APPARENT COEFFICIENT OF RESTITUTION SURFACE OF A PADDLE RACKET

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INTRODUCTION: Skilled players can feel whether a racket has a good response only in a restricted area of the surface or whether it responds favorably to eccentric impacts. Racket physical characteristics should provide high speed ball response upon impacts on a large area of the racket surface.
HATZE (1993) defined the ACOR, and since then, many authors have measured the restitution of tennis rackets at discrete points of their faces (Baker & Putnam 1978, Elliott 1982, Grabiner, Groppel, & Campbell, 1983, Hennig, Milani & Rosenbaum 1993, etc.).The aim of this study was to determine the theoretical ACOR surface for impacts on a free and at rest paddle racket when shooting normal to its impact surface. This theoretical model was then compared with experimental ACOR measurements in two different situations: free racket and hand-held by a player.

Theoretical Model: An impact is considered direct if the object's movements immediately before impact are translations in normal directions to the contact surfaces, and is considered central if the normal impact passes through the center of mass (COM) of each object. Ball-racket impact can be considered a direct and non-central impact. For the purposes of this theoretical model, the *coefficient of restitution* (COR) of the ball-racket impact is considered constant for all of the racket impact area. This simplification is justified because the COR is only a function of the pre-impact velocity and the mechanical properties of the materials. Another simplification for the study of this problem is the use of a bi-dimensional space.

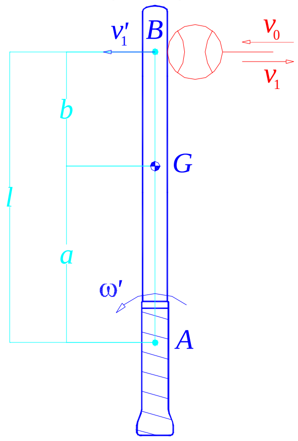

Figure 1

Ball velocities are v_0 and v_1 before and after impact respectively; the racket is initially at rest and rotates around point *A* after impact (Figure 1), so *A* is the pivot for the center of percussion (COP) *B*. The racket angular velocity is ω', with $v_1'=\omega' l$ the liner velocity of *B.* The racket COM is represented by *G*.
The COR of the ball-racket interaction is: COR=$e=v_1-v_1'/v_0$, where *v* represents the absolute value for the velocities. The ACOR was defined in HATZE (1993) as the quotient of ball velocities, after and before impact, COR=$e_a=v_1/v_0$, so, $e_a=e-v_1'/v_0$.
Making an angular momentum balance of the system, before and after impact, with respect to point *A* in Figure 1, results: $mv_0 l = I_A\omega' - mv_1 l$,where I_A is the racket moment of inertia with respect to an axis normal to the plane of Figure 1 passing through point *A* and *m* is the ball mass. After replacing ω', v_0 and v_1 and using the Steiner theorem results:

$$e_a = \frac{e[I_G + m'(l-b)^2] - ml^2}{[I_G + m'(l-b)^2] + ml^2},$$

where *m*' is the racket mass. Distance *l* between the COP and the pivot can be expressed by equation $l = I_B / m'b$ (Brody 1979), where I_B is the racket moment of inertia with respect to an axis normal to the plane of Figure 1 passing through point *B*, and *b* is the distance between COM and COP. Using the Steiner theorem the previous equation can be written, $l - b = I_G / m'b$; replacing *l-b* and *l* and doing some algebra, finally results in:

$$e_a = \frac{e - \frac{m}{m'}\left(1 + \frac{m'b^2}{I_G}\right)}{1 + \frac{m}{m'}\left(1 + \frac{m'b^2}{I_G}\right)}.$$

According to this model, the function ACOR(*b*) (where *b* is the distance between the impact point and the COM) is with good approximation a parabola with the maximum at the COM. According to this result, elliptical paraboloids were adjusted to the experimental data ACOR(*x,y*), since this quadric represents parabolas and ellipses when sectioned by Cartesian planes.

METHODS AND PROCEDURES: Laboratory experiments were carried out using several paddle rackets by means of a set-up intended to measure ball velocity and point of impact, with a *Peak Motus* motion analysis system, based on 180 Hz video cameras and an automatic tracking software. The ball trajectory was reconstructed in 3-D using two synchronized cameras, calculating the average velocity of the ball before and after racket contact, and the ACOR as the quotient of these two velocities. A third camera was used to locate the point of impact. Measurements were done using two pre-impact ball velocities (17 and 30 m/s) and two types of grip support: free and held by a player's arm.

The ACOR shows important changes according to the impact location, so collisions were distributed over the racket face, making possible the construction of a three dimensional mesh to represent it. The theoretical model used for reconstruction of the restitution surface for each of the impact series considered the mechanical system arm-racket-ball symmetrical with respect to the racket longitudinal axis. The generic quadratic equation is: $e_a = B_0 + B_1x + B_2y + B_3x^2 + B_4xy + B_5y^2$. An elliptical paraboloid, symmetrical with respect to the *y* axis has coefficients of the *x* and *xy* terms equal to zero.

The quadratic surfaces were adjusted to experimental data by the mean square method, using the multiple regression module of the *STATISTICA* software, defining ACOR as the dependent variable and y, x^2 e y^2 as the independent ones. According to these analyses, the four coefficients of the elliptic paraboloid, standard error of each coefficient and general standard error of the data were determined.

For clear visualization of ACOR on the racket impact area, restitution surfaces were represented as level curves, scaled to the racket's draw. Figure 2 shows one of these representations done with *AUTOCAD* software.

