

THE ARCHER'S PARADOX

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INTRODUCTION: The archer's paradox is the fact that an arrow does not fly to its mark along the line represented by its axis. The explanation of the archer's paradox was found by means of high speed spark photography which P. E. Klopsteg undertook in order to secure direct evidence of what an arrow does as it leaves the bow [1]. The impulse normal to the axis of the arrow, caused by the release of the fingers from the string, as well as the column-like force of the string on the arrow during its acceleration, results in a significant bending of the arrow shaft as it transits the bow. This allows the arrow to undulate around the bow handle and follow a straight course towards its target without striking the bow handle. P. E. Klopsteg explained this paradox and provided a qualitative understanding of the reasons for matching arrows to a given bow and archer combination [2]. The objective of our study is to develop the mathematical methods of the archer's paradox.

METHODS: A complete mechanical-mathematical model of an archer-bow system is being created. The model takes into consideration the impulse normal to the axis of the arrow, caused by the release of the archer's fingers from the string and the typical special features of the modern sports bow: the non-linear bend of bow limbs, the elastic string, arrow and stabilizers, asymmetric bow arrangement in its flatness [3]. The potential V and kinetic T energy of the transverse motion system in the perpendicular to the bow flatness plane xOy (Fig. 1) is:

$$V = \frac{1}{2} \int_0^l EJ (y'')^2 dx - \frac{a}{2} \int_0^l \left[\rho F \int_0^x (y' - 2u') y' d\chi + m_1 (y' - 2u') y' \right] dx + \frac{1}{2} c (y_2 - y_0)^2, \quad T = \frac{1}{2} m_0 \dot{y}_0^2 + \frac{1}{2} \int_0^l \rho F \dot{y}^2 dx + \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2, \quad (1)$$

where EJ is the bending rigidity, ρ is the mass of a unit volume of material, F is the cross-sectional area, l is the length of the arrow, m_0 is the mass of the arrow's tail with virtual mass of the string (1/3 part [1]), m_1 is the mass of the arrow's point, m_2 is the virtual mass of the bow limbs, c is the virtual rigidity of the bow limbs and the string, u is the initial curvature of the arrow, y is the total deflection of the arrow, a is the longitudinal acceleration of the arrow motion, (\cdot) and $(')$ are derivatives with respect to time t and the longitudinal co-ordinate x . Using the results of videoanalysis [3] it is possible to

approximate the arrow's form as: $y = y_0 + y_3 \frac{x}{l} + y_4 \sin \frac{\pi x}{l}$ (2) where y_0, y_3, y_4 are the independent variables.

Lagrange equations of the second kind for the system are:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}_i} \right) + \frac{\partial V}{\partial y_i} = 0, \quad i = 0, 2, 3, 4 \quad (3).$$

$$M_\alpha = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \alpha_2^{(1)} & \alpha_2^{(2)} & \alpha_2^{(3)} \\ 0 & \alpha_3^{(1)} & \alpha_3^{(2)} & \alpha_3^{(3)} \\ 0 & \alpha_4^{(1)} & \alpha_4^{(2)} & \alpha_4^{(3)} \end{bmatrix},$$

$$M_S = \begin{bmatrix} S_0 + C_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}, M_u = \begin{bmatrix} 0 \\ 0 \\ u_1 \\ 0 \end{bmatrix}, M_R = \begin{bmatrix} R_0 + B_0 \\ R_1 \omega_1 \\ R_2 \omega_2 \\ R_3 \omega_3 \end{bmatrix}, M_v = v \begin{bmatrix} 1 \\ 0 \\ b_3 \\ b_4 \end{bmatrix}.$$

