

# A MATHEMATICAL MODEL OF HUMAN DYNAMIC LOCOMOTION: DEVELOPMENT AND APPLICATION OF THE MODEL

C. Larkins<sup>1</sup> and M. M. Vieten<sup>2</sup>

<sup>1</sup>The University of Michigan  
Ann Arbor, Michigan, USA

<sup>2</sup>University of Konstanz  
Konstanz, Germany

## INTRODUCTION

Human walking is characterized by periods of single and double support phases during which contact with the ground is never broken. In more dynamic locomotion such as running, hopping, and jumping, only a single support phase is utilized to propel the jumper into a ballistic flight phase. Because of the great forces and torques often generated during the support phase, as well as the possibility for intricate aerial dynamic maneuvers, locomotion can be extremely complex. The problem, then, faced by coaches is to devise a systematic procedure which can be used to train athletes to perform such complex dynamic activities.

## BACKGROUND TO THE DEVELOPMENT OF THE MODEL

Coaches have traditionally tackled this problem through trial and error methods. They observe their athletes' performances, qualitatively evaluate the technique, and then through trial and error make adjustments in both technique and training. This process is repeated until they hone in on the best technique. This non-systematic approach is not only stressful on the athlete, it is time-consuming and only minimally productive.

A systematic and, we believe, more efficient procedure is to develop a computer based analytical method that would allow the coach to manipulate the technique systematically without continually stressing the athlete. This procedure is effective only if the coach is familiar with scientific methods. The usefulness depends on whether the model meets the needs of the coach and the validity of the model depends upon whether it can accurately replicate the essential features of the jump including the distance jumped. In addition, a model that is refined enough to accurately model complex movements will include four features: 1) Anatomy, 2) Posture, 3) Dynamic variables, and 4) Control process by intentional muscle contraction or relaxation.

## EXAMPLE OF AN ANALYTICAL MODEL FOR COACHING

One of the most promising analytical methods has been developed by Ramey et al. (1988). As an illustration, they have shown how it can be used to design the running long jump. With their model, in order to attain an increase in distance over a previous jump it is necessary for the coach to first determine a reasonable increase in jump distance; second, determine a reasonable combination of horizontal and vertical takeoff velocities for that distance; and third, determine the style of flight phase appropriate for the jumper. It is then necessary for some qualified individual to determine new force records and segment orientations for the support phase which will yield the required angular momentum and takeoff velocities. This feature allows for coaching strategy and

"...will permit the user to make decisions that reflect an individual user's biases as to how the activity should be performed."

It is essential that an analytical model for coaching allow the coach to make decisions about some of the key performance parameters. For example, in coaching the running long jump the list of essential parameters may vary according to the individual biases of the coach; however, the authors believe that the following must be included: 1) for the pre-support phase, important parameters may include the horizontal and vertical approach velocities, the height of the center of mass at board contact, and leg angle at board contact; 2) key parameters for the support phase may include the appropriate combination of horizontal and vertical takeoff velocities of the athlete, the necessary force-time curve, and some measure of the neural muscular adaptations to training; 3) key flight phase parameters may include the technique used during the flight such as sail style, hang style, or hitch kick style; and 4) the key parameters for the landing phase may include the body position at the instant of final touchdown that the coach believes will give the most advantageous landing position.

### CURRENT MODELS OF HUMAN DYNAMIC LOCOMOTION

A number of other models of human dynamic locomotion have been developed over the years. Although not specifically designed to serve as analytical tools for coaches, they nonetheless are potentially useful. They include simple Hookean spring-mass systems (Blickhan, 1989) and more complex dampened spring-mass systems (McMahon and Green, 1979).

These spring-mass models give a fairly realistic depiction of human dynamic locomotion. The behavior of a spring closely imitates springy legs as are found in nature. A major advantage of these models is that springs possess inherent stiffness that can represent the neural muscular behavior of the support leg. The inclusion of this feature into the coaching model would enable the coach to determine the effects of conditioning and training of the muscles on support phase mechanics. Furthermore, spring-mass models can also determine takeoff velocity vectors and therefore jump distance from initial parameters. This would allow for the design and manipulation of the pre-support phase parameters.

There are, however, a number of limitations to these models. First, a spring-mass system with a massless leg eliminates the ability to describe torque generated between movements of the upper and lower body as well as torque generated to reorient the system during flight. Second, in real human dynamic situations, muscle stiffness is not constant as is the case with current models but varies throughout the support phase. Third, even with the inclusion of damping mechanisms, these models give a grossly oversimplified representation of the neural muscular control. In spite of these limitations, spring-mass models can become a useful tool to the coach with some modifications.

