DYNAMIC ANALYSIS OF ROWING IN MODEL OF MULTI-BODY SYSTEM

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INTRODUCTION

Rowing is a race of the common motion between human body and apparatus. It is a complex motion. It is very difficult to exactly analyze this motion. The author has never seen the report of complete analysis for this motion.

In this paper we base on experimental measure, found the model of rowing in method of multi-body system and define functional relation between technical parameters and moving condition. The result provides theoretical basis for raising sport level.

MECHANICAL MODEL

According to the property of rowing, this system is simulated as a model of six rigid body in planar domain (See Fig.1). B_1-B_4 denote human body, B_1 denotes shanks, B_2 thighs, B_3 hood and trunk, B_4 arms, B_5 oar, B_6 rowboat. They join to each other with a joint. Each joint is respectively O_i (i=1,⋯5). Distance between a joint and its adjacent joint respectively is l_i (i=1,⋯5). Where l_4 is variable, because it isn't real itself, but that it is projection in the vertical plane. Let OA = l_6. The distance between O_i and mass center of B_i respectively is r_i. The mass of each body is m_i. The moment of inertia in mass center is J_i. The origin of coordinate system is fixed in rowlock. The x axis parallel to plane of water. Distance between O and O_1 is a_1 in x axis. Distance between O_1 and end of slide, seat of slide respectively is a_1, $(a-s)$. The oar handle, trunk B_3 respectively cross at β, α to x axis. Vertical swing angle of the rowboat is θ. According to experience of coaches, it is key techniques of rowing to control slide moving and stroking. So that, s (denote slide moving) and θ (denote stroking angle) are defined as controlled functions. For two rowers having same power, if their controlling functions s, β are different, they will have different effect on rowing. Thus,we defined various groups s, β according to measuring data in high speed camera, found differential equations described rowing. Solving these equations, we obtain solved functions v and θ governed by various s, β. Then we can define optimum group s, β as normal technical function, by which judge sport level of athlete. Because rowing is periodic motion, the motion is only analyzed during one period in this paper.

ANALYSIS OF FORCE EXERTED ON SYSTEM

Resultant exerted on system is ***F+f+P+Q***

where F is the force of pushing oar from water (denote total active force) acts on center of oar blade. Its directional vector crosses right angle to oar handle. It is difficult to count its magnitude which may be approximated as

$$F = K_1(\dot{\beta}\, l_6 \sin\beta - v)^2 \tag{1}$$

where v is velocity of boat. The K_1 is assumed constant in this paper, it relate to characteristic of fluid, shape of moving body and surface area of the boat under water. In order to simplify problem, K_1 is defined on basis of article [2]. K_1 =39.6 N/M. f is denoted as resistance on the boat. Its direction is opposite in velocity v. Its magnitude can be approximated as $f = K_2 v$, K_2 =3.27 n/m . P is gravity, acts on mass center of the system. Q is buoyancy acts on center of shape of the boat, which configuration of rowlock (namely origin O). Its magnitude can be approximated as constant, because change of the system moving is very small in direction of y axis. The system is simulated as a model composed of six rigid body, six joint and a single constraint. In consideration of variable l_4, the system is defined as six degree of freedom model. According to the analysis of film, α and β are determined as following constraint relationship:

$$\beta = (14\alpha/5) - (166\pi/180) \tag{2}$$

Thus there is five degree of freedom in this system. Let x, y, θ, β, s be generalized coordinated, $\beta_i = f_i(t)$, $s_i = f_i(t)$ be controlled functions. We will find equations of momentum theorem and angle momentum theorem.

