AN ANALYSIS OF PROJECTION ANGLE OF THE LONG JUMP

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The purpose of this study was to evaluate the optimum projection angle and to determine the characteristics of optimal takeoff techniques. Subjects of this study included 12 male and 12 female elite long jumpers. Their performance was filmed at long jump event of the 8th China National Games. Mathematical methods, including regression analysis were used to determine the relationship of projection angle with projection velocity and projection distance, then to calculate the optimum projection angle. It was concluded that an increase in projection angle, at the expense of a normal loss of projection velocity, would benefit the projection distance according to the capability of Chinese elite long jumpers. It was also concluded that the takeoff leg should extend forward more before active landing in order to avoid extremely small segment angles of the shank.

KEY WORDS: optimum projection angle, projection velocity, segment angle of shank

INTRODUCTION: In the literature on the topic, some Chinese researchers and practitioners have emphasized a minimum loss of horizontal velocity during the takeoff phase. However, some researchers report that the projection angles of Chinese elite long jumpers were too small. In the 3rd IAAF 1991 Tokyo World Championships in Athletics, Mike Powell renewed the oldest world record in Athletics originally set by Robert Beamon. The performances by Powell and Beamon have some similar technique features: a large projection angle and a large loss of horizontal velocity at takeoff. There are some conflicting arguments about the relationship between projection angle and projection velocity. Therefore, the purpose of this study was to investigate, theoretically, the effects of increasing projection angle on projection velocity and the optimum projection angle, then to determine the characteristics of better take off techniques.

METHODS: The subjects of this study included 12 male and 12 female elite long jumpers who competed at the 8th China National Games long jump final. All performances were filmed at 100Hz with one camera placed perpendicular to the runway, about 13m from the takeoff board. The trial that yielded the largest effective distance for each subject was analyzed. Data were smoothed with a Butterworth second-order filter with a cut-off frequency of 10Hz. To investigate the relation between projection angle and projection velocity, statistic regression analysis was used. The equation is:

$$\hat{V}_o = B_0 + B_1 V_s + B_2 \alpha^n \quad \text{(n=1,2,3)} \qquad (1)$$

Where, $\hat{V}_o$ is regression projection velocity. α is projection angle. Angle unit is radian. V_s is approach velocity. In this equation, V_s and α are independent variables and V_o is the dependent variable. Because the correlation between run up speed (approximately equal to approach velocity) and take-off speed are good (Leeds, 1992), and the total momentum, and therefore also the energy of transnational motion, decreases during take-off (Witters, 1992), V_s and α are used to be independent variables. If this equation is rational, B_1should be positive and B_2 should be negative. n is used to determine the rate of decrease of projection velocity as the projection angle increases. The less n is, the stronger athlete's ability is. When V_s is fixed we have:

$$\frac{d\hat{V}_0}{d\alpha} = nB_2\alpha^{n-1} \qquad (2)$$

And, the projection distance equation is:

$$S = \frac{1}{g}\left(\frac{1}{2}V_0^2 \sin 2\alpha + V_0 \cos\alpha \sqrt{V_0^2 \sin^2\alpha + 2g\Delta h}\right) \qquad (3)$$

Δh is the height of center of gravity at takeoff.
In order to calculate extreme value, making use of Eqs. (3) it is necessary to calculate the derivative of *S* with respect to α and making it equal to zero, we have:

$$\frac{dS}{d\alpha} = \frac{V_0}{V_0}\sin 2\alpha + \cos 2\alpha + \left(\frac{V_0}{V_0}\cos\alpha - \sin\alpha\right)\sqrt{\sin^2\alpha + \frac{2g\Delta h}{V_0^2}} +$$

$$\frac{\cos\alpha\left(\frac{V_0^{'}}{V_0}\sin^2\alpha + \sin\alpha\cos\alpha\right)}{\sqrt{\sin^2\alpha + \frac{2g\Delta h}{V_0^2}}} = 0 \qquad (4)$$

