THE BEHAVIOUR OF MUSCLES IN EXTERNAL INSTANTANEOUS FORCE FIELDS

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The purpose of this study is to analyze the behaviour of muscles in an external instantaneous force field. A model is presented which provides a qualitative assessment of what occurs when muscles react to a strong strike or a sudden jerk. In the context of the model, it has been noticed that fine muscles reacted to a strike or jerk differently to massive muscles.

KEY WORDS: muscle deformation and density, dirac function

INTRODUCTION: A muscle is an unstable system whose mechanical characteristics are constantly changing according to temperature change, fatigue, load variation and contraction velocity. Flexibility and the degree of elasticity found in muscular fibres has fixed bounds. A muscular fibre is normally flexible and elastic in bounds of $\pm$ 30 percents of its length in a state of rest. Beyond these limits, it looses these properties and often breaks. Muscle viscosity enhances according to contraction velocity and it is manifested as a resistance to contraction. Resistance reduces its contraction velocity to the "limit" value, thereby protecting it from rupture as a result of rapid contraction (De Vries, 1976).
Specific activity of both athletes and non-athletes, such as jumps, sprints with and without changing direction, throwings, strikes etc., are abundantly characterized by positions of relevant body segments in which muscular injuries occur, caused by stretching and rupture. These injuries are caused by a strike in a muscle and by exceeding stretch capabilities affected by forces of inertia or by an external enormous demand for a large generating force during a short interval of time. Most of those motions produce strong, explosive and passing contractions of muscles in working regimes.
Injuries occur while changing from an eccentric to a concentric phase of contraction of muscles.Muscles can generate enormously powerful forces during a very short interval of time, about 14500 N for 100 milliseconds before rupture, in conditions with the emphasized pliometric component. In these conditions, while motions with accelerations exceed 25 m/sec^2, muscles support enormously powerful forces (14 650 N - Jelen, 1991; Khoury, et all., 1995; Medved, 1981).
1. Fine muscle reactions to strike or sudden jerk
A fine muscle, which length significantly exceeds linear proportions of cross section, can be treated as a one-dimensional structure. Considering the fact that a muscle represents an elastic medium, the element of its mass dm is affected by an elastic force

$$dF_l = v^2 \frac{\partial^2 u}{\partial x^2} dm , \qquad (1.1)$$

where v represents velocity of deformation spreading and u is wave elongation.
Besides that, one has to take into account that muscle oscillations occur in a material environment which, naturally, is resistant to oscillations. It is known that the resistance of environment depends on velocity. Because of that an additional force is introduced which is proportional to the space changes of oscillating velocity $\partial u / \partial t$. The expression for this additional force which is of a friction type, is

$$dF_{tr} = -2W\frac{\partial^2 u}{\partial x \partial t}dm, \qquad (1.2)$$

where the factor of proportionality W has the velocity dimensions.

An external force is very powerful and has a very short time effect in one point. It will be assumed that it is of the form

$$dF_{ext} = A\delta(x - x_0)\delta(t)dm, \qquad (1.3)$$

where δ is Dirac δ -function and A ~ (length^2 /time) regulates the dimensions. Force effect is thereby instantaneously expressed at $t = 0$ and concentrated an the point $x = x_0$.

Using the integral representation for δ - function, the equation isof the form

$$\left(\frac{\partial}{\partial t} + C_1\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - C_2\frac{\partial}{\partial x}\right)u = \frac{A}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dk d\omega\, e^{ik(x-x_0)+i\omega t}. \qquad (1.4)$$

Then, after using appropriate operational calculus the amplitude solution is obtained (for $t > 0$) in the form

$$u = \begin{cases} \frac{A}{2(C_1 + C_2)} = \frac{A}{4\sqrt{\omega^2 + v^2}}; & 0 < \Delta x < C_1 t \\ 0; & \Delta x < 0; \quad \Delta x > C_1 t \end{cases}, \qquad (1.5)$$

here $\Delta x = x - x_0$. If there is not a friction $(\omega = 0)$, then

$$u_0 = \begin{cases} \frac{A}{4v}; & 0 < \Delta x < C_1 t \\ 0; & \Delta x < 0; \quad \Delta x > C_1 t \end{cases}, \qquad (1.6)$$

The absence of friction increases and symmetrises the amplitude.

