# THE MUSCLE CONTRACTION AND THE FORCE PRODUCTION

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We study the muscular force during contraction via linear and quadratic approximations of the resistance force. Using theoretical - mechanics arguments, we shall determine the relevant parameters featuring in the equation of motion. This is very important for the determination of the type of training practice which will transform the muscle for various types of dealing with the medium resistance.

**KEY WORDS:** muscle force, muscle contraction, linear approximation, quadratic approximation

**INTRODUCTION:** The muscle represents an elastic medium whose properties can be changed with the change of temperature, load, fatigue etc., (Pohl, 1955). While confronting the medium resistance, the muscle contracts and the amount of force produced by the muscle is proportional to the contraction. We shall define the muscular contraction as the increase of muscular length in time:

$$y(t) = l(t + dt) - l(t) < 0, \quad (1)$$

where $l$ is the actual muscle length. If we denote the length of the muscle at rest by $L$, then it retains its elastic properties as long as the contraction does not reach one third of the length $L$.

The medium resistance is the function of the contraction rate $\frac{dy}{dx} \equiv \dot{y}$. In the general case, resistance force can be represented in the form of homogeneous infinite series of' rate powers:

$$R = a_1 \dot{y} + a_2 \dot{y}^2 + a_3 \dot{y}^3 + \ldots.., \quad (2)$$

where the linear term is dominant over all other terms in (2) (Pohl, 1955).
Obviously, the medium resistance and the muscular force are oppositely directed.
The resistance of the medium will be treated in the approximation both linear and quadratic in terms of contraction rate. Quadratic approximations leads to more realistic results. Within this approximation one encounters precontraction period which till now lacked the theoretical explanation. Following the laws of the theoretical mechanics, the proportionality coefficient of the resistance force will be determined and it will demonstrate that it depends on intermolecular distances, which can be varied in muscles by physical training. Smaller distances correspond to short - period power production, while longer distances cause the power production in longer time intervals. Theoretical results agree well with experiments performed.
Linear approximation: Within the framework of the linear approximation, using (2) and Newton's second law of motion we have the equation

$$\ddot{y} + \Omega \dot{y} = 0. \quad (3)$$

This equation will be solved with the boundary conditions

$$y(0) = 0; \qquad y(\infty) = -Y. \tag{4}$$

The solution is

$$y(t) = -Y\left(1 - e^{-\Omega t}\right). \tag{5}$$

Since muscle is an elastic medium, the force it produces is proportional to the elongation (contraction in this case). This leads to $F(t) = -Cy(t)$, so that according to (3), we arrive at the expression for the muscle force during contraction

$$F(t) = F_0\left(1 - e^{\Omega t}\right); \qquad F_0 = -CY. \tag{6}$$

One can see from the above result that muscle force increases with time and asymptotically tends towards the maximal values $F_0$.

Another feature of interest is the function $f(t)$ representing time derivative of the muscle force

$$f(t) = \frac{df(t)}{dt} = F_0\Omega\, e^{-\Omega t}. \tag{7}$$

It is obvious that the power produced by muscles depends on $f(t)$.

In the particular case, when $F_{01}\Omega_1 > F_{02}\Omega_2$, it can be easily proved that the characteristic moment of time for the power production is

$$t_0 = \frac{1}{\Omega_1 - \Omega_2} \cdot \ln \frac{F_{01}\Omega_1}{F_{02}\Omega_2}. \tag{8}$$

Within the time range $0 < t < t_0$ power production is higher for higher values of $F_0\Omega\ (f_1 > f_2)$, while in the range $t_0 < t < \infty$, power production increases for lower $F_0\Omega\ (f_1 < f_2)$.

One should also note that power production function $f(t)$, treated as the function of the parameter $\Omega$ has an extremum

$$f_{\max} = \frac{F_0}{et} \tag{9}$$

for $\Omega_0 = \dfrac{1}{t}$.

We have already mentioned in the Introduction that the determination of the parameter $\Omega$ is of great practical importance both for martial arts and sports in general, since by regulating the magnitude of this parameter one can achieve the control of the muscle force according to one's will.

Quadratic approximation. The determination of the precontraction period.

Within the linear approximation, we have assumed that the medium resistance $R = -m\Omega\,\dot{y}$. It turned out in practice that this approximation leads to too high values of the resistance so it should be compensated. This can be achieved by including the quadratic terms too, so that

$R = -m\Omega \dot{y} + a_2 \dot{y}^2$, where $a_2 = mk > 0$. The equation of motion becomes

$$\ddot{y} = -\Omega \dot{y} + k \dot{y}^2 . \tag{10}$$

After a simple procedure we obtain the solution satisfying the boundary conditions (3)

$$y(t) = -Y - \frac{1}{k} \ln\left[1 - \left(1 - e^{-kY}\right) e^{-\omega t}\right] . \tag{11}$$

As previously, we treat the muscle force $F(t)$, as an elastic one. In such a way , we obtain

$$F(t) = F_0 \left( 1 - \frac{1 - e^{-kY}}{kY} e^{-\Omega t} \right) . \tag{12}$$

This leads to the precontraction period

$$T = \frac{1}{\Omega} \ln \frac{kY}{1 - e^{-kY}} . \tag{13}$$

Using the set of parameters $\Omega = 2s^{-1}$, $k = 0.1m^{-1}$ and $Y = 0.2m$ we find the precontraction period to be $T \approx 5ms$ .
This allows us to conclude that the well - known force – time formula for the force results from the quadratic approximation for the medium resistance.

**CONCLUSION:** The muscle contraction was studied theoretically. Medium resistance was treated here both in linear and quadratic approximation in terms of contraction rate. The expressions for the force obtained within above approximation do not differ much and both agree with experiments. However, quadratic approximation should be considered as better since it leads to the appearance of the precontraction period which is missing in the linear approximation.
The most relevant result for practice is the theoretical mechanics based analysis of the proportionality coefficient $\Omega$ in the expression for the medium resistance. This parameter is expressed in terms of the sound velocity in the muscle tissue, masses of the total muscle and a single muscle molecule as well as the average distance of neighboring molecules in the muscle tissue. This average distance can be changed by the physical training (increased or shortened), depending on the particular preparation of the sportsman. If one needs to produce power in shorter time intervals, distance should be shortened (denser muscular tissue) while for long term power production the distance should be increased (muscle tissue less dense).

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