Computational Asoects of 3-dimensional Kinematic Analysis

V. Kohl
Institute of Physics, Czcchoslovak Acadcmy of Scicnccs, Praguc, ČSSR

The biomechanical research group of the Laboratory of elite athletics and the Faculty of Sport in Prague is engaged in the kinematic analysis of the top performances in track and field events from film close-ups for purpose of studying the differences in style of individual athletes.

The aim of this paper is an analysis of the accuracy in individual steps of the processing data. The following hardware is available:

— two phase-locked high speed cameras Photosonics
— film projector
— manual digitizer
— personal computer SHARP MZ 3500

The paper is divided according to the following steps of measurement and processing:

— location of film cameras
— determination of a scale
— digitization
— primary processing data
— calculation of spatial coordinates
— calculation of kinematic quantities

Besides the steps mentioned there are other sources of error, particularly the round-off error of arithmetic operations and the error of the optical systems of both film cameras and the film projector. The aberrations of objectives, even if they belong to some profound errors are not analyzed here.

The imaging of both optical systems we regard as a «black box» even if some of the aberrations, particularly the distortion of image (barell error) can be easily eliminated numericaly. It would be a further reason why it is necessary to determine an axis of the objective during the step of digitalization.

The round-off errors are not considered because, when calculating in FORTRAN with 6 decimal digits or in BASIC programming language with 10 decimal digits the errors are far below the values of the other errors.

LOCATION OF FILM CAMERAS

There is no special reason for the placing of the film cameras in two perpendicular directions or at the same height. Usually the cameras are located in the distance 20 times greater than the real size of the space of the athletic performance (e.g. throwing circle or hurdle). From the similarity of triangles we see that the error in determination of the position of the film cameras and their leveling is therefore 20 times smaller.

If the optical axes are diverted from the ideal perpendicular and horizontal directions more than 10°, the elimination of the distortion of distances near the margin of the image field is advisable. For the sake of illustration I present an example using an objective with focal length of 120 mm: The angular distance between the optical axis and the margin of the window is 2.5°. (Fig. 1).

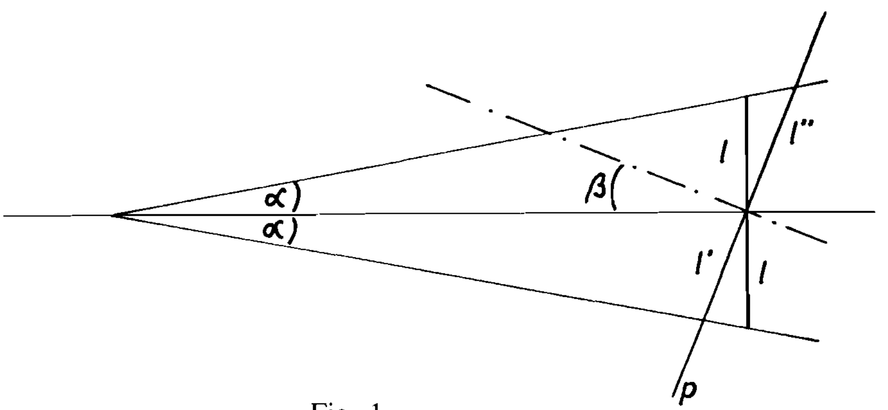

Fig. 1.

where p is object plane
a is angular distance of the objective (2.5°)
β is angle given by the location of a camera
Using the sine theorem for a triangle we can obtain:

$$l' = l\,\frac{\sin(\pi/2-a)}{\sin(\pi/2+a-\beta)} \qquad \text{and} \qquad l'' = l\,\frac{\sin(\pi/2+a)}{\sin(\pi/2-a-\beta)}$$

In the following table the values of the relative length distortion:

$$d(\beta) = (l'' - l') / l$$

are given (in per cent):

TABLE 1

β	(°)	0	10	20	30	40
d(β)	(%)	0	1.5	3.7	5.8	9.6

DETERMINATION OF THE IMAGE SCALE

The previous correction should be involved in the determination of the scale of the close-up. The easiest way to determine the scale is by filming a calibration pattern, (e.g. 1 m-length normal or the diameter of the throwing circle). There is a danger that the calibration pattern is not located perpendicular to the optical axis of the camera or located near the margin of the image field window. In the former case the error is proportional to the cosine of the angular deviation:

TABLE 2

β	(°)	1	2	5	10	20
ε	(%)	0.02	0.06	0.4	1.5	6.0

The latter case is discussed above.