RESULTS: A complex equipment-program method has been developed to measure characteristics of an archer's motions when he executes and prepares a shot. The acceleration field of the archer's body links has been modeled in terms of the quasi-static approach at a moment of the closed kinematic chain of the brachial belt and extremities breaking at the moment of bowstring release. An assumption has been substantiated about the invariability of joint moment in the archer's body at the time of the arrow and bowstring joint movement as a result of a comparison of regularities in man's muscular activity and temporal characteristics of the bow's shot. The archer's paradox model, making clear the process in which the arrow undulates the bow handle, is being substantiated. The sphere of the deflection curve of the arrow and the interdependence of sports arm parameters with shot parameters are assumed. Methods for the correction of bow and arrow parameters in consideration of the archer's individual peculiarities are being developed. The model and methods were approved for one of the best Ukrainian sportsman-archer with data parameters: $l=70\text{cM}$, $h=50\text{cM}$, $m=20\text{G}$, $m_0=4\text{G}$, $m_1=5\text{G}$, $m_2=100\text{G}$, $EJ=6.18\text{ NM}^2$, $c=720\text{N/M}$, $a=3970\text{M/S}^2$, $v=8\text{cM/S}$. The internal data results are:

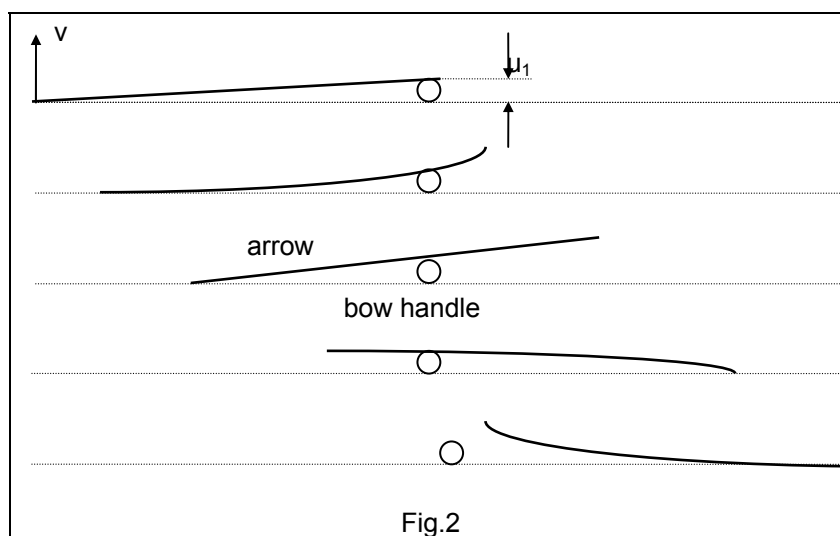
$$\omega_1 = 170\text{S}^{-1}, \omega_2 = 138\text{S}^{-1}, \omega_3 = 284\text{S}^{-1}, \omega_6 = 446\text{S}^{-1}, \alpha_2^{(1)} = 0.199,$$

$$\alpha_2^{(2)} = -0.602, \alpha_2^{(3)} = -0.0981, \alpha_3^{(0)} = 0, \alpha_3^{(1)} = -2.46, \alpha_3^{(2)} = -11.9,$$

$$\alpha_3^{(3)} = -0.628, \alpha_4^{(0)} = 0, \alpha_4^{(1)} = -0.935, \alpha_4^{(2)} = 16.5, \alpha_4^{(3)} = -0.767,$$

$$\alpha_3^{(4)} = -3.98, \alpha_3^{(6)} = -0.120, \alpha_4^{(4)} = 0, \alpha_4^{(6)} = -2.14, \alpha_2^{(0)} = 1,$$

where $\omega_6, \alpha_3^{(4)}, \alpha_4^{(4)}, \alpha_3^{(6)}, \alpha_4^{(6)}$ are belong a free flight when the arrow leaves the string at $t > \tau = 16\text{mS}$. The final results of the computer simulation are present in Fig. 2. The optimal start position of the arrow is $u_1 = -2.78v\tau = 3.5\text{mM}$.



CONCLUSIONS:
 A result analysis enables us to claim that bowstring stiffness influences bend oscillations of the arrow more significantly than the bow's mass, all other conditions being equal. The ability of the arrow to follow a straight course to its target requires

that its bending mode period be matched to the time required by the arrow to leave the bow. When not properly matched, the arrow strikes the bow handle and is deflected off its intended course. A set of experimental investigations of an archer-bow system has been carried out using the developed methods of the mathematical simulation that enable substantiation and confirmed the reliability of the modeling results.

REFERENCES:

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