# DRAG AND FRICTION COEFFICIENTS IN ROLLER SKATING. AN INDIRECT DETERMINATION. SOME SUGGESTIONS ABOUT TRAINING LOADS 

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INTRODUCTION: Only a few biomechanical studies of roller skating are available. There are differences between roller skating and ice skating, and consequently it is not correct to transfer a model from one of the two sports to the other. Roller skaters move against resistance due both to air and to the friction of their skate wheels. No experimental data are available regarding these values, and consequently an estimate of the power required to roller skate at different speeds is not possible. The purpose of this work is to estimate the frictional forces by an indirect method and then use these values to express the mechanical power required to skate.

METHODS AND PROCEDURES: By using Newton's second law for a skater who is slowing down from a given $v_{0}$ under the effect of passive forces (air drag and wheel friction), it is possible to obtain the relationship between $s$ and $t$. These data can be easily collected without any expensive equipment.
In order to fit the s/t data, two coefficients must be calculated: the drag coefficient $\boldsymbol{c}_{\boldsymbol{x}}$ and the friction coefficient $\boldsymbol{k}$. A third coefficient is required to get an estimation of the body surface; this section is described in appendix $A$.
Experimental data of $\boldsymbol{t}$ and $\boldsymbol{s}$ were collected for six athletes from the Italian National team. The test was carried out in Scaltenigo (Venice, Italy), in the 200m rink used for international competition. The rink was divided into four sections 46.77 m , $46.77 \mathrm{~m}, 53.23 \mathrm{~m}, 53.23 \mathrm{~m}$ ) using four lines. Each test consisted of two different parts: in the first part the athletes were required to skate at constant speed through three sections; then they stopped pushing and went on rolling through three more sections. Split times were recorded at each line. The initial speed $\boldsymbol{v}_{0}$ was calculated from the times of the constant speed part by using the numerical first derivative. The other three split times were used in the $t / s$ comparison. For each athlete we measured height and weight and took some pictures in frontal view to estimate the body surface. All athletes but one made two tests with two different kinds of wheels, the most commonly used in high level skating. Type 1 wheels are softer (and therefore generate higher friction values) than type 2; the data obtained from each type generate a "set". Weather conditions were the following: $\mathrm{T}=23^{\circ} \mathrm{C}, \mathrm{p}=$ 1016 hPa , wind speed $<2 \mathrm{~m} / \mathrm{s}$. The value of air density $\rho$ was estimated using the perfect gas equation, obtaining $\rho=1.20 \mathrm{Kg} / \mathrm{m}^{3}$.
Imagine a skater moving at $\boldsymbol{v}_{\boldsymbol{0}}$ at $\boldsymbol{t}=\mathbf{0}$. When he/she stops pushing there are only
two horizontal forces applied, both opposed to $\boldsymbol{v}$. The drag resistance is given by:

$$
\begin{equation*}
F_{a}=\rho c_{x} S v^{2} \tag{1}
\end{equation*}
$$

It is not easy to determine the body surface, that is the surface of the silhouette of the body moving through the air. However, the skater is not a rigid body, so his/her shape changes throughout time. The same applies to $c_{x}$, which takes into account the 3-D shape of the body. It is not possible to evaluate these values at any different position, so we assume that an average value can be used. Appendix $A$ shows how to relate $\boldsymbol{S}$ to the body mass $\boldsymbol{m}$ and the body height $\boldsymbol{h}$.

The second force which slows down the skater is due to wheel friction. Usually the friction force is written as follows:

$$
\begin{equation*}
F_{w}=k_{w} P / r=k_{w} m g / r \tag{2}
\end{equation*}
$$

In this equation the coefficient $\boldsymbol{k}_{\boldsymbol{w}}$ has the same dimensions of $\boldsymbol{r}$, the radius of the wheel, and $F_{w}$ does not depend on speed. To simplify the mathematics, however, we propose a different equation:

$$
\begin{equation*}
F_{w}=k m v \tag{3}
\end{equation*}
$$

The new friction coefficient $\boldsymbol{k}$ can be related to the previous one at a given speed.
Now we can write the $2^{\text {nd }}$ Newton's law (the minus sign takes into account that both forces slow down the athlete):

$$
\begin{align*}
& F_{w}+F_{a}=-m a  \tag{4}\\
& k m v+\rho c_{x} S v^{2}=-m a
\end{align*}
$$