In short, while Ramey has already developed a systematic method for coaching, his model lacks features that the spring-mass models can provide. On the other hand, the existing spring-mass models are not sufficiently refined to deal with complex coaching situations. The purposes of the present study are: 1) To develop a mathematical model of human dynamic locomotion that is more refined than previous spring-mass models; and 2) To use this model as an analytical tool enabling coaches to design, evaluate, and correct technique in a systematic way.

## PRELIMINARY MODEL FOR THE PRESENT STUDY

A preliminary analytical model has been developed by the authors. Our model, similar to the model of Blickhan (1989), consists of a simple spring-mass system. Using Ramey's principle of applying a mathematical model to human dynamic locomotion, we have for the first time used the spring-mass model as a coaching tool. The equations of motion during the support phase consisted of two nonlinear differential equations as shown in Equations (1) and (2) below.

$$y'' = y\omega^2 \frac{1}{\sqrt{(y^2 + z^2)}} - 1 \quad (1)$$

$$z'' = z\omega^2 \frac{1}{\sqrt{(y^2 + z^2)}} - 1 \quad (2)$$

$$\text{where } \omega = \sqrt{k/m}$$

and  $\omega$  is constant since  $k$  is constant

These equations were solved numerically by a Runge-Kutta iterative method on a PC. The flight phase distance was determined through the use of the projectile equations.

In a preliminary study, hypothetical data were used to determine if an optimum value exists for selected pre-support phase parameters when jumping for distance. The pre-support phase parameters tested were the following: horizontal and vertical approach velocity, touchdown angle, leg length at touchdown, and system stiffness.

## RESULTS and CONCLUSIONS

Preliminary results show that an optimum may exist for a number of initial value parameters. For example, Figure 1 below shows a family of curves for the initial value parameter, touchdown angle.

Spring-Mass Model:  $V_y=6,7,8,9,10$  (m/s),  $V_z=0$  m/s,  $k=15,000$  N/m,  $r=1$  m

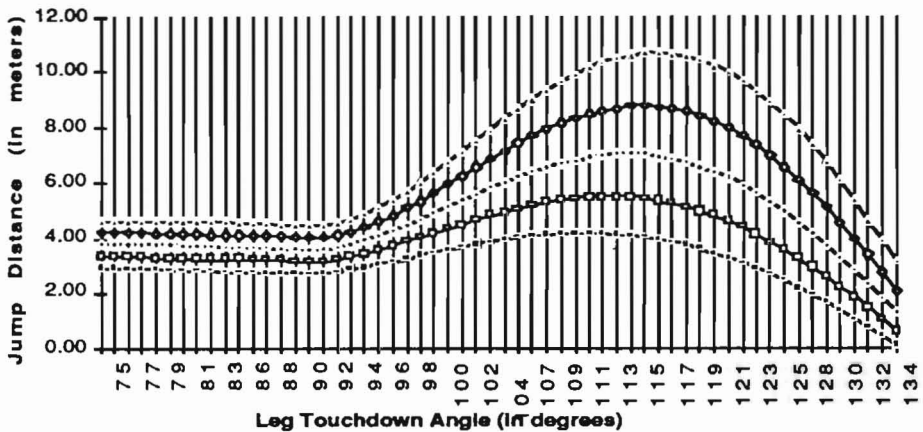


Figure 1. Jump distance as a function of leg touchdown angle.

As can be seen for each case, maximum jump distance was attained when the touchdown angle was between  $110^\circ$  and  $115^\circ$ . Similar results were found for the other initial value parameters.

Hence, while this preliminary mathematical model has proved promising, it was only the first step in the development of a more refined model which combines and expands upon the best of Ramey's model and simple spring-mass models. Since the completion of the preliminary study, we have developed a more refined spring-mass model that can be used as a tool for coaches. It contains all of the features necessary to design a horizontal jump for maximum distance. It also allows the coach to control all essential pre-support parameters as well as support phase parameters. The major refinement of this Advanced Spring-Mass Model is that the system stiffness is calculated as a function of time. A future model that is in development will be refined enough to model angular momentum during both the support and the flight phases by using an N-segment Spring-Mass Model. Ultimately, then, we hope to give the coach full control over how he or she can manipulate the performance of the athlete by being able to work out in the laboratory—and eventually on the track—the best means for each individual athlete to achieve optimum performance—efficiently, quickly, safely.

#### REFERENCES

- Blickhan, R. (1989). The spring-mass model for running and hopping. *J Biomech* 11/12:1217-1227.
- McMahon, T. A. and Green, P. R. (1979). The influence of track compliance on running. *J Biomech* 12:893-904.
- Ramey, M., Waring, W. W., Yang, A. T. (1988). A model to simulate human motion. Final Report to the National Science Foundation. (Grant N. GK41273/73-03903).