where, $V_0^{'} = \frac{dV_0}{d\alpha}$ 。 Substituting $\hat{V}_0(\alpha)$ and $\frac{d\hat{V}_0(\alpha)}{d\alpha}$ into Eq. (4), using optimum seeking method, we can obtain solution α , which is the optimum projection angle.
To evaluate the movement of the take-off leg, we use the following equation:

$$V_{yh}=V_{y(h-k)}+V_{y(k-a)}+V_{y(a-t)}+V_{yt} \qquad (5)$$

where, V_{yh} is the vertical velocity of hip.
$V_{y(h-k)}$ is the vertical velocity of hip relative to knee.
$V_{y(k-a)}$ is the vertical velocity of knee relative to ankle.
$V_{y(a-t)}$ is the vertical velocity of ankle to toe.
V_{yt} is the vertical velocity of toe.

RESULTS AND DISCUSSION: Using data obtained from the analysis of each athlete's best performance, we calculated the regression equation (1). Table 1 shows the corresponding coefficient and multiple correlation coefficient R and test.

Table 1 Coefficient Regression Equation and Test

N		B_0	B_1	B_2	R	Equ Test	Coefficient test	
							B1	B2
1	Male	4.459	0.629	-3.723	0.941	p<0.01	p<0.01	p<0.01
	Female	3.470	0.717	-3.986	0.926	p<0.01	p<0.01	p<0.01
2	Male	3.761	0.635	-5.371	0.937	p<0.01	p<0.01	p<0.01
	Female	2.806	0.714	-5.661	0.922	p<0.01	p<0.01	p<0.01
3	Male	3.470	0.643	-10.163	0.933	p<0.01	p<0.01	p<0.01
	Female	2.607	0.711	-10.638	0.918	p<0.01	p<0.01	p<0.01

B_2 is negative which means that projection velocity would decrease with projection angle increase, which is consistent with the assumption. The results were more accurate as n=3. As n=1, 2, the optimum projection angles were extremely large. As n=4,5, the test of equation and coefficient was p>0.05.

In order to confirm whether the increase of projection angle was beneficial to projection distance, the regression equation (1) was used to calculate the loss value (Rr) of projection velocity as projection angle increased one degree. Projection equation (3) was used to calculate the allowable loss (Ra) of projection velocity keeping the fixed projection distance and the projection angle increased one degree. If Rr < Ra, the increase of projection angle benefits the projection distance. Otherwise, the increase of projection angle decreases the projection distance.

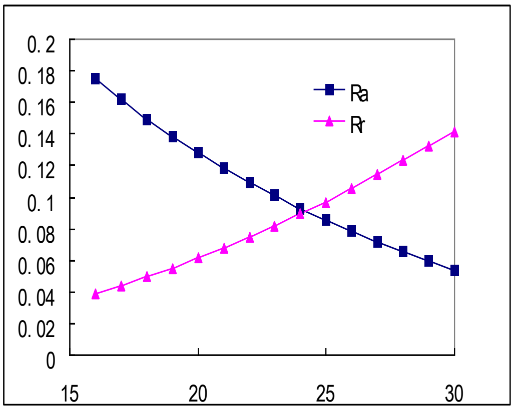


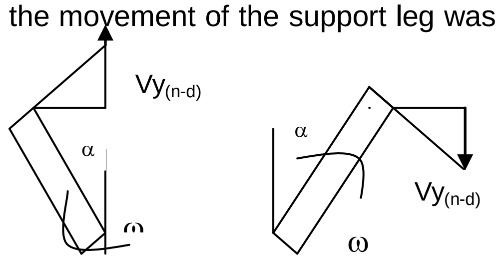
Figure 1 - Ra and Rr at different projectin angle.

Figure 1 shows the men's Rr and Ra, when projection distance was 8m, and the projection angle increased from 15-30°. The average projection angle of Chinese elite athletes was 19°.

