BALANCING WITH REFLEX DELAY

Gabor STEPAN
Dcpartmcnt of Applicd Mcchanics,Tcchnical University of Budapest
Budapcst, Hungary

INTRODUCTION

In sport, unstablc cquilibria of mcchanical systcms oftcn havc to be stabilized by thc human opcrators, i.e. by the sportsmcn. A, typical basic csamplc for this is thc sclf-balancing of thc human body.

Tlic analysis of thc problcm of balancing an invcrtcd pcndulum provcs that thc human opcrator has to apply a quitc complicatcd control stratcgy if hc wants to acliicvc his goal in tlic prcscncc of thc time dclay of his/hcr rcflcxcs. It is a rulc of thumb, that incrcasing timc dclay tcnds to dcstabilizc any dynamical systcm. To avoid thc instability naturally occurring in thc mcclianical systcm and also causcd by thc timc dclay, thc human opcrator lias to choosc tlic control paramctcrs from a narrow rcgion which can bc found only aftcr morc or lcss practicing. Abovc a critical valuc of tlic rcflcx dclay, tlic balancing is impossible.

Stabilization of thc invcrtcd pcndulum is a cliallcnging basic cxaniplc, hcncc, a long scrics of publications lias appcarcd in this linc for tlic last forty ycars (scc c.g. Higdon (1963), Mori (1978), Stcpan (1984), Hcndcrs (1992), Kawazoc (1992)) citlicr about its cxpcrimcntal or thcorctical aspccts. This problcm is intcrcsting not only in biology, but also in robotics, to construct bipcd robots (scc c.g.Hcmami(1978)).

In tlic subscqucnt chaptcrs, tlic stability chart in thc spacc of thc control paramctcrs is constructcd and thc abovc mcntioncd critical rcflcx dclay is calculatcd. Tlic surprisingly simplc analytical rcsults havc intcrcsting physical mcaning and thcy show a good corrclation with simplc cspcrinicntal obscrvations. Thcsc rcsults also providc somc in-vicw into thc work of tlic organ callcd ``labyrinthus" in tlic inncr car which hclps in thc sclf-balancing of thc human body whcn our cycs arc closcd.

MECHANICAL MODEL OF BALANCING

. Considcr tlic simplcst planar mcchanical modcl of tlic invcrtcd pcndulum shown in Fig.1. Its lowcst point slidcs smoothly along thc horizontal linc. This mcchanical modcl is thc simplcst possiblc modcl dcscribing tlic ``man-machinc" systcm whcn somcbody placcs thc cnd of tlic stick on his fingcrtip, and trics to movc this lowcst point of the stick in a way that thc stick is balanced at its uppcr position. Thc systcm has 2 dcgrccs of frccdom dcscribcd by thc gcncral coordinatcs.. Thc angle phi is dctcctcd togcthcr with its dcrivativcs

the horizontal control force Q is determined by them in a way that the upper $\varphi = 0$ position should be asymptotically stable.

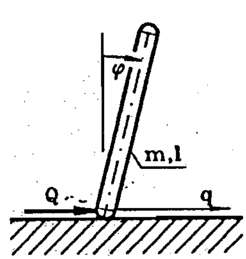

Fig. 1. **Mechanical *model* of stick balancing**

The nonlinear equations of motion assume the form

$$\begin{pmatrix} \frac{1}{3}ml^2 & \frac{1}{2}ml\cos\varphi \\ \frac{1}{2}ml\cos\varphi & m \end{pmatrix}\begin{pmatrix} \ddot{\varphi} \\ \ddot{q} \end{pmatrix} - \begin{pmatrix} \frac{1}{2}mlg\sin\varphi \\ \frac{1}{2}ml\dot{\varphi}^2\sin\varphi \end{pmatrix} = \begin{pmatrix} 0 \\ Q \end{pmatrix}$$

from which the "cyclic" coordinate q can easily be eliminated to be left with the single second order equation

$$(4-3\cos^2\varphi)\ddot{\varphi} + \frac{3}{2}\dot{\varphi}^2\sin(2\varphi) - \frac{6g}{l}\sin\varphi = -\frac{6}{ml}Q\cos\varphi\,. \qquad (1)$$

In the equations, g stands for the gravitational acceleration.