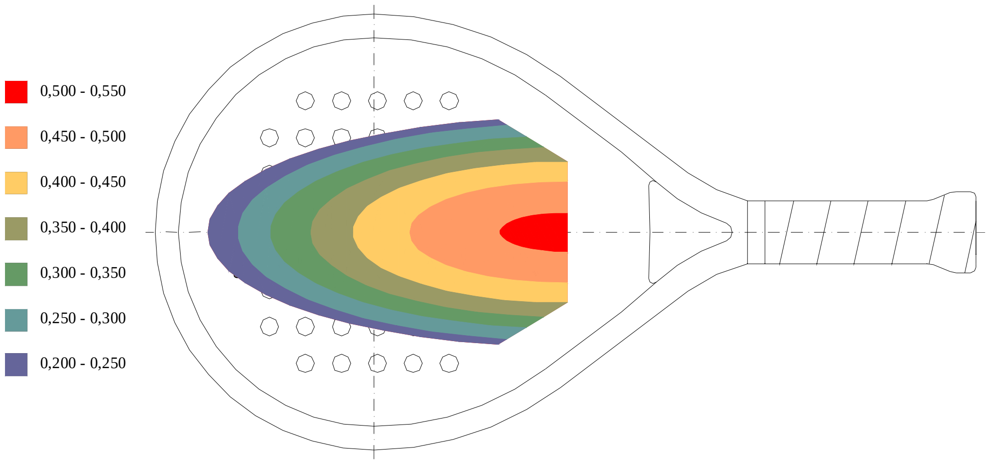

Figure 2 – ACOR of *Tecno Air* racket, hand-held, pre-impact velocity of 17.38 m/s.

RESULTS AND DISCUSSION:

Experimental ACOR data adjusted by the mean square method showed a very good fit to the theoretical surface, and also to the approximated elliptical paraboloid, average error of the fit equal to 0.01. Except for B_0, all coefficients of this elliptical paraboloid are only a function of the racket physical parameters and ball mass. These relations allow us to calculate the restitution surface without the need for costly experimentation, just knowing the masses of the racket and ball, the two principal racket moments of inertia and the restitution coefficient at some point. These methods used for calculation and illustration of the ACOR surface on the rackets makes possible a real and complete evaluation of the racket's response.

Table 1 – Coefficients of elliptic paraboloids representing ACOR surfaces, standard error of each coefficient and general standard error of data.

Racket	Grip	Vel*** (m/s)	N‡	S****	B_0*	s_{B0}**	B_2 E-05	s_{B2} E-05	B_3 E-05	s_{B3} E-05	B_5 E-05	s_{B5} E-05
Extender (2-A)	Free	18.58	19	0.008	0.450	0.003	11	8	-3.9	0.4	-1.27	0.08
Extender (2-C)	Arm	29.09	19	0.013	0.393	0.003	-68	7	-4.7	0.6	-0.79	0.07
Dunlop Impact	Arm	27.06	11	0.010	0.398	0.006	-80	20	-3.5	0.4	-0.49	0.14
Dunlop Impact	Arm	17.55	12	0.010	0.497	0.007	-68	16	-4.8	2.1	-0.76	0.14
Kennex Asym.	Arm	29.38	16	0.007	0.372	0.003	-21	10	-4.3	0.4	-1.00	0.08
Proto 3B	Arm	29.49	15	0.011	0.400	0.005	-18	18	-3.4	0.6	-1.15	0.2
Pro. Fina C1,6	Arm	29.11	15	0.013	0.373	0.006	12	17	-3.7	0.5	-1.04	0.12
Prot Fina C4,5	Arm	15.90	23	0.014	0.442	0.01	116	24	-2.9	0.6	-1.64	0.14
Racket	Grip	Vel*** (m/s)	N‡	S****	B_0*	s_{B0}**	B_2 E-05	s_{B2} E-05	B_3 E-05	s_{B3} E-05	B_5 E-05	s_{B5} E-05
Smashing cinz	Arm	28.72	12	0.004	0.398	0.002	-55	5	-5.1	1.2	-0.89	0.05
Smashing (S1)	Arm	16.30	11	0.008	0.524	0.005	-41	16	-4.0	0.6	-0.96	0.13
Smashing (S1)	Free	17.57	13	0.011	0.484	0.005	53	14	-4.0	1	-1.68	0.15
Smashing (S2)	Arm	16.33	22	0.012	0.505	0.004	-36	14	-4.4	0.6	-1.04	0.12
SmasOca (S0)	Arm	15.94	11	0.007	0.517	0.006	39	18	-4.1	1.2	-1.58	0.15
SmasOca (S0)	Free	18.41	16	0.007	0.498	0.004	26	7	-4.8	1	-1.56	0.08
Smash Oca R.	Arm	29.36	16	0.014	0.432	0.004	-13	12	-3.8	0.6	-1.14	0.1
Steel Amarela	Arm	29.77	16	0.008	0.364	0.003	4	9	-3.0	0.4	-1.11	0.08
Steel Amarela	Free	17.08	16	0.007	0.445	0.003	53	8	-5.1	0.5	-1.43	0.09
Steel Vermelh	Arm	30.02	12	0.016	0.389	0.007	3	18	-4.3	0.9	-1.10	0.17
Tecno A (Oca)	Arm	26.50	11	0.011	0.448	0.007	-48	28	-4.2	0.5	-1.05	0.22
Tecno A (Oca)	Arm	17.38	11	0.007	0.547	0.006	-66	14	-7.3	1.2	-0.81	0.11

* Quadratic coefficients $e_a = B_0 + B_1x + B_2y + B_3x^2 + B_4xy + B_5y^2$.

** Standard error of each coefficient.

*** Average pre-impact velocity in each series

**** General standard error of data.

‡ Number of impacts in each series.

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