Mathematical Correction of the Seismic Wave Propagation

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Efforts to describe objectively the sports activity connected with a top performance lead to the search of new methods. No single method gives a complex picture of a sports activity, it only characterizes it from a special point of view. In sports events, when it is not possible to load the athletes with technical equipment fixed to the body, the set of methods describing the sports activity gets very narrow. Mentioned among these methods could be the seismography utilizing the possibility of earth shock measurements. An analogy can be found between the earth shocks and the shoks occuring during the interaction of the body and the earth support. In compliance with the Huygens principle, the seismic waves propagates in all directions from the place of impact. By seismic wave indication in time, from sensors of defined location it is possible to determine the interaction moments. The quality of the support determines the seismic wave propagation velocity. The homogeneity of the support determines the quality of the recorded signal or the general application of this methodology. The seismic wave propagation velocity ranges in function of the support quality from 500-2000 m/s. The propagation velocities are known for different materials; in spite of that it is not possible to take these values for granted for control. Before any measurement, it is necessary to determine the immediate velocity value. The entire set of problems is related to interferences of seismic waves caused by the non-homogeneous nature of the environment. It is practically impossible to eliminate the interference, i.e. to describe theoretically individual components of the resulting signal. However, it was experimentally proved that the first marked extremes of the recorded courses carry the most accurate information about the time of their onset.

The seismic wave is indicated by a sensor, by an accelerometer sensitive in one direction. Since the seismic wave has predominantly the character or transverse oscillation, the sensitive sensor axis is vertical.

The accelerometer measures the acceleration of the support it is placed on. The recorded signal has the form of damped oscillation whose amplitude is determined by the shock intensity, the distance of the sensor from the shock centre and by the support quality. The support quality determines both, the amplitude and its damping frequency. It follows from the final seismic wave propagation velocity and the non-zero distance between the sensor and the shock sensor. This delay is negligible. It reaches the values of units up to tens of milliseconds.

Knowing the propagation velocity and the sensors location it is possible to eliminate the propagation time and to correct thus the measured time data. The correction is possible only provided the same shock is indicated at least by two sensors from different places. The measured time data lagg behind the real shock onset. Their differences provide information about the relation of the seismic wave propagation times and the shock place to the sensors. In the case of a simplified assumption that three sensors are placed in the same line of the followed action. If the shock occurred outside the sensors, the time difference between the seismic wave arrival to individual sensors is constant. Time correction cannot be determined. In the function of the sensor activation it is only possible to determine whether the shock occurred in front of or behind the sensors. Correction can be carried out if the shock took place between the sensors — as indicated on Figure 1. Since the shock occurs sooner, it is necessary to subtract time differences in relation to other sensors.

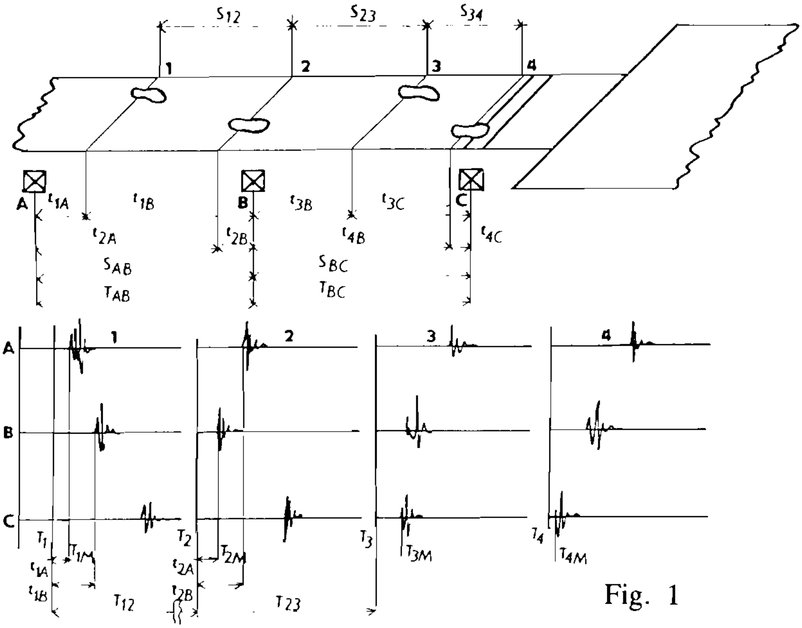

Fig. 1

For the seismic wave propagation velocity it holds:

$$v = \frac{S_{AB}}{T_{AB}} = \frac{S_{BC}}{T_{BC}} \tag{1}$$

For the real interaction time it holds:

$$T_1 = T_{1M} - T_{1A} = T_{1M} - \frac{T_{AB} - \Delta^t_{AB}}{2} \tag{2}$$

It holds similarly for the remaining interactions — step — downs:

For the time interval, i.e. for the step frequency, the expression 3 holds:

$$F_{12} = \frac{1}{T_{12}} \tag{3}$$

If the step-downs occur between the same sensors, the eq. 4 holds:

$$T_{12} - T_2 - T_1 = T_2 - T_{2M} - T_1 + T_{1M}$$

$$T_{12} = T_{2M} - T_{1M} - \frac{T_{AB} - \Delta^t_{1AB}}{2} + \frac{T_{AB} - \Delta^t_{2AB}}{2} \tag{4}$$

$$T_{12} = T_{2M} - T_{1M} + \frac{\Delta^t_{1AB} - \Delta^t_{2AB}}{2}$$

If the step-downs occur between different sensors, eq. 5 holds:

$$T_{23} = T_{3M} - T_{2M} + \frac{T_{AB} - \Delta^t_{2AB} - T_{BC} + \Delta^t_{3BC}}{2} \ / \ s \ / \tag{5}$$

For equidistant sensors $S_{A3} = S_{BC}$ at constant propagation velocity it holds: $T_{AB} = T_{BC}$.