DIGITIZATION OF FILM CLOSE-UP

Several factors must be considered in the process of digitization of the coordinates of anthropometric points, particularly:
— resolution power of the film material
— magnification of the film projector

— operator's capabilities
— resolution power of the digitization device.

The typical resolution power of the film material used is 100 lines/mm (or more precisely 100 boundary lines/mm), then for the size of the film window 7.5 × 10.4 mm it gives 1000 lines/window and if the magnification of the film projector is 50 times we get the final resolution in the image plane (of the digitizer) approximately 1000/500 mm.

A trained human eye is able to distinguish 4 lines/mm. Then the resolution power of a human eye is better than the resolution power of the film material on using such a magnification. But the random error of an operator with the reading measurement of individual anthropometric points remains as the greatest problem. According to the experience of the Prague Laboratory the random errors of the operator can be up to 2 mm, partly due to cursor paralaxis. These 2 mm represent 5 to 25 mm in real size coordinates, depending on the distance of the camera from subject.

Besides the errors already mentioned there is a problem determining the points that are only partially visible, or not visible in the measured image.

To recover such a single missing point we apply the 3-rd order interpolation or the 2-nd order extrapolation in case where the missing point is an extreme one.

By using the Lagrange interpolation formula for the equidistant nodes of the diagram:

$$\overline{t_1 \qquad t_2 \qquad t_3 \qquad t_4 \qquad t_5}$$

a very simple expression for the value of the missing point (t_3) is obtained:

$$f_3 = \frac{1}{6}(4f_2 + 4f_4 - f_1 - f_5) \qquad \text{where } f_3 = f(t_3)$$

We get a similarly simple expression when we use values in 3 adjacent equidistant nodes of the diagram:

$$\overline{t_1 \qquad t_2 \qquad t_3 \qquad t_4}$$

The expression has the form:

$$f_1 = (3f_2 - 3f_3 + f_4)$$

The analog/digital converter of our measurement device has the range of 12 bits, then its resolution is 8 times better than the resolution of the film material. The step of the digitization should include a numerical correction for optical aberrations of the objective of the projector.

PRIMARY PROCESSING COORDINATES

The time sequence of read (measured) coordinates is generaly not sufficiently smooth due to random errors in the process of digitalization. Therefore a smoothing of the coordinates is necessary. In principle two methods are possible:

1) to smooth each from 4 coordinates sequences (2 from each projection) independently,
2) to smooth coordinate sequences with respect to the adjoining ones, either both coordinate sequences on each projection, or first, to perform a spatial reconstruction and then to smooth all three spatial coordinates at once.

The second method is more difficult with the known subroutines very slow, large and clumsy.

Several different ways for smoothing a single coordinates sequence can be used:

1) 3-points or 5-points Gramm formulae
2) a digital filter
3) a smoothing by means of spline functions.

Each method has its advantages and drawbacks. The three methods demand a sufficient density of values. The requirement is in contradiction with limitations created by manual digitalization, requirement for speed of processing, and memory of the computer.

At this point it is necessary to draw attention to a serious disadvantage of a personal computer programmed in BASIC (that stores a floating-point number in 8 Bytes) in comparison with programs written in FORTRAN or other programming language that uses only 4 Bytes of memory. A single calculation can demonstrate the memory requirements: A human body is represented with 23 anthropometric points, each of two projections consists of 2 coordinates/point, that means:

$$23 \times 4 \times 8 = 736 \text{ Bytes / frame}$$

On the other hand the smoothing by Gramm formulae is very simple, quick and does not require any auxiliary arrays.

The most convenient digital filter is the low-pass Butterworth filter of the 2-nd order. This filter is utilised for suppression of the higher frequency part of a signal (e.g. noise). In our case the measured coordinates are considered as the sum of significant values and random errors that represent the signal of higher frequencies. Smoothing by the Butterworth filter may be used repeatedly.

