BIOMECHANICS INSTRUMENTATION

ESTIMATION OF VELOCITIES AND ACCELERATIONS FROM NOISY KINEMATIC DATA

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Introduction

Kinematic analysis of motion often requires the evaluation of quantities not always directly measurable, like velocity and acceleration in order to be complete. (Hatze 1984). Acceleration could be measured by means of accelerometers fixed to the landmarks to be analyzed, but due to the disturbance of the subject it is better to compute them from the landmarks' trajectories evaluated by means of a movement analyser using a passive marker, like the ELITE system (Ferrigno and Pedotti 1985).

In the analysis of sport performances, as well as, in critical pathological studies, maximum freedom of movement is mandatory, otherwise the results obtained will be biased and insignificant. This generates the need for an analysis that is characterized by minimum disturbance of the subject.

The first and second derivatives of the trajectories of the landmarks or of the angles computed between segments identified by markers, can be evaluated by numerical methods from raw spatial coordinates.

The main problem which arises at this point is due to the noise superimposed on the useful signal which heavily affects the outcomes. Whatever the means used to acquire the data, these will be modified by measurement noise. Noise including quantization and distortion due to algorithms, distortion corrections and 3D reconstructions lead to moderately ill-posed problems when attempting to estimate the derivatives (Woltring 1985). An exhaustive review of methods used to compute derivatives from noisy displacement data can be found in Woltring, 1985 and Wood, 1982. For the purpose of this paper only, the main features of the approaches therein already considered will be reported. The method proposed and later described can be classified among those belonging to the frequency domain filters, the parameters of which are estimated directly from the measurement.

Several examples of applications in classic literature data (Pezzack et al. 1977; Lanshammar 1982; Vaughan 1982) and laboratory acquired data referring to sport movements are reported in order to assess the performances of the filter.

Time and Frequency Domain Approaches

Both time and frequency domain approaches have been widely explored in order to produce algorithms for data smoothing and derivative estimation.

The frequency domain approach has been used by Cappozzo et al, 1975, Jackson, 1979; and Anderssen and Bloomfield, 1974, while Pezzack et al, 1977; Winter et al, 1974; Soudan and Dierckx, 1979; Wood and Jennings, 1979; Gustaffson and Lanshammar, 1977; Gasser et al, 1986; Parks and McClellan, 1972; Jetto, 1985 followed the other one. A detailed description of all these methods is not reported herein, but the interested reader can find them in the references. The main problem which arises in both approaches is the bandwidth selection and/or the optimal windowing choice. A compromise between biasing the data in terms of frequency content of the signal and the noise magnification typical of the derivation operators must be found. Several of the cited authors guessed the parameters of the filter, choosing the final value by trial and error, while others had to set (Jackson, 1979; Cappozzo et al, 1975) thresholds based on the rate of reducing the residual error.

More interesting are the methods of the Generalized Cross Validation Criterion (GCVC) applied to spline functions (Craven and Whaba, 1979) and the Optimal Regularization of Fourier Series (Anderssen and Bloomfield, 1974), in which given the criterion, the choice of the optimal filtering window was automatized.

At least two reasons strongly support the automatization of the choice of filter parameters: the time savings and the homogeneity of the results. In fact, if a parameter must be guessed by trial and error, the operator is forced to review the obtained results before proceeding to the next step. This wastes time. Furthermore, different operators could produce different outcomes from the same data.

The aim of this work has been to design an automatic filter able to search its own bandwidth using the least amount of time possible.

Hypotheses

The filter which will be described, as well as those cited above, gives effective results only if five conditions are met: 1) the noise is additive; 2) it is uncorrelated; 3) it is stationary, and 4) the signal has been sampled at equidistant intervals and 5) the sampling rate is at least twice the maximum frequency content of the signal according to the Shannon theorem. In order to make our filter work properly, a further hypothesis must be included: the frequency content of the signal must be reasonably contained within 40 percent of the sampling rate. This constraint is in good agreement with the error formula presented by Lanshammar, 1982b. In fact, according to this formula, the sampling rate must be higher than the Shannon limit in order to obtain low noise derivatives.

