

## COMPUTER SYNTHESIS AND OPTIMIZATION OF JUMPING MOTIONS VIA NONSTATIONARY CONSTRAINTS

Victor A. Sholukha, Anatoly V. Zinkovsky, Alexandre A. Ivanov,  
St. Petersburg State Technical University, Russia

**INTRODUCTION:** This paper considers the results of the authors' research on the goal-oriented computer synthesis of human motions in support and non-support phases. The main attention is paid to the synthesis of pushing phases. In particular, an analysis of running long jump and acrobatic jump sequential optimization results is made. The computer modeling of complex coordination motions is based on the development of an adequate anthropomorphic model (AM). The employment of differentiated non-stationary holonomic and non-holonomic constraints equations proved to be most effective in the developed modeling system for the purpose of goal-oriented motions modeling [1].

**METHODS:** Let us consider the motion equations of a general type with non-stationary constraints. As a rule, the right side of these equations includes the vector-column of generalized forces  $U(t, q, \dot{q})$ , where  $q, \dot{q}$  - vector-column of generalized coordinates and its derivative with respect to time  $t$ , correspondingly. For the goal of the motions' simulation  $U$  can be presented as a sum of three parts. From inverse problem of dynamics  $U$  can be calculated as  $U = U_1(t)$ . Parametric control of the model via selection of characteristics of springs and dampers in joints requires the representation of  $U$  in the form  $U = U_2(q, \dot{q})$ . And, finally, non-stationarity of constraint equations actually requires that equation  $U = U_3 = -P'\lambda$  be satisfied, where  $P'$  - Jacobian matrix of constraint equations,  $\lambda(t, q, \dot{q})$  - vector-column of Lagrange multipliers. Thus, the structure of generalized forces column  $U(t, q, \dot{q})$  should be as follows:

$$U(t, q, \dot{q}) = U_1(t) + U_2(q, \dot{q}) - P'\lambda(t, q, \dot{q}) \quad (1)$$

For descriptions of additional non-stationary items in constraints equations we used parametrically controlled smooth approximation functions which allowed us to synthesize desired motion trajectories, ground reaction forces and kinetic moment increments. For example, they can be represented as polynomial functions of the 5th order, which allows us to set boundary conditions for coordinates and velocities and keep the continuity of accelerations for the adjacent parts of complex non-stationarities. Another kind of function useful for ground reaction force simulations can be presented as a combination of power functions:

$$N_{1y}(t) = A_0 \left[ \frac{(4t^{(q_1+1)}(T-t))^{p_1}}{T^2} - A_1 \frac{(4t^{(q_2+1)}(T-t))^{p_2}}{T^2} \right], \quad (2)$$

where included parameters define the shape and amplitude of the components of the support force and  $T$  is the duration of the support phase of motion. Further, let us consider some types of constraint equations. The most general case is when the absolute or relative motion of certain AM points, for example, their

joints, is preset. In particular, for two arbitrary joints with corresponding numbers  $k$  and  $j$ , the vector constraint equation is:

$$\vec{R}_k - \alpha_k \vec{R}_j - \vec{R}_k^0(t) = 0, \quad (3)$$

where  $\vec{R}_k, \vec{R}_j$  - absolute radius-vectors of joints  $k$  and  $j$ ;  $\alpha_k$  - parameter-indicator of absolute (=0) or relative (=1) motion.

For the preset motion of the center of mass ( $\vec{R}_c$ ) of the system and/or its kinetic moment ( $\vec{k}$ ) the next constraint equations in differential form can be used:

$$\begin{aligned} (\ddot{\vec{R}}_c - \vec{g})M^c - \vec{N}_1(t, q, \dot{q}) &= 0, \\ \dot{\vec{k}} - (\vec{r}_0 - \vec{R}_c) \times \vec{N}_1(t, q, \dot{q}) - \vec{M}_1(t, q, \dot{q}) &= 0 \end{aligned} \quad (4)$$

In these equations  $\vec{N}_1, \vec{M}_1$  - external force and torque applied to the model;  $\vec{r}_0$  - absolute radius-vector of support point.

It should be noted that among the above-listed possibilities of movement simulation, constraint equations allow us to obtain the most constructive results in the synthesis of new motions, as well as adequate modeling of concrete motions. Due to the non-stationary nature of constraints equations, any experimental data on kinematics and/or the dynamics of real motion can be taken into consideration. For the analysis of modeling results we consider the estimates of inter-element control motions distribution in the support phase of jumping motion. The number of anthropomorphic model elements can change with respect to the level of AM adequacy to real human motions.