EQUATIONS

Because motion of the system is very small in direction of y, we only derive the equations of momentum theorem in x and angle momentum theorem:

$$\Sigma\, m_i \ddot{x}_i = F_x; \qquad dH/dt = M \qquad (l = 1,2,\cdots 6) \tag{3}$$

where F_x is resultant in x axis . H is total angle momentum. M is total momentum of external force about origin O. Let $O - x\,y$ plane as complex plane, the r_i is distance from O_i to c_i. Suppose θ is small enough to be neglected its effect for the system in direction of x . Configuration of center c_i of each body is given by :

$$c_1 = a_1 - bi + r_1 e^{i(\theta_1 - \theta)}; \qquad c_2 = s - bi + r_2\, e^{i(\theta_1 - \theta)};$$

$$c_3 = s - bi + r_3 e^{i(\alpha - \theta)}; \qquad c_4 = [\,s - bi + l_3 e^{i(\alpha - \theta)} + l_5 e^{i(\beta - \theta)}]/2;$$

$$c_5 = r e^{i\beta} \tag{4}$$

where θ_1 , θ_2 are expressed in generalizes coordinate s, governed by geometric relation of Fig. (1):

$$\cos\theta_1 = -[\,l_1^2 + (a-s) - l_2^2\,]\,/\,2l_1\,(a-s)$$

$$\cos\theta_2 = [\,l_2^2 + (a-s) - l_1^2\,]\,/\,2l_2\,(a-s) \qquad (5)$$

Then we find out the equation of angle momentum theorem :

$$H = \Sigma\,(m_i + c_i \times \dot{c}_i + J_i\,\omega_i) \qquad (i = 1, 2, \cdots 6) \qquad (6)$$

where ω_i is obsolete angle velocity of each component body :

$$\omega_1 = \theta_1 + \theta;\quad \omega_2 = \theta_2 + \theta;\quad \omega_3 = \alpha + \theta;\quad \omega_4 = \omega_6 = \theta;\quad \omega_5 = \beta + \theta$$

Substitution from (4), (5), (6) into (3), simplifying formulations are obtained :

$$\Sigma\, m_i \ddot{x}_i = \Sigma\, m_i\, \mathrm{Re}(\ddot{c}_i) = K_1\,(l_6\,\dot{\beta}\sin\beta - \dot{x})^2 - K_2\,\dot{x}^2 \qquad (7)$$

$$d\,[\,\Sigma(m_i\, c_i \times \dot{c}_i + J_i\,\omega_i)\,]\,/\,dt = \Sigma\, m_i\, g \cdot \mathrm{Re}(c_i) + fb - Fl_6 - K_2\int_0^{l} (l\dot{\theta})\, dl$$

where last item is moment of viscosity resistance, l is distance from O to dl.

The known constants were measured by $a_1 = 0.38$ m ; $b = 0.16$ m ; $a = 0.08$ m. J_i were obtained by Hanavan's report [1] and measuring parameter of rower Xu : $J_1 = 0{,}26$ kg-m^2; $J_2 = 0.48$ kg-m^2; $J_3 = 4.9$ kg-m^2; $J_4 = 0.31$ kg-m^2; $J_5 = 3.06$ kg-m^2 ; $J_6 = 98.29$ kg-m^2. Refer to article [2], $\beta(t)$, $s(t)$ can be approximately described as linear, cosine (sine), square functions during the stroking. They are give by:

$$\beta_1 = 5\pi t/6 + \pi/6;\quad \beta_2 = 5\pi\sin(\pi t/2)/6 + \pi/6;\quad \beta_3 = \pi/6 + 5\pi t/3 - 5\pi t^2/6;$$

$$s_1 = -0.6t + 0.7;\quad s_2 = 0.6\cos(\pi t/2) + 0.1;\quad s_3 = 0.6t^2 - 1.2t + 0.7 \qquad (8)$$

Let $0 \le t \le 0.8$, $v_0 = 0.5$ m/min, $\theta_0 = 0$ and substitute (8) into (7). We obtain nine group solutions v, θ after counting. (See Fig.2)

The result are obtained by analyzing as following: If β, s are defined as linear functions, the variation of θ is small. It means that the boat sailing is smoother, but increasing is slower. If s is square function, the curve of velocity v will decrease during special time. It is unfavorable for rowing. After analyzing the data of each group, the conclusion is clear: $\beta_3\, s_2$ is defined as optimum group.

CONCLUSION

According to above analysis, we obtained these conclusions as following:

(1) It is effective to research rowing in the method of multi-body mechanics.

