# MULTI BODY SIMULATION OF HUMAN BODY MOTION IN SPORTS 

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## INTRODUCTION

This paper presents a complete methodology for the analysis of the human body motion during sports activities. The final objective is to simulate in a computer the behaviour of a mechanical model of the human body when performing a physical activity, so as to obtain from the simulation various mechanical results like velocities, accelerations, internal forces or external reactions... that cannot be measured directly and that are of interest in the study and improvement of the activity performance.

The study of the human body considered as a multi body system (MBS) is relatively new in the field of biomechanics, but the latest advances in MBS formalisms, the increasing power of computers and the wide-spread use of MBS codes bring a new tool for the analysis of the human body motion.

The first computational solutions to dynamic problems of biomechanics were presented by Chao and Rim (1973), and the first computational studies on the human walking motion published by Arkivar and Seireg (1975). More modern works are those of Buford et al (1990) which present a mechanical model of a hand, and Delp (1990) which presents a model of the lower limbs. Koering (1984) and Maciejewski (1985) are the first to use multi-articulated models of the human body, but do not carry out mechanical simulations and only make a graphical animation of the motion. A powerful model for the mechanical simulation of the bone-muscle system has been presented by Chao, Lynch and Vanderploeg (1993), where the motion data are captured by means of electromagnetic position sensors and video systems. Properties of geometry and mass are obtained from data scanned from real human bodies. Bones are modelled as rigid bodies and ligaments as spring and dampers. The method, which calculates the reaction forces at the joints and the forces required at the muscles, is oriented to medical diagnosis and includes animation facilities.

Finally it is worth mentioning the work of Gutierrez (1990) and Alvarez et al (1993) in which the behaviour of the human body is sports using a MBS approach is studied, but with limitations in the definition of the mechanical model and in the type of dynamic simulations carried out. A similar approach was used in the development of the DYNAMAN tool for ESA/ESTEC, but with further simplifications because of real time response constraints, and limited to kinematic simulation (see Celigüeta, 1993). The work presented here uses the same philosophy of
considering the human body as a multi body system with a pre defined topology : i.e. an assembly of rigid elements, representing the different human members, connected by joints that restrict the relative motion between them. The main difference from the previous works resides in the fact that the dimensions of the mechanical model do not need to be determined in advance, but are calculated as a function of the data measured during the real motion. This allows the building of a MBS that is consistent with motion and whose dynamic simulation provides results which are much more coherent. The proposed methodology is of general purpose and can be applied to other fields of biomechanics although sports have been the most natural field of application.

## MOTION CAPTURE

The first step in the present methodology is the capture of the actual motion of the human body during the activity to simulate. This capture is carried out by following a standard method of video recording, which is today widely used in biomechanics applications : a number of fixed video cameras (usually 2 ) are located covering the complete exercise area. First of all, the cameras are calibrated using an appropriate three dimensional structure of known size. The activity of the athlete is then recorded at a standard rate of 25 images per second, with all the cameras synchronised so as to obtain the frames at the same time value.

The recorded video motions are digitized using a manual process. In this process every frame is presented on the computer screen, where the user has to identify with the mouse pointer a set of key points of the human body. These points are called basic points and will be used later on for two purposes: to define the size of the multi body system used to represent the human body; and to define the input motion applied to such a MBS. The digitisation process is done at a frequency of 50 Hz because of the division of every recorded frame into its even and odd lines. Manual digitisation is normally used for sport activities because of the difficulty of putting optical marks on the athlete's body. However, the analysis methodology allows the use of automatic digitisation whenever possible. In fact the process is normally carried out by using commercially available software programs, greatly facilitating the task of point identification and determination of the 3D coordinates of the points selected on the computer screen. As a result of the motion digitisation, the 3D trajectories of the basic points of the human body are obtained by using the well known direct linear transformation technique (Abdel Aziz, Karara, 1971). These trajectories are stored on disk files in an format appropriate for further processing.

The digitization process is very error prone for several reasons : operator picking errors, small image size, pixel size...making the trajectories unsuitable for input to a multi body solver. To overcome this problem, the digitized trajectories are conditioned in a two step process. First, a filtering technique is applied to every trajectory in an independent way, so as to eliminate high frequency components originated by the motion capture operation. For this filter operation a classical Butterworth filter of third order is used, with a two pass technique. This simple technique has proven to be very efficient for obtaining smoothed motions. The second part of the data conditioning is carried out during the construction of the mechanical model.