**RESULTS AND DISCUSSION:** Further let us consider some results of running long jump simulation for the modeling of support and flying phases of the motion (Figure 1). Mass-inertia characteristics of 16-links model (Figure 2) are presented in Table 1, where  $M$  - mass,  $L$  - length,  $A$  - local position of center of mass,  $J_c$  - central moment of inertia assessed from the regression analysis formulas application for the sportsman with total mass  $M^c=90.2\text{kg}$  and height 1.85m. As follows from

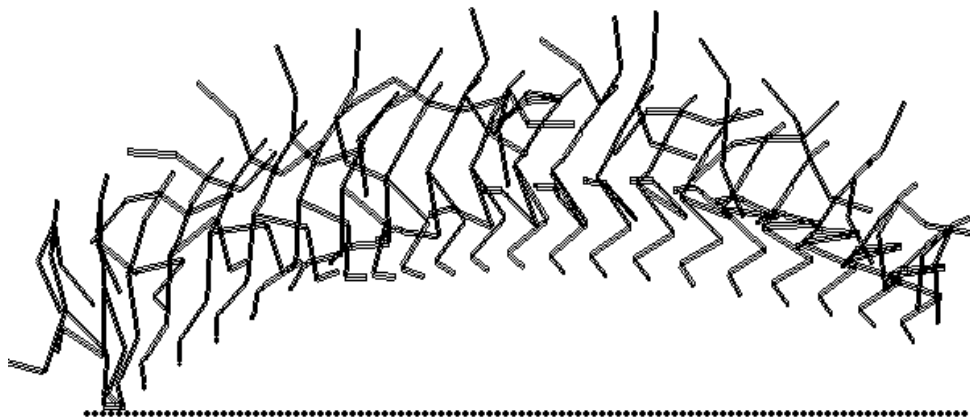


Figure 1. Kinematic scheme of the 16-element model support and free-fall phases of the running long jump.

Figure 1, all phases of the motion were simulated as one motion due to application of constraint equations (2)-(4) types. For the flying phase we specified  $\vec{N}_1 = \vec{M}_1 = 0$ . Different lengths of jumping (6m, 7.25m, 8.8m) were simulated due to the variation of duration of the stance phase and the amplitudes and shapes of support reaction force components. The initial and final positions and velocities distribution were also varied in order to provide support force realization.

The trajectories of the swinging leg ankle and arms motions were preset with variation of positions of these joints at the end of the motion. The main difficulties of synthesis, besides the short period of observation (0.10 - 0.14sec), were associated with combination kinematic constraint equations (regulating limb motion) and force ones (ground reaction, moment of momentum). Not significant, at first glance, variation of constraints leads to the impossibility of motion realization and essentially non-monotonous distribution of relative angular velocities.

During the process of motion synthesis it turned out that increase of the maximum value of the ground reaction force, along with decrease of the time period of the support phase, should be coupled with an increase in angular velocity of the swinging leg. By analyzing this fact we can conclude that the increase of swinging leg angular velocity leads to the increase of the inertia force pressing the model down and, therefore, allows us to increase the work of the support impulse. For the considered motion, the problem of control optimization during the support phase of the running jump for non-confining constraints on inter-element moments and constraints on

N	M (kg)	L (m)	A (m)	J <sub>c</sub> (kg*m <sup>2</sup> )
1	0.21	0.033	0.020	0.538E-3
2	1.00	0.167	0.100	0.538E-2
3	0.65	0.070	0.035	0.522E-2
4	3.25	0.333	0.166	0.522E-1
5	13.1	0.506	0.311	0.275E+0
6	10.3	0.150	0.106	0.764E-1
7	15.0	0.278	0.125	0.119E+0
8	14.2	0.251	0.124	0.954E-1
9	5.48	0.200	0.124	0.337E-1
10	2.46	0.319	0.144	0.154E-1
11	1.94	0.434	0.200	0.150E-1
12	2.46	0.319	0.144	0.154E-1
13	1.94	0.434	0.200	0.150E-1
14	13.1	0.506	0.195	0.275E+0
15	3.90	0.413	0.167	0.522E-1
16	1.21	0.200	0.102	0.538E-2

