimensionless drag quotient  $\kappa_n$  (see Eq.5) was considered:

$$\kappa_n = \kappa / \exp(-a/a^*) \quad (11)$$

### Results

The results obtained make possible determining the influence of the wind velocity and mean altitude. Table 1. gives the results of numerical analysis for three high-performance attempts in rial conditions and recalculated to the normalized conditions (zero wind / zero elevation).

Table 1.

Parameters	R.BEAMON (1968)	C.LEWIS (1991)	M.POWELL (1991)
Official distance (m)	8.90	8.91	8.95
Effective distance (m)	8.90	8.91	8.98
Actual conditions			
wind (m/s)	2.0	2.9	0.3
elevation (m)	2200	0	0
Drag quotient $\kappa_n$	0.015	0.011	0.012
Normalized result (m)	8.31	8.62	8.92
Result in vacuum (m)	9.27	9.20	9.68

Changing wind velocity  $U$  and mean altitude  $a$  values, using calculated quotient  $\kappa$  in equation (6) as personalized athlete's aerodynamics profile, the forecast of results and comparable values, derived from the results obtained under specific conditions can be performed. Fig.2 present results of computer simulation.

### Discussion

The comparison of the actual and normalized data indicate that the absolute value of the results in long jump during last 24 years might be reconsidered.

As another application of the developed model, a series of iso-energetic trajectories might be emulated, in order to enhance the performance by optimizing the flight aerodynamics under average conditions.

Long jump distance as a function of mean altitude  
(zero wind velocity)

Long jump distance as a function of wind velocity  
(zero elevation)

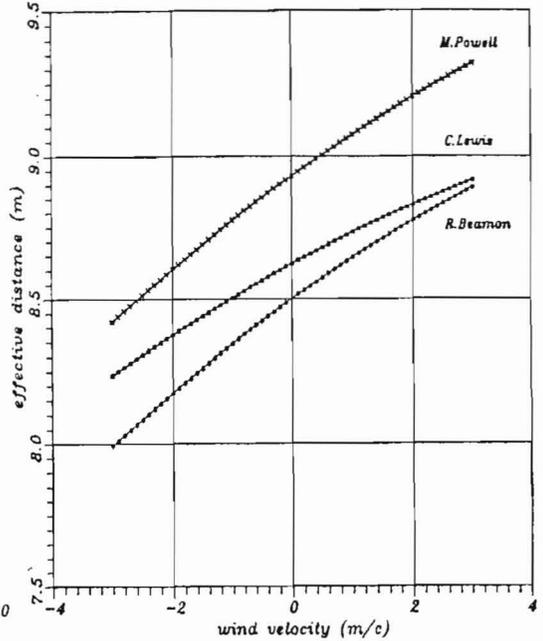
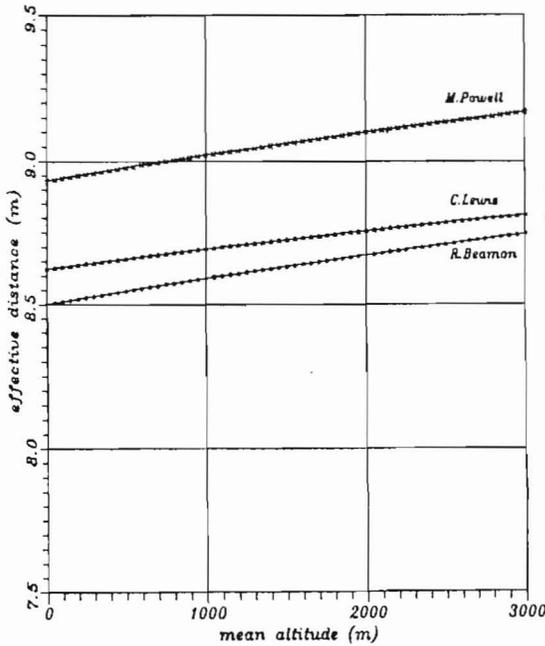


Fig.2 Results of the computer simulation.

#### References

- Bruggemann, P., Nixdorf, E., Ernst, H. (1982). Biomechanische Untersuchungen beim Weitsprung. *Die Lehre der Leichtathletik*, 4, 36-40.
- Goltstein, H. (1980). *Classical mechanics*. Second edition, Addison-Wesley Publishing Company, Inc. Inc.
- Hay, J.G., Nohara, H. (1990). Techniques used by elite long jumpers in preparation for takeoff. *International Journal Sport Biomechanics*, 23, 229-239.
- Hay, J.G. (1986). The biomechanics of the long jump. *Exercise and sport sciences reviews* 14.
- Luthanen, P., Komi, P.V. (1979). Mechanical power and segmental contribution to force impulse in long jump take-off. *Eur. J. Applied Physiol*, 41, 267-274.
- Susanka, P., Stepanek, J. (1986). Dependence of resultant sports performance on running speed in the long jump. Prague.