Then 5 after modification is equal to 4.

It follows from the preceding equations:

In case of equidistant sensors, correction of time difference between step-downs can be carried out without knowing the seismic wave propagation velocity.

With a sufficient sensor sensitivity it is suitable to use only two sensors placed outside the time corrected step-downs.

The step frequency calculated from the measured times differs significantly from the step frequency calculated from the time corrected data. If the step frequency of the last three steps before the take-off lies within the range of 3-7 steps/s, the correction is only a few tenths of a step.

If the followed action does not take place in a straight line but rather in a plane, the preceding simplifying conditions are not satisfactory. The time correction of the seismic wave propagation is, however, possible even in the plane. The plane is defined by three sensor points. They are distributed in the plane so that the action takes place between them. This condition, however, needs not to be categorically satisfied since, unlike the straight line correction, in the plane it is possible to correct the time data outside the triangle delimited by the sensors.

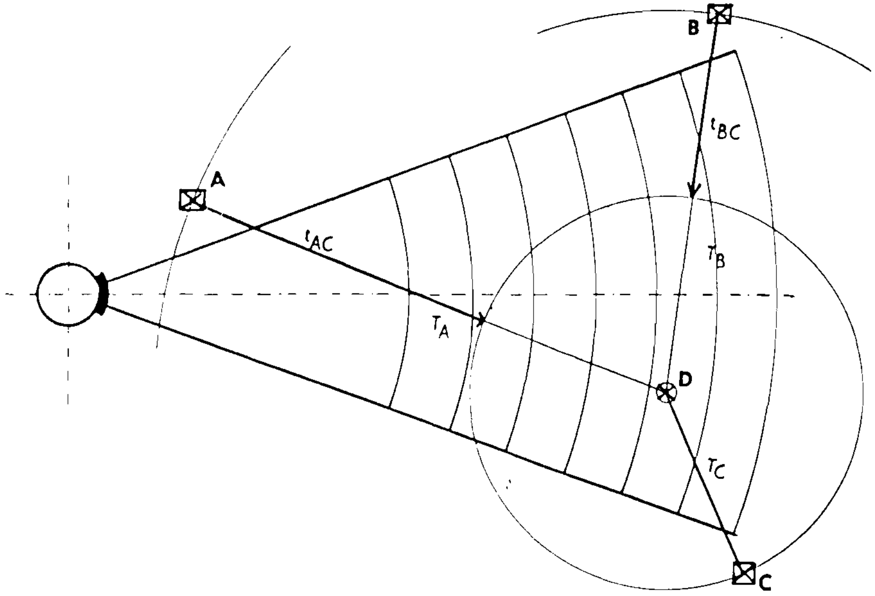

Fig. 2

From the place of the impact, the seismic wave propagates in all directions within an area characterized by a circle (Figure 2). With the exception of one case, the impact has a gradual effect on individual sensors. The shock precedes the time of the first sensor response. Knowing the seismic wave propagation velocity, it is possible to derive the time correction from the difference in reaction time between the sensors 1 and 2 and the sensors 1 and 3, respectively. The measured time will be corrected by the time value necessary for the seismic wave transition from the shock place to the nearest sensor. This value is characterized by the circle radius with a centre at the place of the sensor.

By solving a set of three circle equations with centres at the places of the sensors, we can obtain the intersection of the above circles. Such a procedure is possible, but rather complicated. For that reason, a mathematical procedure was chosen. The basic idea of such an iteration is a comparison of real and model time differences. On the basis of such differences, triangles are designed whose gravity points approache the place of the impact (Figure 3).

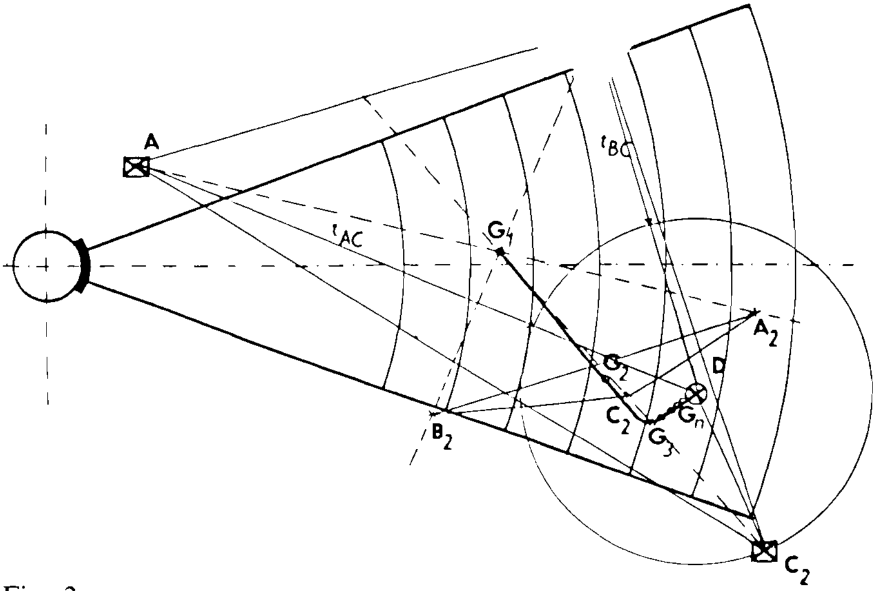

Fig. 3

Theoretically, it is possible to determine the shock place knowing the time difference between individual sensors activation. The precision of time determination, based on seismographic records, restricts shock origin place to be determined more accurately than to the tenths of centimeters. The seismographic method using the mathematical correction makes the determination of the shock time more accurate.