The known subroutine SMOOTH by Ch. Reinsch or some of its modification is useful for a smoothing by means of spline functions. The level of smoothing, i.e. the number S must be given as an input data for the subroutine. The meaning of the value S is:

$$S = \sum_{i=1}^{N} (f_i - \bar{f}_i)^2$$

where N is the number of values in the sequence and $\bar{f}_i$ is the smoothed value of the function in the node i.

In comparison with the previous methods the spline smoothing requires a memory for working arrays but on the other side enables the smoothing of sequence with non-equidistant nodes. The method seems to be the best from the mathematical point of wiew since it gives a polynomial of the 3-rd order, unfortunately it is also the most complicated and the slowest.

For illustration I present the times necessary for the smoothing by individual methods (only relative values):

3-points Gramm formula	1.6
5-points Gramm formula	2.6
double low-pass Butterworth filter	10.6
subroutine SMOOTH (own modification)	69.3

It is difficult to compare the individual methods. Generally it is possible to assert that the smoothing of a separate sequence of coordinates is not sufficient if the sequence is not dense enough. It demonstrates the following example: The coordinates of 2 anthropometric points (left elbow and wrist) were randomly choosen from 14 frames. The coordinates were smoothed by the 3-points Gramm formula and by the double Butterworth filter, then their distance (i.e. the length of a forearm) calculated. The following table shows a great dispersion of the calculated values at individual frames (in mm):

TABLE 3
DISPERSION OF CALCULATED VALUES IN MM

frame	no smooth.	3-p. Gramm	Butterw.
1	324	318	323
2	263	274	285
3	255	272	277
4	317	287	290
5	305	292	297
6	283	277	290
.....			
13	338	282	283
14	286	297	288
∅	286.3	275.9	280.2
σ	7.6	5.1	4.8

In the second to the last line is the arithmetic average while the last line has the quadratic mean deviation. The maximal deviations in several columns are 18, 15 and 15%.

From the example it can be seen that even smoothing procedures are unable to eliminate the random errors due to the operator's mistaken reading if a sequence is not dense enough (e.g. 50 frames/s in this example). Therefore the measured coordinates must be checked by a computer and their measurement must be repeated if necessary. More convenient is the reading coordinates on a device connected on-line to a computer. I regard this step as a most complicated part of the whole process.

3-DIMENSIONAL (SPATIAL) RECONSTRUCTION

It is useful to apply a method of analytical geometry either plane or in space for calculation of spatial coordinates. The former can be used if the elevation differences between the film cameras with respect to the central point of the athletic performance are small. The point is usually an assumed center of mass (CM) of an athlete's body. The optical axes of both film cameras should be directed to the athlete's CM, otherwise it is necessary to apply the correction of coordinates mentioned above.

If the optical axes are not horizontal or are not perpendicular to each other it is necessary to rectify coordinates, i.e. to apply the correction of the division by the cosine of the angular deviation.

A spatial point is determined as an intersection of two straight lines. Vectors of the straight lines are the differences between the coordinates of a camera and the coordinates measured from the film. In the general case the straight lines have no intersection and it may be replaced by the center of their shortest distance. The magnitude of the distance can be used to check coordinates from both projections.

If the optical axes of the film cameras are horizontal, the shortest distance is always vertical. Then it is possible to perform only plane calculation of the intersection in a horizontal plane and to calculate a vertical spatial coordinate as an arithmetic average.

CALCULATION OF KINEMATIC QUANTITIES

Centers of mass of segments of an athlete's body are calculated according to formulae:

$$X_{T(i,j)} = X_i + c_{T(i,j)} (X_j - X_i)$$

where $c_{T(i,j)}$ are coefficients describing the segment (i, j).

Other derived quantities are velocities and accelerations of anthropometric points and centers of mass, several angles, distances, etc. either in original time nodes or between them, and besides that also tangential and normal quantities for throwing events.

