HIGH JUMP DIRECT DYNAMIC SIMULATION

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INTRODUCTION

High jumps are composed of a complex sequence of movements whose single contribution to the whole body motion cannot be intuitively predicted. While most of the movements are well learned by the elite athletes in order to reach the result, few are not effective or negative from the mechanical point of view and are probably performed for an erroneous feeling of their effects. The direct dynamic simulation of jumping can be useful in order to more fully understand the mechanics of high jump techniques; to explain to the athletes the consequences of some errors; and to test possible evolution of the exercise. When the external forces and the relative movements of the limbs are known, the calculation of the whole body trajectory and orientation is a direct dynamic problem.

METHODOLOGY

A mathematical method, recently developed by our group in cooperation with the University of Brescia (Casolo and Legnani, 1991), has been used to build the software package for the direct dynamic simulation. It is based on the use of six 4 x 4 generalized matrices containing both linear and angular components of position, velocity, acceleration, external actions, momentum and inertia, respectively (M, W, H, f, G, J).

Input data for the simulation of the aerial phase of the high jump are: initial position and velocity matrices of the whole body and the law of motion ($\theta = f(t)$) of all the joints involved in the exercise. The body model consists of 14 rigid bodies mutually connected by means of a sequence of revolute pairs (37 d.o.f.). The anthropometry and the mass distribution can be calculated by means of the program ANTHROPM. This program, based on the regression equations from McConville et al. (1980) can estimate the individual inertia matrices using as input as many parameters as are available from a minimum of two (height and weight) to a maximum of 52.

The application here presented concerns the simulation of changes in the kinematics of actual jumps. Therefore, for the first simulation, the initial conditions of the aerial phase, as well as the joint's L.O.M., have been extracted from the images of the competition. In the second phase they have been varied in order to try to improve the performance.

INITIAL CONDITION FOR THE DIRECT SIMULATION

The evaluation of the initial velocity matrix W_{01} (0 = inertial frame; 1 = trunk) of the trunk is a critical question. We used the following two approaches:

a) The first consists only of the numerical calculation of the derivative M' of the position matrix:

$$M'_{<t0>} = \frac{M_{<t0+dt>} - M_{<t0-dt>}}{2dt}$$

Even if the orientation of a body in space cannot generally be described by a vector whose derivative is equal to the rigid body angular velocity (Nappo, 1979), our method allows us to express the velocity matrix with the following simple relationship:

$$W_{<t0>} = M'_{<t0>} M_{<t0>}^{-1}$$

This formula does not introduce numerical problems because the inversion of the matrix M can be done without executing any division. However, manual digitation and low sampling rate cause, as in our practical example, relevant errors when this approach is used.

b) In order to reduce those errors we adopted a technique for the calculation of $W_{(t0)}$ which is based on the analysis of whole airborne phase. During this phase, the athlete's angular momentum g calculated with respect to the CG is constant, while momentum r varies because of the gravity. For the airborne phase we can write in matrix form:

$$\Gamma_{(G)<t>} = \Gamma_{(G)<t0>} + mH_g Dt \qquad (1)$$

where $\Gamma_{(G)}$ is the momentum matrix of the whole body calculated with respect to the CG, m is the mass of the body and H_g is the gravity acceleration matrix (4 x 4 whose only non-null terms are the components of the gravity acceleration in the chosen reference frame. Solving the inverse dynamic problem from the collected data, we can compute the $\Gamma_{(G)}$ matrix at each time step. Therefore, by means of (1) we can evaluate $\Gamma_{(G)<t0>}$ from each interval of the airborne phase. Consequently its average value $\Gamma_{(G)<t0>}$ can be used to determine the initial velocity matrix of the trunk. This average matrix is then referred to the main reference frame (0) as follows:

$$\Gamma_{(G)<t0>} = M_{OG<t0>} \Gamma_{(G)<t0>} + M^t_{OG<t0>}$$

where $M_{OG<t0>}$ is the position matrix of the CG reference frame at time <t0>.