## HUMAN BODY MODEL

The key part of the methodology corresponds to the creation of a mechanical model of the human body. This is carried out by using the natural coordinates concept, proposed by García de Jalón et al (1986) : the position and orientation of the joints and elements of the mechanism are defined by the cartesian components of a set of points and unit vectors located at the joints.

The construction of the final mechanical model of the human body requires different steps, that are described in the next paragraphs. It is based on the use of the recorded trajectories as initial data, and on the use of symbolic expressions that relate these data to define the model. The symbolic expressions are processed by a symbolic manipulator, that builds up the complete mechanical model. The process is completely general purpose, but is explained here for a typical model.

First, the model of the primary mechanism is created. This model consists of all these points whose trajectories have been digitized and conditioned. They are referred to as primary points. Normally 22 primary points are used for sport activities. Their names can be seen in figure 1 ( $L$ and $R$ stand for left and right, and all have the suffix $P$, to mean point).


Figure 1. Primary points of the human body model.
The primary mechanism cannot be used for simulation, because it does not contain all the information that is required. Firstly, not all the points necessary to define the mechanism exist as primary points, mainly because most of them cannot be identified on the video images. Secondly, the primary points do not contain information on the orientation of the joints and this orientation must be defined by unit vectors located on such joints, so these unit vectors are not initially defined. All these entities that do not exist among the primary ones are called derived points and derived vectors. Only a small number of derived points exist normally, but all the vectors are derived vectors, as the joint orientation is very difficult to measure.

The position of all the derived points and vectors is defined as an algebraic function of the position of other points and vectors whose position is already known. This procedure is applied recursively until arriving at a primary point, whose position as a function of time is always known. For example, the definition of a derived point (called CentreTrunkP) located at the centre of the trunk is obtained as a function of three primary points (see figure 1) by means of the following algebraic expression

> CentreTrunkP $=(- \text { LowerNeckP }+ \text { LHipP } / 2+\text { RHipP } / 2)^{*} 0.75+$ LowerNeckP

The definition of derived vectors is more involved. It normally requires the use of cross products of two segments each defined by two basic or derived points and/or unit vectors. For example the definition of a unit vector describing the orientation of the left knee, called $L K n e e V$, is obtained as a function of the points located at the hip (LHipP), at the ankle (LAnkleP) and at the knee itself (LKneeP) (see figures 1 and 2). Two segments joining the knee with the hip and the ankle are defined first, whilst the cross product of both segments defines the unit vector :

$$
\text { LKneeV }=\text { unit }((\text { LHipP }- \text { LKneeP }) \wedge(\text { LAnkleP }- \text { LKneeP }))
$$

Other vectors require the use of previously defined vectors. For example the vector defining the lateral rotation between the neck and the trunk (LowerNeckV) is defined in two stages :

$$
\begin{aligned}
& \text { FrontalV = unit }((\text { LowerNeckP }- \text { LHipP }) \wedge(R H i p P-\text { LHipP })) \\
& \text { LowerNeckV }=\text { unit }((\text { UpperNeckP }- \text { LowerNeckP }) \wedge \text { FrontalV })
\end{aligned}
$$

Figure 2 shows all the vectors defined in this way for the human model. Up to 18 vectors are used in a typical model for sports.

With this process the so called derived mechanism is constructed : it consists of the trajectories of all the primary and derived points and vectors.

It must be noted that when using natural coordinates, points and vectors must be located at the joints, so the points and vectors of the derived mechanism, already define the joints that can be used in the model.


Figure 2. Unit vectors defining the orientations of the joints

The next step is the definition of the rigid elements that represent the human body members. These rigid elements are defined in terms of the points and vectors located at the joints. Figure 3 shows the models used for the right leg and arm and figure 4 for the head and trunk. It should be noted that most of the main human member are modelled with two coincident mechanical elements (for example the right arm is modelled with elements RArmint and RArmExt). This is due to the necessity of representing the rotation of the member with respect to itself.

