# A MODELLING METHOD FOR DISCRETE LOW SAMPLING FREQUENCY TEMPORAL SERIES ON THE EVALUATION OF INTRA- CYCLIC SWIMMING SPEED FLUCTUATIONS 

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## INTRODUCTION

Profiles of intra-cycle swimming speed fluctuations has been widely used as a highly informative parameter on swimming biomechanics. Methods described in literature include: (i) free swimming and (ii) linked swimming approaches.

In a previous study (Vilas-Boas. 1992), we described a photo-optical method for the assessment of free swimming intra-cyclic velocity fluctuations. This method was characterized by a low sampling frequency and by a least adequate mathematical modelling method, since conventional polynomial regressions do not fully respect the cyclicity of the phenomena itself: both extremes of the model curve don't fit with each other, imposing-sudden velocity descontinuities that could only be explained through infinite accelerations and forses.
The purpose of this paper is'to describe a modelling method for discrete intra cycle velocity fluctuation analysis with reduced sampling frequencies.

## METHODOLOGY

Data acquisition: Intra-cyclic velocity (v) / time (t) pairs of values were obtained from a intermittent light-trace photographic method (Vilas-Boas, 1992). The method consists in the photographic registration. with prolonged exposure, of the trace produced by a pulse-light device attached to the waist of the swimmer, at a middle distance between the two hip joints. Photos (Canon T70, 35mm, Kodacolor 1000 ASA film) w.re digitized using a Calcamp digitizing table, the Sigma Scan software and a PC computer.
The modelling method: The first step consists in the superimposition of three consecutive breaststroke cycles, sampled with reduced frequency. This was performed subtracting, or adding, a estimated cycle period (T) to each $t$ value of the extreme cycles sampled in each photograph. The initial T value was estimated from the time interval between two consecutive absolute v minimums. This first step was performed in order to: (i) increase the number of points to define the final model; and (ii) allow the individual model of the stroke cycle velocity / time curve to be calculated on more than one isolated stroke cycle. Once translation was accomplished, one first 8th degree polynomial regression was calculated:


In order to allow the polynomial equation to respect the cyclical nature of the phenomena, two constrains were imposed to the regression, both on the initial $(t=0)$ and the final $(\mathrm{t}=\mathrm{T})$ instants of the stroke cycle model:
(i)
$v(0)=v(T)$
and
(ii)

$$
\left.\frac{d v}{d t}\right|_{t=0}=\left.\frac{d v}{d t}\right|_{t=T}
$$

This was accomplished as follows:
Taking into account the equation (1),

$$
\begin{equation*}
\mathrm{v}(0)=\mathrm{a} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
v(T)=a+{ }_{i=1}^{1} b_{i} * T \tag{5}
\end{equation*}
$$

Changing (4) and (5) into equation (2),

$$
\begin{equation*}
a=a+{ }_{i=1}^{1} b_{i *} T \tag{6}
\end{equation*}
$$

then:

$$
\begin{equation*}
{ }_{i=1}^{l} b_{i} * T^{1}= \tag{7}
\end{equation*}
$$

In the other hand, once:

$$
\frac{d v}{d t}={ }_{i=1}^{I} i * b_{1} * T^{-1}=b_{1}+2 b_{2} t+b_{3} t^{2}+\ldots+I * b_{1} * t^{I-1}
$$

(8)
the first derivatives of the velocity in order of time for the first $(t=0)$ and last $(t=T)$ points of the stroke cycle can be described as follows:

$$
\begin{equation*}
\left.\frac{d v}{d t}\right|_{t=0}=b_{1} \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{d y}{d t}\right|_{t=T}=b_{1}+{ }_{i=2}^{I} i * b_{i} * T^{-1} \tag{10}
\end{equation*}
$$

Considering the second imposed constraint (3), is possible to note that:

$$
\begin{equation*}
b_{1}=b_{1}+\sum_{i=2}^{i} i b_{i} * T^{-1} \tag{11}
\end{equation*}
$$

and then:

Starting from (12) is then possible to calculate the $\mathbf{b}_{\mathbf{l}}$ coefficient of the regression equation, taking into account the optimized estimations of the coefficients $\mathbf{b}_{\mathbf{2}}$ to $\mathbf{b}_{\mathbf{I}-\mathbf{1}}$, performed through the Marquardt (1963) algorithm.
If:

$$
\begin{equation*}
2 \mathbf{b}_{2} T+\mathbf{b}_{3} \mathrm{~T}^{2}+\ldots+\mathrm{b}_{1} \mathrm{~T}^{\mathrm{T}-1}= \tag{13}
\end{equation*}
$$

then:
$2 b_{2} T+b_{3} T^{2}+\ldots+(I-1) * b_{1-1} T^{1-2}+I *\left(-\frac{b_{i}}{I-1} \frac{\mathrm{~b}_{1}}{T^{1-1}}\right) T^{\mathrm{T}-1}=$
It means:
$2 b_{2} \mathrm{~T} \quad 3 \mathrm{~b}_{3} \mathrm{~T}^{2} \ldots \quad(\mathrm{I}-1) \quad \mathrm{b}_{\mathrm{I}-\mathrm{I}^{\mathrm{T}}} \mathrm{T}^{\mathrm{T}}-1\left(\frac{\mathrm{~b}_{1}}{\mathrm{~T}^{\mathrm{I}-1}} \frac{\mathrm{~b}_{2}}{\mathrm{~T}^{\mathrm{I}-2}} \cdots \quad \frac{\mathrm{~b}_{\mathrm{I}-1}}{\mathrm{~T}}\right) \mathrm{T}^{\mathrm{I}-1}=$
or:
$2 \mathrm{~b}_{2} \mathrm{~T} \quad 3 \mathrm{~b}_{3} \mathrm{~T}^{2} \cdots(\mathrm{I}-1) \quad \mathrm{b}_{\mathrm{I}-1} \mathrm{~T}^{\mathrm{I}-2}-\left(\mathrm{Ib}_{1} \quad \mathrm{I} \mathrm{b}_{2} \mathrm{~T} \quad \cdots \quad \mathrm{Ib} \mathrm{l}_{\mathrm{I}-1} \mathrm{~T}^{\mathrm{I}-2}\right)=$
Working on, we obtain:

$$
\begin{equation*}
-\mathrm{Ib}_{1}(2-1) \mathrm{b}_{2} \mathrm{~T}(3-\mathrm{I}) \mathrm{b}_{3} \mathrm{~T}^{2} \cdots(-1) \mathrm{b}_{\mathrm{I}-1} \mathrm{~T}^{\mathrm{I}-2}= \tag{16}
\end{equation*}
$$

and:

$$
\begin{equation*}
\mathrm{Ib}_{1}(\mathrm{I}-2) \mathrm{b}_{2}^{\mathrm{T}}(\mathrm{I}-3) \mathrm{b}_{3} \mathrm{~T}^{2} \ldots \mathrm{~b}_{\mathrm{I}-1} \mathrm{~T}^{\mathrm{I}-2}= \tag{17}
\end{equation*}
$$

Developing in order to bl we have:

$$
\begin{equation*}
b_{1}=\frac{1}{I}\left[(I-2) b_{2} T+(I-3) b_{3} \mathbf{T}^{2}+\ldots+b_{I-1} T^{T-2}\right. \tag{1.9}
\end{equation*}
$$

or:

$$
\begin{equation*}
b_{1}=\frac{1}{I}_{i=2}^{I-1}(I-1) b_{i} T^{1-1} \tag{20}
\end{equation*}
$$

Starting from (11) it is also possible to calculate the last coefficient of the regression equation (bI).
If:

$$
\begin{equation*}
\mathbf{b}_{1} \mathbf{T}+\mathbf{b}_{2} \mathbf{T}^{2}+\ldots+\mathbf{b}_{\mathrm{I}-1} \mathrm{TI}= \tag{21}
\end{equation*}
$$

then:

$$
\begin{equation*}
b_{1}=-\frac{1}{T^{( }}\left(b_{1} T+b_{2} T^{2}+\ldots+b_{I-1} T^{I-1}\right) \tag{22}
\end{equation*}
$$

and:

## RESULTS AND DISCUSSION

Figure 1 presents velocity / time curves for three breaststroke techniques performed by 6 Portuguese male swimmers at 200 m race pace. It is possible to note the general coherence of the models.


Figure 1. Individual velocity / time curves obtained for 6 swimmers, each one performing three breaststroke techniques at 200 m race pace.
Using PC-Matlab (3.13) for integration and derivation of the special polynomial equations, it is possible to assess acceleration curves, and per phase resultant impulses (Vilas-Boas, 1994), as well as duration and horizontal distance covered per phase.
Results were highly compatible with previous reports (Vilas-Boas, 1993) and pointed out that: (i) mean minimum velocity associaied with the recovery of the legs (vl) was $.40(\mathrm{SD}=.035) \mathrm{m} . \mathrm{sec}^{-1}$; (ii) mean maximal velocity associated with the leg kick (v2)
was 1.43 ( $\mathrm{SD}=.039$ ) $\mathbf{m} . \mathrm{sec}^{-1}$; (iii) mean minimum intermediate velocity associated with the transition phase between leg and arm strokes (v3) was 1.07 ( $\mathrm{SD}=.027$ ) m.sec$\mathbf{1}$; (iii) mean peak velocity associated with the armstroke (v4) was 1.26 ( $\mathrm{SD}=.038$ ) $\mathbf{m} . \mathbf{s e c}^{-1}$; (iv) mean acceleration and resultant impulse between v 1 and $\mathbf{v} \mathbf{2}$ were 3.03 ( $\mathrm{SD}=. \mathbf{3 1 4}$ ) $\mathbf{m} . \mathrm{sec}^{-2}$ and $61.40(\mathrm{SD}=3.726) \mathrm{Ns} ;(\mathrm{v})$ between $\mathbf{v} \mathbf{2}$ and $\mathbf{v} \mathbf{3}$ were -1.08 $\mathbf{m} . \mathbf{s e c}^{-2}$ and $-21.43(\mathrm{SD}=3.478) \mathrm{Ns}$; (vi) between $\mathbf{v 3}$ and $\mathbf{v} 4$ were $.69(\mathrm{SD}=.084)$ $\mathbf{m} . \mathbf{s e c}^{-2}$ and $11.32(\mathrm{SD}=1.853) \mathrm{Ns}$ and (vii) between $\mathbf{v} \mathbf{4}$ and $\mathrm{vl}^{\prime}$ were $-2.24(\mathrm{SD}=$ $.026) \mathrm{m} . \mathrm{sec}^{-2}$ and $-51.13(\mathrm{SD}=.962) \mathrm{Ns}$.

## CONCLUSIONS

Results of this study showed that the described mathematical method is feasible for modelling discrete low sampling frequency velocity / time curves of breaststroke swimmers. Nevertheless, further research on procedure fidelity should be conducted in the future.

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