Remembering that the whole body G matrix can be written by means of the velocity matrices W_{0i} and of the inertia matrices J_i of all the segments i of the multi-body system

$$\Gamma_{(0)} = \Sigma(W_{0i} J_i - J^t_i W^t_{0i}) = \text{skew}\{\Sigma(W_{0i} J_i)\} \qquad (2)$$

and the absolute velocity matrix of a body segment can be written as:

$$W_{0i} = W_{01} + W_{1i}$$

where W_{0i} is the velocity matrix of the trunk and W_{1i} is the relative velocity matrix of the segment with respect of the trunk; and then the (2) can be easily rewritten in function of the trunk velocity:

$$\begin{aligned}\Gamma_{(0)<t0>} &= \text{skew}\{\Sigma(W_{01} J_i) + \Sigma(W_{1i} J_i)\} \\ &= \text{skew}(W_{01} J^*) + \Gamma^*_{(0)<t0>} \qquad (3)\end{aligned}$$

or

$$\text{skew}(W_{01} J^*) = \Gamma_{(0)<t0>} - \Gamma^*_{(0)<t0>} \qquad (4)$$

where Γ^* and J^* are known:

$$\Gamma^* = \text{skew}\{\Sigma(W_{1i} J_i)\} \qquad J^*_i = \Sigma J_i$$

It follows that the corrected initial velocity of the trunk W_{01} (that is the only unknown), can now be obtained by solving the 6 x 6 linear system which derives from the manipulation of the above matrix equation (4). Moreover, from G, the initial velocity of the CG can be calculated.

RESULTS AND DISCUSSION

The direct dynamic simulation of the winning high jump of the1992 Olympic Games shows the adequacy of the model and of the method used. Initial conditions and joints' L.O.M. have been extracted from the data collected during the competition while the whole body trajectory and orientation during the flight phase have been computed by the program.

Figures 1 and 2 shows the good agreement between the original jump and the output of the direct simulation with the L.O.M. and the initial condition extracted by the same jump.

At this stage it is now possible to change the performance acting on the L.O.M. of some joints. As a preliminary example, it was suggested by a coach that we postpone the flexion of the takeoff leg, acting on the original law of motion of knee flexion as shown in Figure 3. While obviously the trajectory of the CG does not vary, in the new simulation (Figure 4), the trunk orientation remains rather vertical for a longer time and starts to rotate around the bar when the CG is higher. This change alone is not optimal because it also affects the body twist. Over the bar the left hip is lower than the right one (Figure 5) and consequently the recovery of the lower leg is more difficult. This effect can be compensated for by also changing the arm movement.

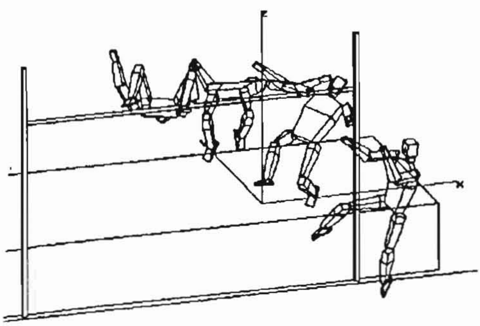


Figure 1. Original jump.

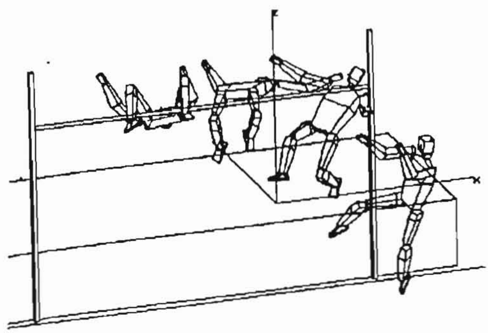


Figure 2. Simulated jump without changes.

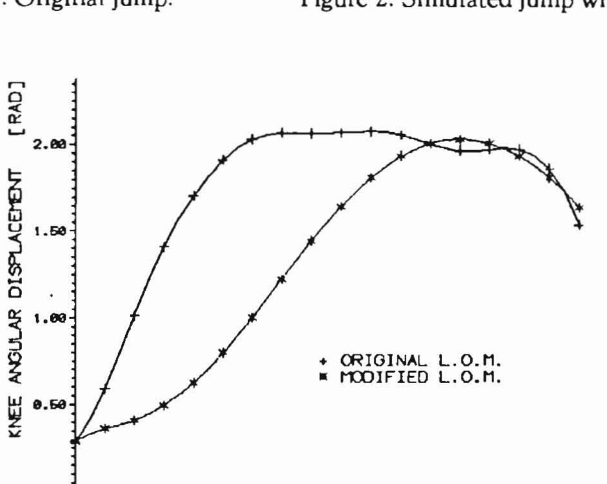
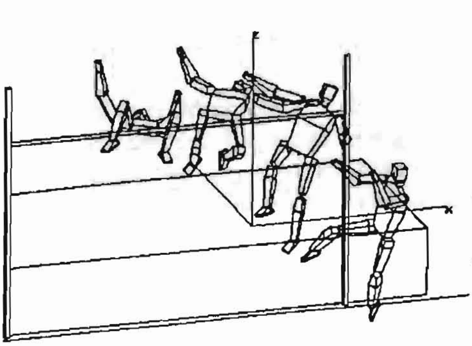


Figure 3. Original and new L.O.M. for knee flexion.