Figure 5 shows the complete human model, that contains 32 rigid elements, connected by 32 joints : 28 revolute joints located at the main articulations; 2 Cardan joints located at the ankles; one more at the throat (to model the 2 rotations existing at each of such points); and finally one floating joint is located between the lower trunk and the ground. This last joint has 6 degrees of freedom and defines the absolute position of the mechanism with respect to the ground.


Figure 3. Detailed models of the right arm and leg


Figure 4. Trunk and head model


Figure 5. Complete human model

The points and vectors used to represent the mechanism joints define the mechanism topology, but is also necessary to define the dimensions of the different elements. In natural coordinates this is done by associating a local reference frame to each element of the mechanism and calculating the coordinates of the points and the components of the vectors that define the element with respect to such a local frame. When a point or vector is shared by two elements, its local coordinates in both elements must be given.

The local coordinates of points and vectors are calculated taking the trajectories of the points and vectors, either primary or derived, as a basis again. The problem is that the distances between the trajectories are not constant,
because of the errors contained in the latter. The solution is to average the distances between the trajectories first and then use the averaged values to calculate the local coordinates of the points. This averaging procedure corresponds to the second phase of the numerical conditioning of the recorded trajectories, previously mentioned.

This procedure is called conditioning of the local reference frame and is also carried out by using symbolic expressions that relate the already existing entities. For example, to define the local frame of the lower trunk element that has 3 points (RHipP, LHipP, CentreP) and a vector (FrontalV) 5 symbolic expressions are used (figure 6). First the sides of the triangle ( $a, b, c$ ) are calculated by averaging the trajectories of the 3 points. This is done using the function MeanLength of the symbolic manipulator. Second the triangle height ( $h$ ) calculated, supposing that side $a$ is fixed and averaging the height generated by sides $b$ and $c$. Finally the local reference frame is located by defining the local coordinates of the 3 points and the vector as a function of the previous geometric magnitudes.
$\mathrm{a}=$ MeanLength(RHipP,LHipP)
$b=$ MeanLength(RHipP,CentreP)
$c=$ MeanLength(LHipP,CentreP)
$h=0.5^{*}\left(\operatorname{sqrt}\left(b^{*} b-a^{*} a / 4\right)+\operatorname{sqrt}\left(c^{*} c-a^{*} a / 4\right)\right)$

| LocalPoint=RHipP | $X=-a / 2$ | $Y=-h / 3$ | $Z=0$ |
| :--- | :--- | :--- | :--- |
| LocalPoint $=$ LHipP | $X=+a / 2$ | $Y=-h / 3$ | $Z=0$ |
| LocalPoint=CentreP | $X=0$ | $Y=2^{*} h / 3$ | $Z=0$ |
| LocalVector=FrontalV | $X=0$ | $Y=0$ | $Z=1$ |



Figure 6. Lower trunk local reference frame

Once the dimensions of the elements have been determined, the definition of the mechanism is complete. The resulting model is called the consistent model, because its dimensions are consistent with the dimensions of the actual human body and constant, as required by the MBS solver.

The last step in the creation of the multi body system corresponds to the definition of its degrees of freedom and the values adopted by such degrees of freedom during the simulation. The degrees of freedom used for the consistent model are:

- the angles at all the joints, that define the relative position of one element with respect to the others. A typical model contains up to 34 angles (28 at the revolute joints and 6 at the 3 Cardan joints)
- the position of the reference element (lower part of the trunk) with respect to the ground. This position is defined by the 3 absolute coordinates of a point and the rotations of the 3 Bryant angles used to locate the orientation of the lower trunk.

The values of the angles as a function of time that must be applied as inputs to the degrees of freedom to simulate the motion, are again obtained from the trajectories of the points. As the angles are defined between 3 points, it is very easy
to obtain their values from the dot product equation of the two segments defined by the 3 points. The sign indeterminacy is solved, when necessary, by using the cross product equation. No extra symbolic expressions are required, as all the necessary information is contained in the angle definition.

The values of the angles are obtained in tabular form, with as many values as the trajectories. Finally, the tabular values are fitted using cubic splines, to obtain a smoother variation. This is necessary because the time increment between the tabulated values (normally $1 / 25 \mathrm{sec}$ ) can be too big for simulation purposes and can create convergence problems.

## MULTI BODY SIMULATION

For the simulation of the mechanical model of the human body, the formulation corresponding to the natural coordinates has been used (see Garcia de Jalón and Bayo, 1993 for more details). Two types of simulation are of interest for the human body motion : kinematic and inverse dynamics.