2. Massive muscles.

It is of intereest to consider two dimensional case (stomach muscles) as well as three dimensional case. The friction will be neglected ($\vec{W} = 0$.)

Analogously to the one dimensional case the elongations can be found: For two dimensinal case

$$u = \begin{cases} \frac{A\,\mathrm{sgn}\,t}{8\pi R v}\frac{1}{\sqrt{\frac{v^2t^2}{R^2} - 1}}; & \frac{vt}{R} > 1 \\ 0; & \frac{vt}{R} \le 1 \end{cases}, \qquad (2.1)$$

where $R = \sqrt{(x - x_0)^2 + (y - y_0)^2}, \quad R = vt$ and in the three dimensional case

$$u = \begin{cases} \frac{A}{8\pi vR^2}\delta\left(\frac{vt}{R} - 1\right); & t > 0 \\ \frac{A}{8\pi vR^2}\delta\left(\frac{vt}{R} + 1\right); & t \le 0 \end{cases} \qquad (2.2)$$

where $R = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$, $R = vt$

In both cases muscles are breaking. The breaking lines in the two dimensional case are circles $(x - x_0)^2 + (y - y_0)^2 = v^2t^2$ and in the three dimensional case are spheres $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = v^2t^2$.

3. Muscular tissue density during strike or sudden jerk

This will indicate that the linear density (of fine muscle) and a volume density (of massive muscle) satisfy the wave equation, which is also satisfied by the deformation (elongation).

Linear density can be presented as

$$\rho = \frac{m}{L - u(x,t)} , \qquad (3.1)$$

where L is a muscle length in the state of rest, u is elongation and m is a mass which is a constant.

One obtains

$$\rho_{tt} - v^2\rho_{xx} = \frac{\rho}{L - u}\left(u_{tt} - v^2u_{xx}\right) + \frac{2}{\rho}\left(\rho_t + v\rho_x\right)\left(\rho_t + v\rho_x\right) . \qquad (3.2)$$

Note, the elongation satisfies $u_{tt} - v^2u_{xx} = 0$ and, because the mass is constant, ρ satisfies the continuity equation: $\rho_t + v\rho_x = 0$. Thus, relation (3.2) becomes

$$\rho_{tt} - v^2\rho_{xx} = 0 , \qquad (3.3)$$

It follows that the volume density is of the form

$$\rho = \frac{M}{\Phi - V(\vec{r},t)} , \qquad (3.4)$$

where M is the muscle mass which is constant, Φ is the volume of a muscle in the state of rest and V is the volume deformation of Φ .

Operating in the same way as for linear density (only $\frac{\partial}{\partial x}$ is substituted with the Hamilton operator ∇), a conclusion can be drawn that volume density satisfies the wave equation

$$\frac{\partial^2 \rho}{\partial t^2} - v^2 \Delta \rho = 0 \quad , \tag{3.5}$$

since the volume deformation V satisfies the equation of the same type and due to the constant mass, the continuity equation

$$\frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla) \rho = 0$$

holds.

CONCLUSION: The analysis done in Sections 2 and 3 shows that a fine muscle obtains a stable rigid form after strong strike or a sudden jerk, while a massive one ruptures along lines (circles in the two dimensional case) and surfaces (spheres in the three dimensional case) which can be determined through the given solutions.

It should be pointed out that the muscle density can be determined in a similar way to elongation.

Also, It should be pointed out that homogeneous parts of equations require more attention and the analysis with more details. Such problems will be addressed in further investigations.

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