An interpolation formula of the 3-rd order is convenient for calculation of mentioned quantities in an arbitrary time instant (node). Because the values are known in equidistant time nodes, the formula has the following simple forms:

for the diagram:

$$\overline{\quad t_1 \qquad t_2 \quad t \qquad t_3 \qquad t_4 \quad}$$

$$P_3(t) = -\frac{f_1}{6h^3}(\tau^3 - 6\tau^2 h + 11\tau h^2 - 6h^3) + \frac{f_2}{2h^3}(\tau^3 - 5\tau^2 h + 6\tau h^2) -$$

$$-\frac{f_3}{2h^3}(\tau^3 - 4\tau^2 h + 3\tau h^2) + \frac{f_4}{6h^3}(\tau^3 - 3\tau^2 h + 2\tau h^2)$$

where $h = t_2 - t_1$, etc. is the equidistant time step or better the inverse value of the frequency of frames; $\tau = t - t_1$

or for the following diagram:

$$\overline{t_1 \quad t_2 \quad t \quad t_4 \quad t_5}$$

$$Q_3(t) = \frac{-f_1}{12h^3}(\tau^3 - 8\tau^2 h + 19\tau h^2 - 12h^3) + \frac{f_2}{6h^3}(\tau^3 - 7\tau^2 h + 12\tau h^2) -$$

$$- \frac{f_4}{6h^3}(\tau^3 - 5\tau h^2 + 4\tau h^2) + \frac{f_5}{12h^3}(\tau^3 - 4\tau^2 h + 3\tau h^2)$$

By the differentiation of the formula $P_3(t)$ we obtain:

$$P'_3(t) = \frac{-f_1}{6h^3}(3\tau^2 - 12\tau h + 11h^2) + \frac{f_2}{2h^3}(3\tau^2 - 10\tau h + 2h^2) -$$

$$- \frac{f_3}{2h^3}(3\tau^2 - 8\tau h + 3h^2) + \frac{f_4}{6h^3}(3\tau^2 - 6\tau h + 2h^2)$$

for the calculation of velocities and

$$P''_3(t) = \frac{-f_1}{h^3}(\tau - 2h) + \frac{f_2}{h^3}(3\tau - 5h) - \frac{f_3}{h^3}(3\tau - 4h) + \frac{f_4}{h^3}(\tau - h)$$

for the calculation of accelerations.

Even simpler are the formulae for calculation $Q_3(t_3)$, $Q'_3(t_3)$ and $Q''_3(t_3)$ for the value of the left-out point $t_3 = (t_2 + t_4)/2$,
e.g. $Q''_3(t_3) = (f_1 - f_2 - f_4 + f_5) / 3h^2$

The calculation of the value $P_3(t)$ or $Q_3(t)$ suffers from the error of an interpolation, in our case:

$$E(t) = f^{(iv)}(\xi)\prod(t)/24$$

where the function:

$$\prod(t) = (t - t_1)(t - t_2) \ldots (t - t_{max})$$

vanishes in arbitrary point $t_1, t_2, \ldots, t_{max}$
and $f^{(iv)}(\xi)$ is the value of the 4-th derivative in some ξ in the interval $< t_1, t_{max} >$.

The error of the derived quantities calculated by means of the differentiated functions can be estimated similarly, only the function $\prod(t)$ must be replaced by its derivative.

A great part of the errors that occur during the processing of the film close-ups cannot be evaluated analyticaly, but it is possible to determine them experimentaly, i.e.

1. to repeat the individual steps, including the placing and the adjusting of the hardware and the measurement
2. to change and compare computational methods
3. to calculate errors statisticaly, e.g. the mean quadratic deviation.

As soon as we have values of the errors, their propagation during further computation can be calculated by means of the known formula:

$$\delta f(x_1, \ldots, x_n) = \sum_{i=1}^{n} E_i \frac{\partial f}{\partial x_1}$$

where E_i is the error of the variable x_i, f is a function of n variables $x_i, \ldots, x_n$ and δf is the resulting error.

It can be concluded that processing the film close-ups cannot be considered as final. Besides the errors of used optical systems not discussed here the greatest drawback occur predominantly in the process of point reading and in checking and smoothing measured data.

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