- Kinematic simulation consists of the calculation of the mechanism motion for some known values of all its degrees of freedom, with independence of the forces required to produce such a motion. In our case the motion is completely defined by the values of all the angles, so kinematic simulation is straightforward and leads to the restitution of the original motion. Let $\mathbf{q}(t)$ be a vector that contains the coordinates of all the points and vectors, and the angles used to model the multi body system. Only the angle values are known in $q$, so the problem is to find the values of the coordinates of points and vectors. The different terms of $\mathbf{q}$ are not independent, but are related by a set of constraint equations, that can be written as:

$$
\begin{equation*}
\phi(\mathbf{q})=0 \tag{1}
\end{equation*}
$$

In natural coordinates, the constraint equations arise from different facts : first because of the rigid body conditions that must be fulfilled by the different points and vectors that define a rigid body; second by the constraint equations originated at the joints between the points and vectors that define them and third by the equations that define the angles at the joints. The non-linear constraint equations are solved iteratively by using a Newton-Raphson method, in a least squares sense. The equation for iteration (i) is:

$$
\begin{equation*}
\Phi_{\mathbf{q}}^{(i) T} \Phi_{\mathbf{q}}^{(i)} \Delta \mathbf{q}^{(i)}=-\Phi_{\mathbf{q}}^{(i) T} \phi\left(\mathbf{q}^{(i)}\right) \tag{2}
\end{equation*}
$$

where $\Phi_{q}^{(i)}$ is the Jacobian matrix of the constraints at iteration (i).

- The velocity problem allows us to calculate the velocities of the remaining elements of the MBS when the velocities of the input elements are known. The equation now is the first time derivative of equation (1) :

$$
\begin{equation*}
\Phi_{\mathrm{q}} \dot{\mathrm{q}}=0 \tag{3}
\end{equation*}
$$

In our case, since the angles at the joints are defined as cubic spline functions of time, the known terms of $\dot{\mathbf{q}}$ are very easy to evaluate by numerical differentiation and equation (3) is very easy to solve.

- The acceleration problem allows the computation the accelerations of the system elements $\ddot{q}$ when the accelerations at the angles are known. The equation for this problem is :

$$
\begin{equation*}
\Phi_{q} \ddot{q}=-\dot{\Phi}_{q} \dot{\mathbf{q}} \tag{4}
\end{equation*}
$$

- The inverse dynamics simulation calculates the internal efforts that must be applied at the MBS to obtain the prescribed motion. The motion equations of the MBS, can be written as :

$$
\begin{equation*}
\mathbf{M} \ddot{\boldsymbol{q}}+\Phi_{\mathbf{q}}^{T} \lambda=\mathbf{Q} \tag{5}
\end{equation*}
$$

where $\mathbf{M}$ is the inertia matrix, $\mathbf{Q}$ the vector of external forces and $\lambda$ the vector of the Lagrange multipliers associated with the constraints. As the accelerations $\ddot{q}$ are known from the solution of the acceleration problem this equation allows the calculation of $\lambda$. With the values of $\lambda$ it is simple to compute all the internal efforts. These internal efforts are : the reactions (forces and torques) at the joints, and the motor efforts that must be applied at the angular degrees of freedom to produce the motion. These are magnitudes of maximum interest for the biomechanical study of the motion, and allow the comparison between different styles, calculation of maximum values, etc.

In addition to the internal forces, other useful magnitudes can be also obtained from the inverse dynamics simulation : energy distribution, angular velocity and angular momentum of the elements, contact forces with the ground, etc.

## IMPLEMENTATION

The methodology presented here has been implemented in a completely new version of the software program called Compamm/Sport. The program is of general purpose and can deal with any motion that has been recorded and where the trajectories of a set of primary points have been digitized and are expressed in a tabular form.

The most important component of the program is the symbolic manipulator previously mentioned that allows to construct the multi body model of the human body. Like the rest of the program, it has been developed in C++ and includes the following major functionalities: C-like syntax, mixed operations between scalars and 3 component vectors, logical and relational operations (>, $=,<$, and, or...), control and loop statements (if, for, while..), a complete mathematical library, and the capability of working with point trajectories. The use of this manipulator greatly facilitates the definition of any model of the human body because it avoids the need of writing specific programs that must be changed for every topology or size of the elements. By using only symbolic statements written in a file the user can create his own models and modify them easily.