The control force Q is considered in the simplest form of a PD controller with constant gains P and D chosen by the operator appropriately:

$$Q(t) = D\dot{\varphi}(t-\tau) + P\varphi(t-\tau)\,. \qquad (2)$$

In this formula, the human reflex delay is also modelled by a constant time lag (or dead time) τ. Clearly, the trivial solution φ a 0 in (1-2) describes the equilibrium to be stabilized.

STABILITY ANALYSIS

A stability analysis of the $\varphi = 0$ position is required to find suitable control parameters P, D in (2). The variational system of the motion equation (1) with (2) at the trivial solution assumes the form of a linear retarded differential difference equation:

$$\ddot{\varphi}(t) - \frac{6g}{l}\varphi(t) + \frac{6}{ml}\left(D\dot{\varphi}(t-\tau) + P\varphi(t-\tau)\right) = 0\,. \qquad (3)$$

Theorem 1. If there is ao delay in the system, i.e. $\tau = 0$, then the trivial solution of (3) is asymptotically stable in Lyapunov sense if and only if

$$P > mg \quad \text{and} \quad D > 0\,.$$

'This statement can easily be proved by means of the well-known Routh-Hurwitz criterion since (3) becomes a simple ordinary differential equation in this case.

Theorem 2. Let the time delay'be positive in (3), i.e. $\tau > 0$. The trivial solution of (3) is asymptotically stable if and only if

$$mg = P_{min} < P < P_{max}(\omega) = \left(mg + \frac{ml}{6\tau^2}\omega^2\right)\cos\omega\,, \qquad (4)$$

where w is the only value satisfying Dw $= P\tau$ tanw in the interval $(0, \pi/2)$.

The proof of this theorem can be based on the analysis of the corresponding transcendental characteristic function

$$\lambda^2 - \frac{6g}{l}\tau^2 + \frac{6}{ml}\tau D\lambda e^{-\lambda} + \frac{6}{ml}\tau^2 P e^{-\lambda}\,,$$

whether all its infinitely many characteristic roots satisfy $\mathrm{Re}\lambda < 0$. This analysis is supported by the method presented in Stépán (1989).

The corresponding stability chart in the plane of the gain parameters P, D for constant delays $\tau_1 < \tau_2 < \ldots$ is presented qualitatively in Fig. 2. The encircled numbers show the number of those characteristic roots λ having positive real parts. For example, if too great proportional gain P is applied by an untrained operator, *2* complex conjugate characteristic roots turn up in the right half of the complex plane. This refers to a Hopf bifurcation resulting periodic motion mound the desired equilibrium. The recent experiments of Kawazoe *(1992)* also show a strong periodic component in the angle signals produced by untrained operators when balancing an inverted pendulum.

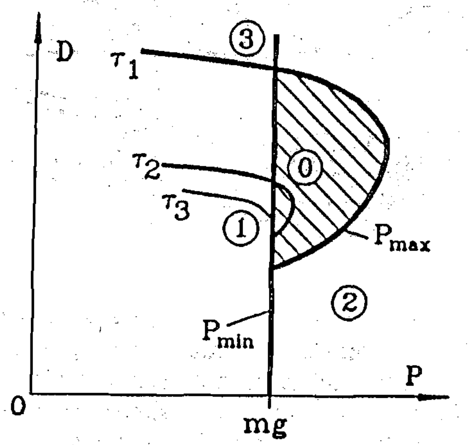

Fig. 2. **Stability chart**

CRITICAL REFLEX DELAY

The stability chart in *Fig.* 2 also shows that the shaded stability domain shrinks as the time delay τ increases and. a certain critical value it disappears. This critical delay is calculated in

Theorem 4.6. There always exist parameters P, D such that the trivial solution of (3) is asymptotically stable if $\tau < \tau_{cr}$, and the trivial solution is always unstable if $\tau > \tau_{cr}$, where the critical value is given by

$$\tau_{cr} = \sqrt{\frac{l}{3g}}\,. \qquad (5)$$

This simple result *can* be proved by checking the condition $P_{min} < P_{max}(\omega)$ for the existence of any stability domain for P in (4).