Algorithm

The block scheme of the designed algorithm is reported in Figure 1. Raw data f(t) is used in order to evaluate the probable useful bandwidth of the signal itself and is preprocessed in order to avoid edge shape distortions due to implied periodicity. After these two parallel operations, the filter extracts from the signal the part which is characterized by a reasonable signal to noise ratio. This part, denoted as f^ (t) is then derived twice producing the first (velocity) and second (acceleration) derivatives of f^ (t): f^' (t) and f^"(t).

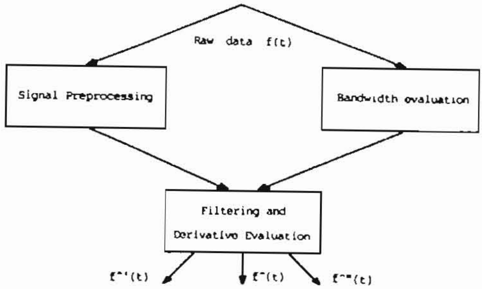

Figure 1 - Algorithm block scheme

Bandwidth evaluation:

In order to evaluate the bandwidth of the signal, its spectrum must be estimated. Theoretically, the bandwidth of a landmark coordinate is unlimited, but practically, in particular when derivatives must be computed, a limit must be imposed in order to obtain a useful signal to noise ratio. The chosen cut off frequency is the one for which the signal falls below the noise level. In order to find this value, the spectra of the signal and of the noise should be well known. When this is not the case, a certain number of hypotheses and approximations must be used to find the values. The hypotheses have been stated before and they will be used here, in order to build up the algorithm. The first step is the estimation of the signal plus the noise (measurement) power spectrum. There are various methods which can be used for this purpose, a review can be found in Kay and Marple, 1981; and Makhoul, 1975. Due to the shortness of the data records typical of many fast sportive acts, and the necessity of obtaining sharp well defined spectra, the autoregressive (AR) forward backward algorithm (Kay and Marple, 1981) has been chosen. This algorithm fits the measurement data by a model which assumes that each sample of the signal is a linear combination of its (P) preceding or future values. The coefficients of this combination are the parameters of the AR model of the order P as follows:

$$\hat{s}_f(n) = -\sum_{k=1}^{P} a(k)s(n-k) \qquad P \leq n \leq N-1 \qquad (1A)$$

$$\hat{s}_b(n) = -\sum_{k=1}^{P} a^*(k)s(n+k) \qquad 0 \leq n \leq N-1-P \qquad (1B)$$

where $\hat{s}(\cdot)$ represents in (1A) the forward prediction estimate and in (1B) the backward one, $s(\cdot)$ is the measurement and N represents the number of samples. The * represents the complex conjugate of the parameters. By using the coefficient $a(\cdot)$ a Power Spectrum Densisty (PSD) estimate is easily derived:

$$\hat{S}(z) = \frac{\delta^2 \Delta t}{1 + \sum_{k=1}^{P} a(k)z^{-1}} \qquad (z=\exp(j\omega t)) \qquad (2)$$

where $\hat{S}(z)$ is the PSD estimate, Δt the sampling interval, and δ^2 the variance of the modellization noise as follows:

$$\delta^2 = \sum_{n=P}^{N-1} \left| s(n)-\hat{s}_f(n) \right|^2 + \sum_{n=0}^{N-1-P} \left| s(n)-\hat{s}_b(n) \right|^2 \qquad (3)$$

The noise superimposed on the signal is evaluated, according to one of the hypotheses, as the average value of the power of frequencies between .8 FN and FN, where FN represents one half of the sampling rate. The cut off frequency has been set where the PSD overcomes 20 times the noise level, in order to attain a high signal to noise ratio. In order to avoid slow oscillating effects on the high order derivatives due to sharp truncation of the signal spectrum, the power of the signal is assumed to decrease linearly from the cut off frequency to twice this value. Summarizing, the Discrete Fourier Transform (DFT) of the measurement is windowed by a function unitary up to the cut off frequency and decreasing (Wiener weighed) up to twice this value. The way in which the DFT of the measurement is obtained will be described in the next subsection. The order P of the model has been determined by the analysis of the practical data and by choosing the value for which the cut off frequency shows an asymptotic behavior. This value range is between seven and ten. We have chosen the latter on the basis of the residual whiteness test results.