The program also includes an advanced visualization system, that allows the display of the evolution of the multi body system while the simulation is being performed. For this purpose a geometrical model of the human body is used; it is composed of a set of polygonal facets associated with every element of the mechanism., with appropriate size and graphical properties. It is also possible to
include the graphic representation of most of the computed magnitudes (for example vector magnitudes as reactions forces or angular momentum are shown as solid arrows), as well as the trajectories of points of interest.

## EXAMPLES

## Walking motion

This example represents a simple walking motion of a 75 Kg person at a 6.1 $\mathrm{Km} / \mathrm{h}$ speed.

The recorded motion lasts 1.15 sec (from $t=9.5 \mathrm{sec}$ to $t=10.65 \mathrm{sec}$ ) and includes only two steps, with three contacts with the ground : right foot between $t=9.5 \mathrm{sec}$ and $t=10.1$, left foot contact between $t=10.1$ and $t=10.6$ and again right foot contact at $t=10.6 \mathrm{sec}$. The motion was filtered with a 2 Hz cutoff frequency. Figure 7 shows 6 key frames of the motion.


Figure 7 . Walking motion

A kinematic simulation of the recorded data allows us to determine the values of the different angles involved in the motion. For example, figure 8 shows the rotation of the two knees.


Figure 8. Knee rotation angle during the motion of walking
The inverse dynamic analysis of the motion allows the calculation of the forces involved and particularly the contact forces with the ground. To validate the computed values, the contact force between the left foot and the ground has been measured by means of a force platform in the same conditions of speed, mass, etc. Both the measured and the computed value for such a left foot contact force are displayed in figure 9 , where a good correlation can be observed.

Force (N)


Figure 9. Ground contact force during walking.

## High jump :

This example corresponds to the winner jump ( 2.02 m ) by H. Henkel, at Barcelona'92. The data for this example where obtained in the frame of the ABAT project, organized and sponsored by the IOC. Figure 10 shows a view of the jump that lasts from $t=9.78 \mathrm{~s}$ to $\mathrm{t}=10.76 \mathrm{~s}$. The instant where the athlete leaves the ground is $\mathrm{t}=10.06 \mathrm{~s}$.


Figure 10. High jump motion

Figure 11 shows a useful information obtained from the analysis in the form of the distribution of the errors in length for the different elements of the multibody system, as originated by the digitisation process. It can be seen that the small elements such as feet or head have up to $15 \%$ error, while others are in the $5-10 \%$ range. It should be noted that the motion was recorded with cameras located at some distance from the athlete, because of access restrictions during the games, so the image quality is decreased.


Figure 11. Error in members length (\%)

The kinematic simulation of the motion allows the computation of the velocity of the centre of the trunk. Its three components are shown in figure 12.


Figure 12. Velocity of the centre of the trunk during the high jump.
Figure 13 shows the vertical reaction force during the last contact with the ground, before starting leaving from the ground. Its value reaches up to 2700 N . Figure 14 shows the modulus of the internal reaction force at the athlete's right hip (point RHipP).


Figure 13. Vertical reaction force during the high jump


Figure 14. Reaction force at the right hip during the high Jump (modulus).

## Football motion : head shot

This example corresponds to a head shot during training of player J.M. Bakero. The motion lasts 1.24 sec with a total of 31 frames. The input data was filtered with a cut-off frequency of 5 Hz . Some representative frames of the motion are included in figure 15. The contact with the ball occurs at about $t=10.8 \mathrm{sec}$.


Figure 15. Head shot motion


Figure 16. Energy distribution during the head shot
The dynamic simulation of the recovered motion allows to compute the energy distribution as a function of time for the whole player (figure 16). Figure 17 shows the motor efforts applied at the player's neck during the shot.


Figure 17. Torques at the player neck during the head shot

## Football : foot shot

This example corresponds to a shot of player R. Koeman recorded during a match. The total motion lasts 1.5 sec and some of its key frames are shown on figure 18. The contact between the foot and the ball happens between $t=9.8$ ant $t=10 \mathrm{sec}$.


Figure 18. Football shot.