CONCLUSIONS

In spite of the fact, that the model is simplified, and formula (2) of the control force describes only the basic components of the human operator's behaviour, the above results are quite reliable even quantitatively. The delay of our reflexes is in the range of 0.1 second through, our eyes and arms. Formula (5) means that after a short practice, everybody is able to balance a stick of length $l = 0.3$ meters, when the critical delay ie $\tau_{cr} \approx 0.1$ second. Anybody *can* experience that the longer the stick is, the easier it is to equilibrate it, since τ_{cr} becomes greater. It is impossible to balance short sticks like pencils, etc. Finally, if one is a bit tipsy, the long stick cannot be equilibrated either, because the delay of'the reflexes becomes too great. This may cause problems even in the self-balancing of the human body.

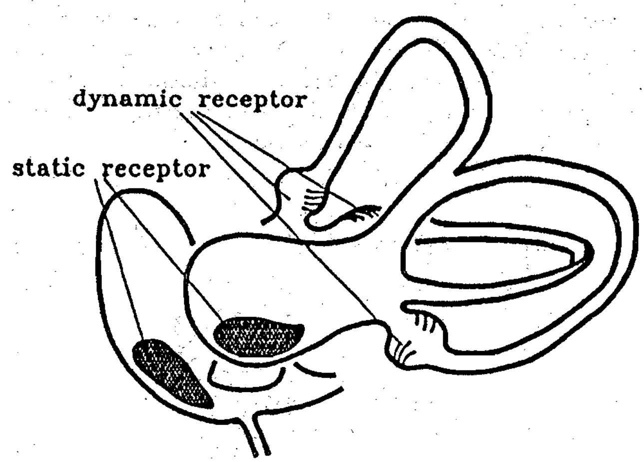

Fig. 3. Dynamic and static receptors in *inner ear*

The self-balancing of human beings is, of course, a very complicated phenomenon. The body is controlled by us to stabilize it in a position which is physically unstable with a lot of degrees of freedom. However, even a simple inverted pendulum cannot be balanced by means of a single position signal or a single velocity signal. As Fig.2 shows, there is no stability if either $D=0$ or $P=0$ in (3). The human brain also has to use both signals, and the ear does provide them. Roughly speaking, the semicircular canals sense the angular velocity, while the attitude is scnscd by means of the otolith organs as shown in Fig3. (see also Steele, 1979).

These conclusions may provide a good basis for developing ncw tests to check the sportsmen's reflex delays and balancing abilities (scc also Bretz (1994)).

ACKNOWLEDGEMENT

This rcsearch was supported by thc Hungarian Scientific Rescarch Foundation under grant N. OTKA 5-328.

REFERENCES

Bretz, K., Kaskc, R. J., 1994. Some parametcrs of multiloop biofeedback control of posture. {\it Proc. XII Int. Symposium on Biomechanics in Sport) (Si\'ofok, 1994).

Hcndcrs, M.G. \& Sondack, A.C. 1992, 'In-the-large' bchaviour of an inverted pendulum with linear stabilization. {\it Int. J. of Nonlinear Mechanics} {\bf 27}, 129-138.

Hemami, H., Weimer, F. C., Robinson, C. S., Stockwell, C. W., \& Cvetkovic, V. S., 1978, Biped stability considerations with vestibular models. {\sl IEEE Trans. Automatic Control) {\bf 23) 1074-1079.

Higdon, D.T. \& Cannon, R.H. 1963, On the control of unstablc multiple-output mechanical systems. {\it ASME Publications) {\bf 63-WA-48}, 1-12.

Kawazoe, Y. 1992, Manual control and computer control of an inverted pendulum on a *cart.* {\it Proc. 1st Int. Conf. on Motion and Vibration Control} (Yokohama, 1992), 930-935.

Mori, S., Nishihara, H. \& Furuta, K. 1976, Control of an unstable mechanical system. {\it Int. J. Control} {\bf 23}, 673-692.

Steele, C. R., 1979, Studies of the ear. {bl Lectures in Applied Mathematics} {\bf 17) 69-91.

St\'ep\'an, G. 1984, A model of balancing. {\it Periodica Polytechnica} {\bf 28}, 195-199

St\'ep\'an, G. 1989, {\it Retarded Dynamical Systems). Longman, Harlow.