Signal Preprocessing

The purposes of the signal preprocessing are to: avoid edge effects, prepare the signal to be transformed by a fast algorithm (Fast Fourier Transform or FFT) and to perform the transformation.

The edge effects are prevented by extrapolating the measurement one second before and one second after using the prediction model obtained in the previous section and by cosine tapering these extensions to the average value (AV) of all the extrapolated values. In order to apply FFT, the signal is padded up to the nearest power of two by the AV. The preprocessing ends by transforming the measurement in the frequency domain by the FFT algorithm.

Filtering and Derivative Evaluation

The filtering of the measurement is performed by multiplying the DFT obtained in the previous subsection by the window already determined. After filtering, the derivatives are computed by multiplying each windowed DFT component by jw and $-w^2$ respectively. The antitransformation of the three frequency domain sequences leads to a time domain representation of the filtered signal and its estimated derivatives which represent the final result of the proposed algorithm.

Results

In order to test the algorithm and permit a comparison with other methods reported in the literature (see references), we have processed the raw data presented by Pezzack et al, 1977 and the same with added noise reported by Lanshammar 1982a. These data refer to the angular displacement between hand and forearm during two abduction-adduction phases, the first executed slowly and second quite quickly. As reported in Figure 2, the filtered data provide an excellent fit to the raw ones, as proven by the residual error of 0.002 radians. The second derivative conforms closely to the data of the accelerometer reported in the same paper showing similar amplitude peaks and low noise in the low amplitude phase.

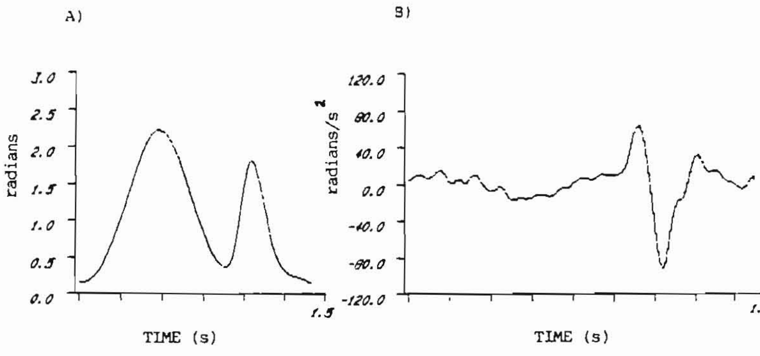

Figure 2 - A) Raw angular displacement data from [Pezzack et al. 1977]
B) Second derivative of data in A), cut off frequency 5.1 Hz

These data are, according to Lanshammar 1982a, too good. In fact he estimated a noise standard deviation of only 0.0013 radians which is a value lower than in many practical cases in biomechanical measurements. In order to create a set of data more realistic, this author increased the noise synthetically by adding a gaussian white noise with zero mean and 0.006 radians of standard deviation, thus achieving a new signal more difficult to process. By using these data we obtained the results of Figure 3. In this case, as expected, the second derivative appears to be more smooth than the previous one. In fact, the algorithm set a lower cut off frequency in order to reduce noise magnification, but despite the lower cut off frequency, the information contained in the second derivative has not been degraded.