The equation (4') is a relationship between $\boldsymbol{v}$ and $\mathbf{a}$; both these variables are time functions, so that the equation (4') can be rewritten in differential form:

$$
\begin{equation*}
k m d s / d t+\rho c_{x} S(d s / d t)^{2}=-m d^{2} s / d t^{2} \tag{5}
\end{equation*}
$$

After double integrating the equation (5), under the boundary conditions $t=\mathbf{0}, \boldsymbol{v}=\boldsymbol{v}_{\boldsymbol{o}}$, $s=0$, we have:

$$
\begin{align*}
& v=k m v_{0} /\left[\left(k m+v_{0} \rho c_{x} S\right) e^{k t}-v_{0} \rho c_{x} S\right]  \tag{6}\\
& s=m /\left(\rho c_{x} S\right) \ln \left[1+v_{0} \rho c_{x} S\left(1-e^{-k t}\right) /(k m)\right] \tag{7}
\end{align*}
$$

The equations (6) and (7) are rather complicated, even if the starting assumptions are quite simple. In any case, they contain only two unknown coefficients ( $\boldsymbol{c}_{x}$ and $\boldsymbol{k}$ ) and set up the relationship between $t$ and $s$ we were looking for.

A computer program was written to get the best-fit solution for the two unknown coefficients. In a first stage all data were used together, regardless of the kind of wheels used, to obtain an average value for the $\boldsymbol{c}_{\boldsymbol{x}}$ and $\boldsymbol{k}$. Later, using the two sets of data and without allowing any further change to $\boldsymbol{c}_{\boldsymbol{x}}$, two $\boldsymbol{k}_{\mathbf{1}}$ and $\boldsymbol{k}_{\mathbf{2}}$ values were calculated. The program minimizes the residual error of the model, expressed as $\Sigma\left(1-\mathrm{s}_{\text {calc }} / \mathrm{s}_{\mathrm{exp}}\right)^{2}$.

| Definitions |  |  |  |
| :--- | :--- | :--- | :--- |
| $A$ | Acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Air pressure $(\mathrm{hPa})$ |  |
| $B$ | Body area coefficient (dimensional. | $P$ | Body weight $(\mathrm{N})$ |
|  | $\boldsymbol{h}$ and $\boldsymbol{m}$ in IS units) | $\rho$ | Air density $\left(\mathrm{Kg} / \mathrm{m}^{3}\right)$ |
| $C_{x}$ | Drag coefficient (dimensionless) | $\rho_{b}$ | Body density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| $F_{a}$ | Drag resistance (N) | $r$ | Wheel radius $(\mathrm{m})$ |
| $F_{w}$ | Wheel resistance $(\mathrm{N})$ | $s$ | Distance from the start (m) |
| $G$ | Gravity acceleration $\left(=9.81 \mathrm{~m} / \mathrm{s}^{2}\right)$ | $S$ | Body surface $\left(\mathrm{m}^{2}\right)$ |
| $H$ | Body height $(\mathrm{m})$ | $t$ | Time $(\mathrm{s})$ |
| $I$ | Surface index $(\text { Kg m })^{1 / 2}$ | $T$ | Temperature $\left({ }^{\circ} \mathrm{C}\right)$ |
| $K$ | Wheel friction coefficient $(1 / \mathrm{s})$. | $v$ | Speed $(\mathrm{m} / \mathrm{s})$ |
|  | Proposed form | $v_{0}$ | Speed at the start (m/s) |
| $K_{w}$ | Wheel friction coefficient $(\mathrm{m})$. | $V$ | Volume $\left(\mathrm{m}^{3}\right)$ |
|  | Standard form | $W$ | Body width $(\mathrm{m})$ |
| $M$ | Body mass $(\mathrm{Kg})$ | $W$ | Mechanical power $(\mathrm{W})$ |