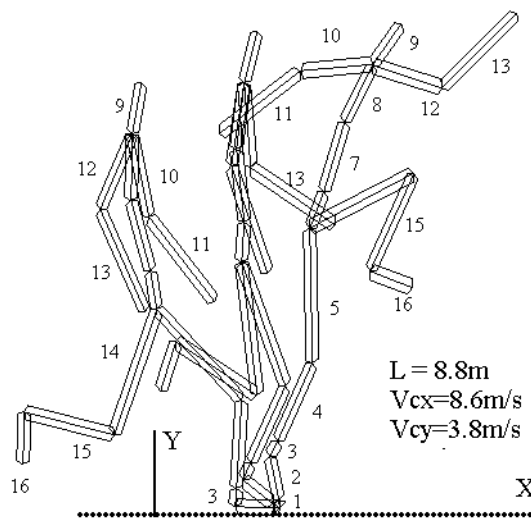


Figure 2. Kinematic scheme of the 16-element model support phase motion.

kinematics was actually solved. It is essential to note that for the considered approach it is possible to obtain a continuous picture of inter-element moments behavior for increasing jump length. Some results of this kind of optimization are presented in Figures 3-5. The problem of energy loss assessment was solved by calculating the so-called biomechanical work (Abio), as a result of integration per time of generalized power absolute values sum. From all the figures it can be seen that the significant distinction is the swinging leg torque behavior (curve 14).

**CONCLUSIONS:** The analysis of synthesized inter-element control moments values showed that the most significant influence on the value of the ground reaction and, therefore, on the pushing off velocity was the motion of the swinging nonsupport leg. Variation of the parameters values of ground reaction and resulting value of the kinetic moment allowed us to synthesize AM motion in the support phase so that it would ensure the desired trajectory of the AM motion in the flying phase of acrobatic motions.

Research showed the necessity of employing non-stationary constraint equations in the synthesis of complex coordination human motions. Such an approach to motion control synthesis minimizes the number of parameters to be varied and gives a relatively stable solution with respect to small variations of AM structure.

**REFERENCE:**

Zinkovsky, A. V., Sholuha, V. A., Ivanov, A. A. (1997). Mathematical Modelling and Computer Simulation of Biomechanical Systems. Singapore: WSP.

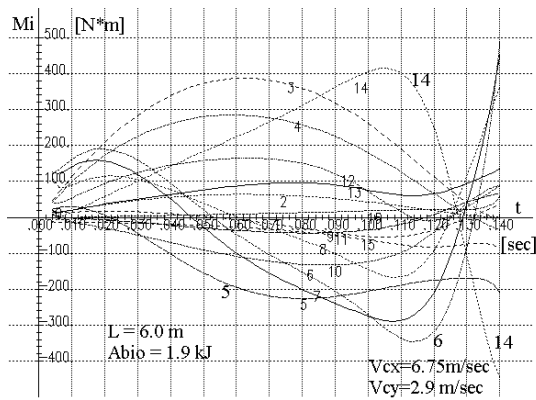


Figure 3. Torques distribution (L=6.0m)

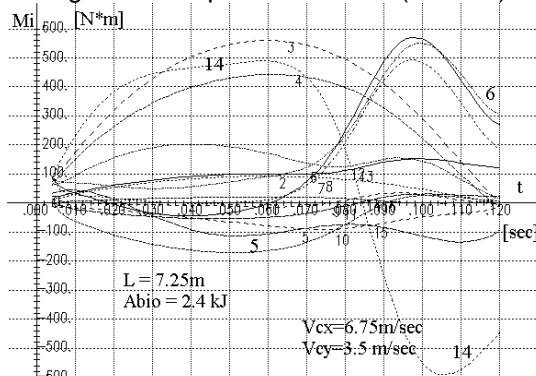


Figure 4. Torques distribution (L=7.25m)

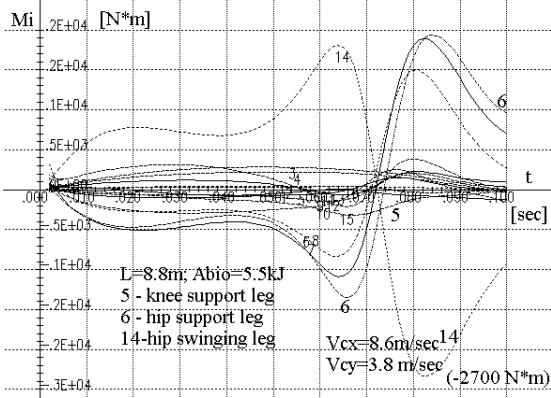


Figure 5. Torques distribution (L=8.8m)