A SIMPLE MATHEMATICAL MODEL FOR ESTIMATING FORCE AND TORQUE TO THE WRIST, ELBOW AND SHOULDER IN SERVING, SPIKING AND BLOCKING IN VOLLEYBALL

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INTRODUCTION: Volleyball spiking, blocking and high speed serving may cause shoulder dislocation and subluxation, elbow contusions, sprained wrists, finger sprains, fractures and dislocations. Such damage may result from the impulse caused by a simple spike with a velocity of, e.g., 25m/s. If contact time is about 0.01 second, then according to the second law of Newton we can estimate the force of impulse as:

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = \frac{25 \times 0.25}{0.01} = 625N$$

The ball velocity at the instant of impact can be considered as zero. This force can apply torque to the wrist, elbow and shoulder; we can estimate the force applied to the muscles in the joints. This is feasible if we can measure contact time dt by any means and the velocity of the ball VBo. We considered a mechanical model for which we used a mathematical relationship with which we can easily estimate the velocity of the ball, force and torque applied to the upper extremity.

METHODS: In order to estimate ball velocity in spiking or serving, for example, we designed a ballistic pendulum consisting of a large wooden block of mass $M_p = 10Kg$, suspended by cords of length l=7m. This pendulum is essentially used for determining the velocity of bullets(D. Hallidy and D. Resnick 1992). A bullet of mass m traveling with a horizontal speed Vi strikes the pendulum and remains embedded in it. The collision time in this case is considered very small compared to the time of swing of the pendulum; the supporting cords remain approximately vertical during the collision. This in mind, we adopted the rule of the pendulum, in order to determine the velocity of the ball.
In our case the ball is not, as in the in the case of a bullet, embedded in the pendulum and also is not similar to the case of a collision between two ivory or glass balls. We had therefore to consider the coefficient of restitution, e, in our calculations (J.G. Hay 1986).

Mathematical model
a) ball velocity

According to restitution coefficient definition as

$$\frac{V_B}{V_{Bo}} = -e \qquad (1)$$

the conservation of momentum can be written as

$$m_B V_{Bo} + m_B e V_{Bo} = M_p V_P \qquad (2)$$

we can, on the other hand write

$$\tfrac{1}{2} M_P V_p^2 = M_P gh \qquad (3)$$

combining equations (2) and (3) we obtain

$$V_{Bo} = \frac{M_p \sqrt{2gh}}{(1+e) m_B} \qquad (4)$$

where V_{Bo} is the ball initial velocity, Mp and m_B are pendulum and ball mass respectively, e, the restitution coefficient and h, the vertical displacement gained by the impact.
b) pendulum velocity
At impact, the pendulum gets kinetic energy $\frac{1}{2} M_P V_p^2$ which, neglecting air resistance, is transferred to its potential energy, Mpgh.
In fact, from (3) we can get for Vp as

$$V_{Po} = \sqrt{2gh} \qquad (5)$$

e) Force exerted on ball
In order to estimate the force exerted on the ball, we use Newton's second law as

$$F = dp / dt \qquad (6)$$

from which we can deduce this force as

$$F_B = m_B V_{Bo} / \Delta t \qquad (7),$$

in which Δt is the contact time between hand and ball. This impact time was measured by a timer with a precision of 10^{-3} seconds, which was especially designed for this purpose, (Shahbazi Moghaddam et al. 1996).
d) Force exerted on pendulum
From equation (2) and using Newton's second law, we can easily get for the force exerted to the pendulum as

$$F_P = \frac{m_B V_{Bo} (1+e)}{\Delta t_P} \qquad (8)$$

Δt_P is the time of impact between ball and pendulum measured by the timer designed.

Experimental
This pendulum was hung from the ceiling and the ball was hung in front of the pendulum and was positioned on its longitudinal axis. The players smashed the ball and caused the pendulum to deviate and reach its maximum height. The

displacement of the center of gravity, S, and the height it could gain by the transfer of kinetic energy, h, can be obtained from two triangles; one made of pendulum length l and displacement S and the height it could gain by the transfer of kinetic energy, h, can be obtained from tow triangles; one consisting of pendulum length, l, and displacement, S, and the other consisting of the displacement, S, and the height, h.These two triangles form two sides perpendicular to each other, therefore they are equivalent and we obtain

$$S / l = h / S \cong \Theta \qquad (9)$$

whereas $$S = \sqrt{l.h} \qquad (10)$$

By filming at the appropriate wide angle and displaying, we could get Θ and then the *S be* and h could be easily obtained. These data can be used for determining *Vý* Bo and *V* p, which in turn can make the other parameter to be obtained. The impact tim $\Delta\ tB$ and Δtp were achieved by the timer especially designed (Shahbazi et al. 1996).

We used aluminum foil on the hand and on the ball and also on the pendulum in order to get the impact time.

RESULTS: By projecting the video tape on the wall with the video projector, we could get the angle Θ with enough precision. The impact time of the hand-ball and ball-pendulum were measured by the timer. The impact time of the hand-ball, depending on the skills of the players, varied from 0.005 sec. to 0.012 sec., while for the ball-pendulum it varied between 0.011 sec. And 0.017 sec. With these data we could easily find the mechanical parameters which are given in Table 1.

Table 1. Mechanical parameters versus deviation angle Θ

Θ ***Rad.*** — **Mech.Para**	**0,0175**	**0.035**	**0.053**	**0.071**	**0.088**	**0.105**
h **cm**	0.22	0.86	2.04	3.42	5.43	7.72
S **cm**	12.31	24.50	37.02	49.21	61.62	73.50
***V*Bo** **m/s**	4.75	9.38	14.31	18.65	23.51	28.11
Vp **m/s**	0.21	0.41	0.63	0.81	1.03	1.23
FB **N**	84.82	167.52	255.36	333.04	419.82	501.79
Fp **N**	70.0	136.67	210	270	343.33	410

CONCLUSION: The method proposed is simple and practical and can be applied to other sports such as football and handball. A simple camcoder of 60 Hz would be enough to go through the measurements. Of course a camcoder with a higher speed would offer greater precision. Considering the mechanical aspect is essential.

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