At that point, Rr =0.055m/s was far smaller than Ra=0.138 m/s. Using the regression equation, the optimum projection angles were obtained (23.6 - 24.6° for males, 21.5 - 23.3° for females. It was concluded that an increase in projection angle, at the expense of a normal loss of projection velocity, would benefit the projection distance according to the capability of Chinese elite long jumpers.

Comparing vertical velocity of the hip with vertical velocity of center of gravity, it was found that the movement of the support leg provided 80% of vertical velocity of center of gravity. Therefore, using the relative velocity of each segment, the movement of the support leg was analyzed. Every relative velocity can be written as:

$$V_{y(n-d)} = -1\omega \sin\alpha \quad (6)$$



(a) (b)

Where, ω is negative because of the clockwise rotation of each segment; α is the angle between vertical line and segment. As showed in Figure 2 (a), α is positive; As showed in Figure 2(b), the segment exceeded the vertical line, α is negative. Because 1 is fixed, only ω and α can affect V_y.

To characterize the techniques of take-off with quite large projection angles, male athletes were divided into two groups: Group A with large angles of projection and Group B with small angles of projection. Their average angles were 21.1° and 18.3°, respectively. Table 2 shows their relative velocity at take-off. The most difference came from $V_{y(k-a)}$. As shown in Table 2.3, there was a great difference between the two group's segment angle of the shank, which caused the difference of $V_{y(k-a)}$ and V_y.(sin(-21.7°)=-0.369, sin(-33.5°)=-0.552). The extremely small segment angles of the shank at take-off were caused by extremely small segment angles of the shank at touchdown. On the other hand, To increase the projection angle, long jumper must have larger segment angles of the shank, and stronger ability to resist impulse force during the support phase.

Table 2 Relative Velocity at Take-Off (m/s)

	V_y	V_{yh}	$V_{y(h\text{-}k)}$	$V_{y(k\text{-}a)}$	$V_{y(a\text{-}t)}$	V_{yt}
Group A	3.45	2.847	-0.303	-0.961	1.478	2.628
Group B	2.85	2.327	-0.397	-1.366	1.357	2.733
Difference	0.6	0.52	0.094	0.405	0.121	-0.105

Table 3 Segment Angle of Group A (°)

Time	Thigh	Shank	Foot
Touchdown	42.6	23.6	76
Maximum knee flexion	26.8	-6.5	68
Take-off	-12.8	-21.7	39.9

Table 4 Segment Angle of Group B (°)

Time	Thigh	Shank	Foot
Touchdown	38.8	18.2	73.2
Maximum knee flexion	16.8	-20.8	55.5
Take-off	-17.4	-33.5	19.6

The landing motion was also analyzed. An active landing helps to minimize the loss in the horizontal velocity of athlete's center of gravity (Timothy & Hay, 1992). Elite athletes whose projection angle was large didn't sacrifice active landing for large segment angles of the shank. Before active landing, Group A's segment angle of the shank was larger than those of Group B. (31.5°and 25.3°) and at touchdown, two groups' relative velocities $V_{x\,(a\text{-}k)}$ and $V_{x\,(k\text{-}h)}$ are similar (5.85m/s, 1.14m/s and 5.7m/s, 1.05m/s). According to equation (6):

$$V_{x(a\text{-}k)} = 1\cos\alpha$$

because the more α is, the lesser $\cos\alpha$ is, to obtain same $V_{x(a\text{-}k)}$, higher ω is necessary. In other words, long jumpers need stronger flexors of the knee to land more quickly when they have larger segment angles of the shank.

CONCLUSION: In order to improve the performance, Chinese elite long jumpers should increase their projection angle, which could benefit projection distance. To increase projection angle, they should: (1) extend their take-off leg forward before active landing to avoid extremely small segment angles of the shank; (2) have stronger flexors of the knee to increase angular velocity of flexion knee.

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