(2) To improve technique of controlling slide and stroking, it enables sport level to raise in a big margin.

(3) It is the most effective as β is denoted by square curve, s by cosine curve.

(4) The theoretical results corresponds to feeling of coaches

Fig1 Mechanical Model of Rowing

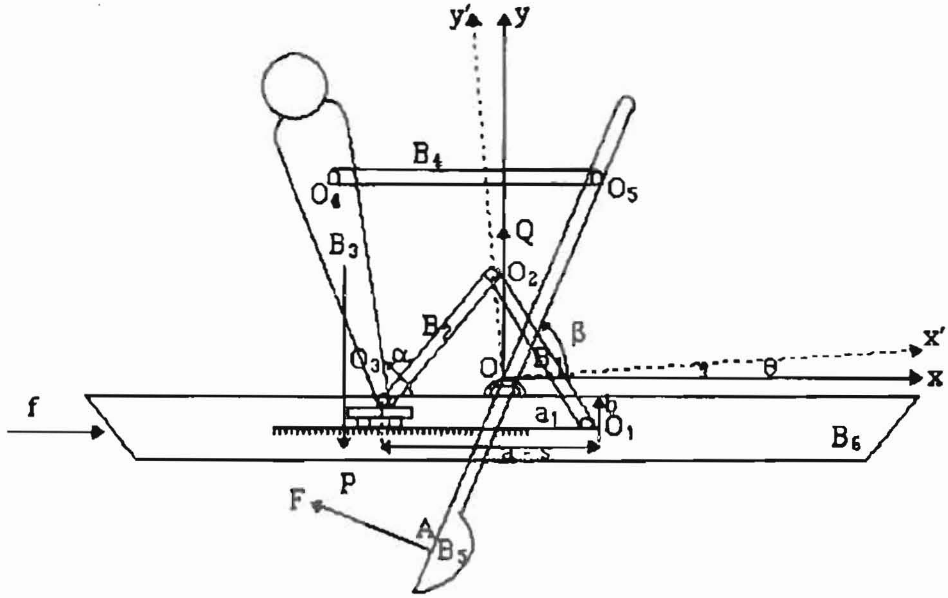
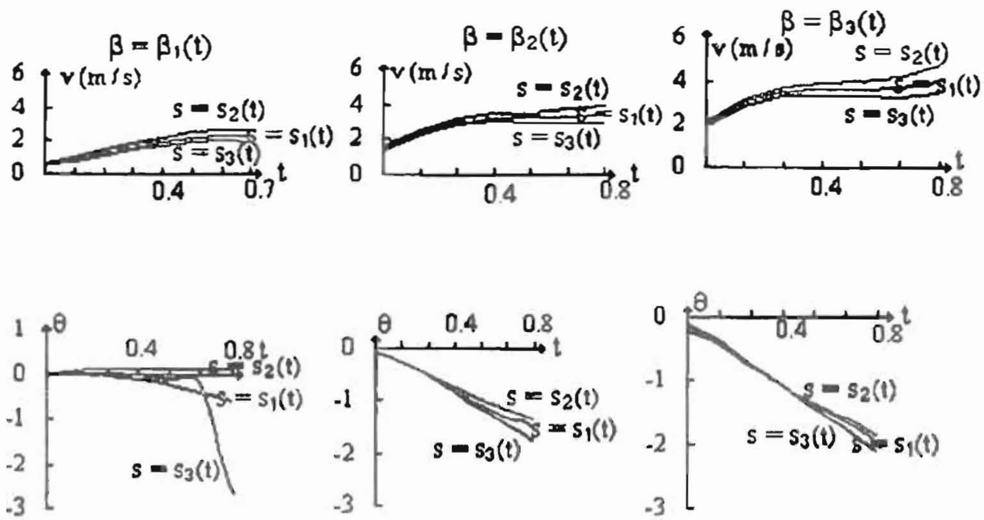


Fig2 Velocity V of Boat and Vertical Swing Angle θ



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[2] Shen Hong yu, (1986). The experiment of resistance for rowing, J. Sports Scince Research