RESULTS AND DISCUSSION: The proposed equation and the experimental data were used to obtain the drag coefficient and the wheel friction coefficient under the test conditions.
The numerical solution gave the following values:

$$
\begin{aligned}
\boldsymbol{c}_{x} & =0.318 \\
\boldsymbol{k} & =0.0137 \mathrm{~s}^{-1}
\end{aligned}
$$

$$
k_{1}=0.0145 \mathrm{~s}^{-1}
$$

$$
k_{2}=0.0130 \mathrm{~s}^{-1}
$$

In our method there were three facts which are not completely correct: considering the wheel friction dependent on $\boldsymbol{v}$; considering $\boldsymbol{c}_{\boldsymbol{x}}$ and $\boldsymbol{S}$ as constant during skating; the wind speed was not 0 during data collection. Even so, the results were quite good.


After getting the drag and friction coefficients, it is possible to draw a graph showing the relationship between speed and the required mechanical power $\boldsymbol{W}=\boldsymbol{F}$ $\boldsymbol{v}$. The following graph refers to three hypothetical athletes, representing a young skater (A), a high level female (B) and a high level male skater (C).


As we can see, in the range $8-12 \mathrm{~m} / \mathrm{s}$, which is close enough to the actual test conditions, the power curve is very steep: $10 \%$ speed increase from $10 \mathrm{~m} / \mathrm{s}$ to $11 \mathrm{~m} / \mathrm{s}$ leads the athlete $C$ from 270 W to 340 W (26 \% increase).
Two suggestions can be given to the coaches: when using skates, keep the speed as high as possible to avoid sudden reductions of training load; when not using skates, choose different exercises requiring mechanical power in the range of 200-500 W , depending on the athlete.

CONCLUSION: With the model proposed, it was possible to calculate the external forces required for a given skater in a known environment for skating at different speeds. It was possible to compute the mechanical external power required to roller skate. In this way it is possible to compare the roller skating to other sport activities. It was also possible to make important suggestions for coaches about the kind of training loads to use.

## APPENDIX A - ESTIMATING THE BODY SURFACE

The only available data from an athlete are the height $\boldsymbol{h}$ and the mass $\boldsymbol{m}$. We suggest assuming the body surface to be proportional to the height and the "width" $\boldsymbol{w}$ of the body. If we consider a cylinder (diameter $=\boldsymbol{w}$, height $=\boldsymbol{h}$ ) having an average density of $\rho_{b}$, its mass is given by:

$$
\begin{equation*}
m=\pi w^{2} / 4 h \rho_{b} \tag{8}
\end{equation*}
$$

The "width" can be calculated by solving the equation (8) as follows:

$$
\begin{equation*}
w=\left[4 m /\left(\pi h \rho_{b}\right)\right]^{1 / 2} \propto[m / h]^{1 / 2} \tag{8’}
\end{equation*}
$$

The body surface can be supposed to be proportional to the vertical section of such a cylinder:

$$
\begin{align*}
& S \propto h w \propto(m h)^{1 / 2}  \tag{9}\\
& S=B(m h)^{1 / 2}=B i \tag{10}
\end{align*}
$$

The term $\boldsymbol{i}=(\boldsymbol{m} \boldsymbol{h})^{1 / 2}$ contains both data about the "size" of the athlete and is defined as a body index.
Experimental data were recorded by measuring $\boldsymbol{h}$ and $\boldsymbol{m}$ for several athletes (the six mentioned in the main part of this work, plus a group of 4 young boys and girls in the range of 12-18 years, to have information at smaller sizes). For each athlete one or two pictures were taken in a frontal view, having in sight a square of known size ( $1 \mathrm{~m} \times 1 \mathrm{~m}$ ). The athletes simulated different skating attitudes, so that the picture could be used to evaluate $\boldsymbol{S}$.


Each picture has been digitized and a computer program gave the real surface. It is no wonder if the same athlete could have two different values of $\boldsymbol{S}$, depending on his/her position. For each athlete the $\boldsymbol{i}$ index was calculated as above Eventually, the $\boldsymbol{B}$ coefficient was calculated as the average value, obtaining $\boldsymbol{B}=\mathbf{0 . 0 3 7}$. The last graph shows the comparison between experimental and calculated $S$. The mean square error is about $9 \%$, which is lower than the difference a single athlete can show (16\%).

## REFERENCES

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