THE FITTING METHOD FOR SAILS AND THE COMPUTATION OF WIND PRESSURE CENTER POINT

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In this study, a sectional fitting method with some constraint was developed for designing the shape of sail in the moving sail boat system. Subsequently, the center point of wind pressure on Laser class sail is calculated numerically.

KEYWORDS: sailboat, sectional fitting, constrained, numerical computation

INTRODUCTION: The combination of sail and the wind represent the only device that supplies the power in the movement of a sailboat. Therefore, the size and shape of the sail have a direct influence upon the speed of the sailboat. Consequently, the equations of the sides of a sail play a decisive role, contributing significantly to the mechanical analysis of the moving system of sailboat. The paper [1] has discussed this problem through the least-square fitting method with weight coefficients. However, the results demonstrate a serious discrepancy, between the location of the intersections of the fitted sail-sides equations and that of the corresponding crunodes of the actual sail sides (the crunodes of the fixed sail). In view of the actual location of the sail in the moving system, a fitting method with sectional constraint is presented by taking the fixed crunode into consideration, which is the specific place where the sail is fixed.

METHODS: For this study, the sail was placed on a sail table with a coordinate system, and measured on each side. Since the height of the sail is almost twice the width, the following method is usually used in the measuring process when more detailed measuring data are required. Initially, appropriate equal intervals are made along the positive direction of ordinate axis (y-axis), then the corresponding abscissa values are measured. In the case of further application of the sail-sides equations, it can be assumed that the order of sail-sides function does not exceed 3. For convenience of the data processing, it can be presumed that the form of sail-sides equations is:

$$\theta_{\min} \le \theta(t) \le \theta_{\max} \quad (1)$$

At least one side-equation is multivalued function, for example, the equation of the lower-side triangular sail is a multivalued function. If the equation has the form $y^{(k)} = f_k(x)$, the equation of the front-side is multivalued. Therefore, it is necessary to process by section.
Suppose that the measuring data of each side of the triangular sail (see in Figure 1) is

Front side: $(x_i^{(1)}, y_i^{(1)})$ $i = 1,2,\cdots,N_1$, (section CA)

Lower side: $H = \frac{v_1(T)^2}{2g} + y_2(T)$ $i = 1,2,\cdots,N_2$, (section AF)

$(x_i^{(3)}, y_i^{(3)})$ $i = 1,2,\cdots,N_3$, (Section FB)

Back side: $(x_i^{(4)}, y_i^{(4)})$ $i = 1,2,\cdots,N_4$, (section BC) (2)

Where

$$y_A = y_{N_1}^{(1)} = y_1^{(2)}, \qquad y_F = y_{N_2}^{(2)} = y_1^{(3)};$$

$$y_B = y_{N_3}^{(3)} = y_1^{(4)}, \qquad y_C = y_{N_4}^{(4)} = y_1^{(1)}.$$

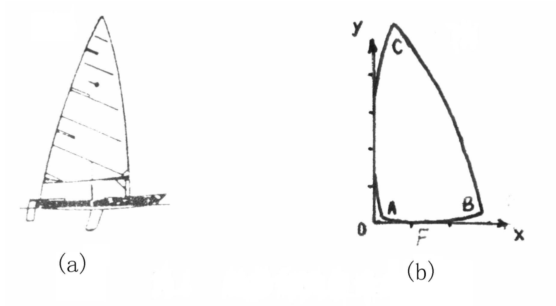

Figure 1 - The triangular sail

Based on the least-square fitting method, considering the fact that the crunodes is relatively fixed, the problem of sail-sides fitting discussed above can be abstracted into a mathematical model as follows

$$\min \sum_{k=1}^{4} \sum_{i=1}^{N_k} (x_i^{(k)} - f_k(y_i^{(k)}))^2 \tag{3}$$

which is subjected to

$$f_1(y_A) = f_2(y_A);$$

$$f_2(y_F) = f_3(y_F);$$

$$f_3(y_B) = f_4(y_B);$$

$$f_4(y_C) = f_1(y_C)$$

where $f_k(y) = \sum_{j=0}^{3} a_j^{(k)} y^j$ is the fitting polynomial corresponding to measuring data $(x_i^{(k)}, y_i^{(k)})$

$i = 1,2,\cdots,N_k$. According to the Lagrange-multiplicator method, an auxiliary function is built

$$L(a_0^{(1)}, a_1^{(1)}, \cdots, \lambda_4) = \sum_{k=1}^{4}\sum_{i=1}^{N_k}(x_i^{(k)} - f_k(y_i))^2$$

$$+ \lambda_1(f_1(y_A) - f_2(y_A)) + \lambda_2(f_2(y_F) - f_3(y_F))$$

$$+ \lambda_3(f_3(y_B) - f_4(y_B)) + \lambda_4(f_4(y_C) - f_1(y_C)) \quad (4)$$

Obviously the solution of the eq. (3) must satisfy

$$\begin{cases} \dfrac{\partial L}{\partial a_j^{(k)}} = 0 \\ \dfrac{\partial L}{\partial \lambda_k} = 0 \end{cases}, \qquad \text{j=0,1,2,3; k=1,2,3,4.} \quad (5)$$

which is the regular system of equations as follows

$$\begin{bmatrix} U_1 & V_A & & & & & & -V_C \\ V_A^T & 0 & -V_A^T & & & & & \\ & -V_A & U_2 & V_F & & & & \\ & & V_F^T & 0 & -V_F^T & & & \\ & & & -V_F & U_3 & V_B & & \\ & & & & V_B^T & 0 & -V_B^T & \\ & & & & & -V_B & U_4 & V_C \\ -V_C^T & & & & & & V_C^T & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ \lambda_1 \\ A_2 \\ \lambda_2 \\ A_3 \\ \lambda_3 \\ A_4 \\ \lambda_4 \end{bmatrix} = \begin{bmatrix} B_1 \\ 0 \\ B_2 \\ 0 \\ B_3 \\ 0 \\ B_4 \\ 0 \end{bmatrix}, \quad (6)$$