The dynamic analysis of the motion gives the internal forces appearing at the joints. Figure 19 shows some of the most representative : the torque that is applied at the different joints of the right leg in order to generate the motion.


Figure 19. Torques at the right hip, knee, and ankle during a foot shot.

## CONCLUSIONS

The method presented allows computer simulation of the behaviour of a mechanical model of the human body, when performing real motion, previously recorded in video during actual activity.

The mechanical model consists of an articulated multi body system, defined by natural coordinates. It is created by taking the trajectories followed by a number of primary points, identified in the video records, as initial data. The rest of the model is created as a function of the initial trajectories by using a symbolic description, that can include complex geometrical relations of vector nature. The symbolic manipulator is very easy to use and allows users to define their own models and to modify them by using only geometrical and mechanical concepts, without requiring any programming operation. This permits to create the models that are best adapted to the particular sport activity, by changing the level of modelling detail of the different parts of the human model.

The model, so created, is consistent with actual recorded motion and its simulation allows the recovery of initial motion and the calculation of all mechanical magnitudes involved in it, like forces, accelerations, energy, etc.

The model construction and its kinematic and dynamic simulation have been implemented in a software program called Compamm/Sport, providing a powerful tool for the analysis of human body motion. The program includes an advanced visualization module that permits representation of the simulated motion and its comparison with the actual motion. It can deal with any human motion; as such it has been applied to different sports activities, and has demonstrated a good correlation with real data. It can also be applied to other fields like ergonomics.

The general purpose nature of the method and the open architecture of the software allow its extension in the future, by the inclusion of new functionalities like: the distribution of the calculated internal forces among the different muscles; more detailed modelling of muscles and articulations; the inclusion of control laws, and the use of motion modification and optimization techniques.

## REFERENCES

Abdel Aziz, Y.I., Karara, H.M., (1971). Direct Linear Transformation for Comparator Coordinates into Object Space Coordinates in Close-range Photogrammetry. Symposium of the Society of Photogrammetry, Falls Church, VA.

Alvarez, G., Gutierrez, A., Serrano, N., Urban, P., García de Jalón, J., (1993). Computer Data Acquisition, Analysis and Visualization of Elite Athletes Motion, 4-th Int. Symposium on Computer Simulation in Biomechanics, Paris.

Arvikar, R.J., Seireg, A. (1975). The Prediction of Muscular Load Sharing and Joint Forces in the Lower Extremities During Walking, Journal of Biomechanics, 8, 80-102.

Buford, W.L., Meyers, L.M., Hollister, A.M., (1990). Modelling and Simulation System for the Human Hand, Journal of Clinical Engineering, 15, 445-451.

Celigüeta, J.T. (1993), DYNAMAN : Real Time Kinematic Simulation Tool of the Human Body in Space - Final Report, European Space Agency, Noordwijk.

Chao, E.Y., Lynch, J.D., Vanderploeg, M.J., (1993). Simulation and Animation of Musculoskeletal Joint System, ASME Journal of Biomechanical 'Engineering, 115, 562-568.

Chao, E.Y., Rim, K., (1973). Application of Optimization Principles in Determining the Applied Moments in Human Leg Joint During Gait, Journal of Biomechanics, 6, 497-510.

Delp, S.L., Loan, P., Hoy, M. G., Zajac, F. E., Torp, E. L., Rosen, J. M., (1990). An Interactive Graphic-Based Model of the Lower Extremity to Study Orthopaedic Surgical Procedures, IE+W Trans., Biomechanical Engineering. BME, 37, 757-767.

Delp, S.L., (1990). A Computer-Graphics System to Analyze and Design Musculoskeletal Reconstructions of the Lower Limb, Ph. D. Thesis, Department of Mechanical Engineering, Stanford University.

García de Jalón, J., Unda, J., Avello, A., (1986). Natural Coordinates for the Computer Analysis of Three Dimensional Multibody Systems, Computer Methods in Applied Mechanics and Engineering, 56, 309-327.

García de Jalón, J., Bayo, E., (1993). Kinematic and Dynamic Simulation of Multibody Systems - The Real Time Challenge, Springer-Verlag, New York.

Gutierrez, A., (1990). Comportamiento Mecánico del Cuerpo Humano en el Deporte de Alta Competición. Ph. D. Thesis, Escụela Superior de Ingenieros Industriales, University of Navarra. San Sebastián.