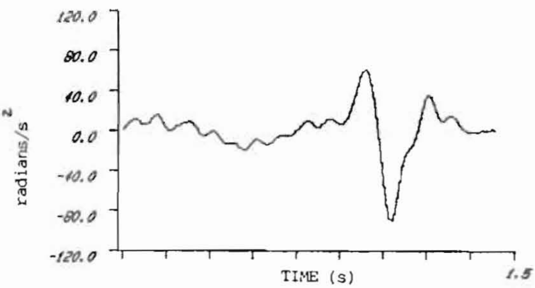

Figure 3 - Second derivative of the data from [Lanshammar 1982a]
Cut off frequency 4.5 Hz

Another set of data that we used to test the method is the one, referring to a free falling ball, provided by Vaughan 1982. These data point out a limitation of our algorithm due to the extrapolation used. In fact the AR model, instead of trying to model the parabolic behavior of this trajectory, interpolates it with a slow varying sinusoid. This wrong interpretation generates strong edge effects which diffuse deeply inside the useful signal as shown in the second derivative reported in Figure 4. It can be seen in this figure that the acceleration oscillates around its true value of -g.

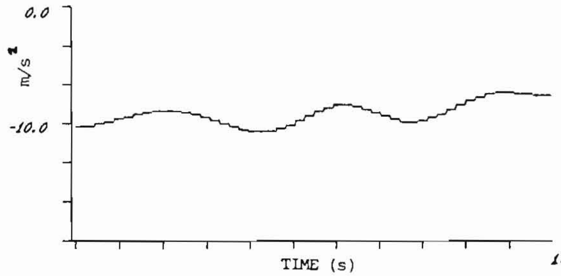

Figure 4 - Free falling ball acceleration estimate

In order to show that the problem of Figure 4 is due to the particular signal and not to its edges which are strongly different, we report an analysis performed on a sinusoid corrupted with a gaussian white noise with a standard deviation of 0.015 and featuring edges with different values. The results reported in Figure 5 show the very good behavior of the filter, in fact, no edge effects propagated inside the useful signal. Two other examples are reported in Figures 6 and 7 where data acquired by the ELITE system of the Centro di Bioingegneria of Milan are reported. These data refer to an Abalakov test (Figure 6) and a sprint start (Figure 7). The data of Figure 6 refer to the trajectory of a landmark positioned on the hip of the athlete. Three peaks are evident in the second derivative along the vertical axis: the first one refers to the prestretching phase, the second to the take off and the third to the landing. Due to the high amplitude of these peaks the chosen bandwidth causes oscillations around the -g value during the flying period. Would someone be interested in analyzing this part? It should force a lower bandwidth paying the cost of a drastic reduction in the amplitude of the peaks.

Figure 7 shows the movement of the hip marker during the pushing phase of the start sprint, together with the computed acceleration.

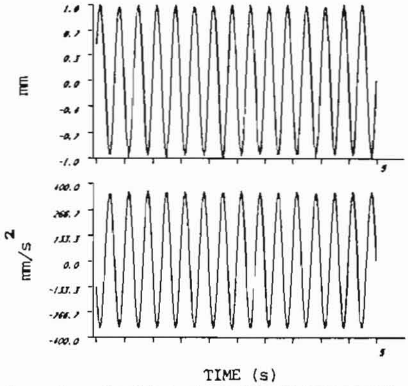

Figure 5 - Test performed with a sinusoid: A) raw signal, B) second derivative

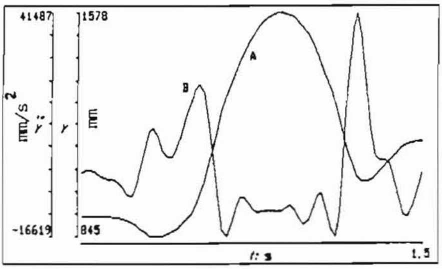

Figure 6 - Abalakov test: A) hip marker coordinate vs. time, B) 2nd derivative

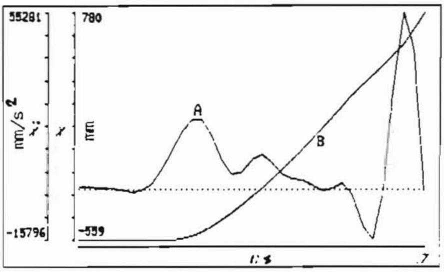

Figure 7 - Sprint start: A) hip marker coordinate vs. time, B) 2nd derivative

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