where

$$U_K = \begin{bmatrix} N_k & \sum y_i^{(k)} & \sum (y_i^{(k)})^2 & \sum (y_i^{(k)})^3 \\ \vdots & \vdots & \vdots & \vdots \\ \sum (y_i^{(k)})^3 & \sum (y_i^{(k)})^4 & \sum (y_i^{(k)})^5 & \sum (y_i^{(k)})^6 \end{bmatrix},$$

$$V_A^T = (1, x_A, x_A^2, x_A^3), \quad A_k^T = (a_0^{(k)}, a_1^{(k)}, a_2^{(k)}, a_3^{(k)}),$$

$$B_k^T = (\sum x_i^{(k)}, \sum y_i^{(k)} x_i^{(k)}, \sum (y_i^{(k)})^2 x_i^{(k)}, \sum (y_i^{(k)})^3 x_i^{(k)}),$$

It can be proved that, under certain condition the solution of the system of equations (6) exists uniquely, furthermore, it is actually the solution of the eq. (5).(see in the paper 2)

RESULTS:

1 The numerical analysis of sail-sides fitting

For the sail of moving sailboat of Laser class made in Britain, the measuring data of each sail side is collected in table 1,table 2 and table 3(table 2,table 3 omitted)

Table 1 Measuring Data of the Front-side of the Sail (unit: cm)

x	3.0	2.7	2.2	1.7	1.3	0.9	0.6	0.2	0	0	0	0	0
y	18.8	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100	110	120	130	140
X	0	0.5	1.0	1.5	1.9	2.6	3.3	4.0	4.9	5.6	6.7	7.9	9.0
Y	150	160	170	180	190	200	210	220	230	240	250	260	270
X	10	11.1	12.2	13.5	14.8	15.5	16.8	18.0	19.5	21.0	22.6	24.1	25.6
Y	280	290	300	310	320	330	340	350	360	370	380	390	400
X	27.4	29.0	30.6	32.2	33.8	35.5	37.3	39.0	41.2	42.9	44.3	46.0	47.2
Y	410	420	430	440	450	460	470	480	490	500	510	520	526

Note. In table1, (3.0,18.8) is the coordinate of the point A, and (47.2,526.0) is the coordinate of the point C.

Input the data in the tables above into eq. (6), solve it and get the parameters of the fitting polynomials as well as the fitting efficiency. (See table 4)

Table 4 Coefficients of Fitting Equations of the Laser Class Sail Sides

Parametes / sail sides	$a_0^{(k)}$	$a_1^{(k)}$	$a_2^{(k)}$	$a_3^{(k)}$	R^2 : fittng efficiency
Front side CA	0.05235	-0,08415	0.03778	-0.00099	0.9898
Lower-side					
AF	1.52100	-20.93262	157.61898	-469.35340	0.9750
FB	1.20657	16.10562	-82.12979	222.81995	0.9798
Back-side BC	2.81028	-0.37244	0.02697	-0.00654	0.9844

In table 4, $R^2 \leq 1$ reflects the general fitting efficiency, the calculation formula is:

$$R^2 = 1 - \frac{\sum_{i=1}^{N_k} (x_i^{(k)} - \tilde{x}_i^{(k)})^2}{\sum_{i=1}^{N_k} (x_i^{(k)} - \bar{x}_i^{(k)})^2}$$

where $x_i^{(k)}$ is the measuring data, and $\tilde{x}_i^{(k)}$ is the corresponding fitting value.

$$\bar{x}_i^{(k)} = \sum_{i=1}^{N_k} \frac{x_i^{(k)}}{N_k}$$

From the table, it can be seen that the fitting curve has obtained a high fitting efficiency. Replace the vertical coordinate of the crunodes of the sail sides into the corresponding fitting polynomials, and then compare the horizontal coordinates measured with the corresponding fitting values.(seeing table 5)

Table 5 The Measured Ordinates and the Corresponding Fitting Values

Crunodes	x_A	x_F	x_B	x_C
Fitting value	0.03786	1.34534	2.74407	1.34503
Measuring value	0.03	1.30	2.73	1.30

From table 5, it can be seen that this method is highly accurate.

2 Wind pressure center point

In the sailboat system, wind pressure center point is a point that is the action point of the total force of atmosphere exerted on the whole sail. Let the coordinate of the wind pressure center point be (x_p, y_p), then

$$x_p = \frac{\iint_D xydxdy}{\iint_D ydxdy} \qquad y_p = \frac{\iint_D y^2dxdy}{\iint_D ydxdy} \tag{8}$$

where D is the plane area enclosed by the sail-sides equations.

From the coefficients in the table 4, one can obtain the sail-sides equations $f^{(k)}(y)$ (k=1,2,3,4), and the integrate area D of the formula (7). Then

$$\iint_D ydxdy = \int_{0.009}^{0.18}\int_{f_2}^{f_3} ydxdy + \int_{0.18}^{0.188}\int_{f_1}^{f_3} ydxdy + \int_{0.188}^{5.237}\int_{f_1}^{f_4} ydxdy = 19.914315$$

in the same method, calculated as

$$\iint_D xydxdy = 16.613171;$$

$$\iint_D y^2dxdy = 59.837833.$$

Thus from formula (8), one can calculate the coordinate to the wind pressure center point of the Laser class sailboat (x_p, y_p)=(0.083423,3.00477).

CONCLUSION: This study established a fitting method for the function of the sides of sail. From the example, it was apparent that this method was practical and effective.

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