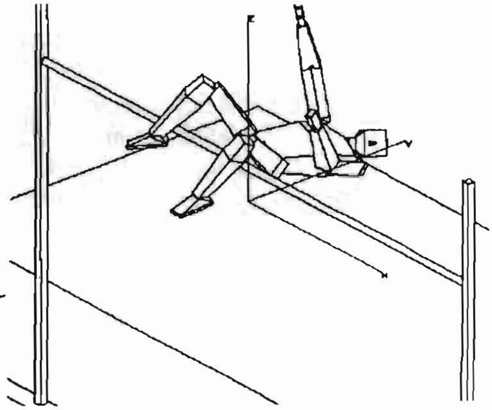


Figure 4. Simulated jump with new L.O.M.   Figure 5. Simulated jump over the bar.

The preliminary tests demonstrated the adequacy of the software developed to simulate the high jump. The initial condition for the simulation of the airborne phase can be estimated with good precision by means of the average momentum matrix technique.

Future versions of the programs, that are now under development, will provide more help to the technician in the choice of the correction of the L.O.M. and of the initial conditions. They also will check if the new movements are compatible with the athlete's physiology.

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APPENDIX

For the 3D kinematic analysis of chains of rigid bodies besides the well known transform matrix, we use two other matrices (Casolo and Legnani 1991) having a very similar structure:

$$M_{ij} = \begin{vmatrix} X_x & Y_x & Z_x & T_x \\ X_y & Y_y & Z_y & T_y \\ X_z & Y_z & Z_z & T_z \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad W_{ij} = \begin{vmatrix} 0 & -\omega_z & \omega_y & \omega_x \\ \omega_z & 0 & -\omega_x & v_y \\ -\omega_y & \omega_x & 0 & v_z \\ 0 & 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} \omega & & & V_0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$H_{ij} = \begin{vmatrix} G & & & A_0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \qquad \text{where } G = \omega^2 + \omega'$$

M is the position (or transform) matrix of frame j with respect to frame i. W is the generalized velocity matrix containing the angular components and the velocity of a point (pole) of the body j. Analogously the generalized acceleration matrix H contains both linear and angular components of the body acceleration.

For the dynamics we introduced the action matrix ϕ, the momentum matrix G and the inertial pseudo-tensor J:

$$\phi_{k(s)} = \begin{vmatrix} 0 & -c_z & c_y & f_x \\ c_z & 0 & -c_x & f_y \\ -c_y & c_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{vmatrix} \quad \Gamma_{k(s)} = \begin{vmatrix} 0 & -\gamma_z & \gamma_y & \rho_x \\ \gamma_z & 0 & -\gamma_x & \rho_y \\ -\gamma_y & \gamma_x & 0 & \rho_z \\ -\rho_x & -\rho_y & -\rho_z & 0 \end{vmatrix} \quad J_{k(s)} = \begin{vmatrix} I_{xx} & I_{xy} & I_{xz} & q_x \\ I_{xy} & I_{yy} & I_{yz} & q_y \\ I_{xz} & I_{yz} & I_{zz} & q_z \\ q_x & q_y & q_z & m \end{vmatrix}$$

where $q_x = mx_g$ $\qquad I_{xx} = \int x^2 dm$ $\qquad I_{xy} = \int xy\, dm$

f represents the resultant of the forces F and torques (or couples) C applied to a body k with respect to the frame (s). In a similar way, Γ is the momentum tensor of the body k with respect to a frame (s); r represents the linear momentum of the body and g is its angular momentum. The fundamental dynamic relationships can be written as:

$$\Gamma_k = W_{0k}\, J_k - J^t_k\, W^t_{0k} = \text{skew}\{W_{0k}\, J_k\}$$
$$\phi_k = H_{0k}\, J_k - J^t_k\, H^t_{0k} = \text{skew}\{H_{0k}\, J_k\}$$

where '0' denotes an inertial frame. The first equation shows the link between the momentum of a body k, with respect to a frame '0', and its velocity; the second equation is the dynamic equilibrium equation of the body k having absolute acceleration H and subjected to the action ϕ. The skew operator is defined, for any square matrix X, as follows:

$$\text{skew}\{X\} = X - X^t$$