EVALUATION OF A RUNNER'S PHYSIOLOGICAL PARAMETERS USING DIFFERENT SPRINT MODELS

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INTRODUCTION: Hill's model of sprinting, based on Newton's second law of motion, uses two physiological parameters to characterize the sprinter, the maximum propulsive force per unit mass and the resistance-to-motion parameter related to the runner's internal energy losses. Furusawa *et al.* (1927) suggested a resistive force law linear in the running speed. Later Keller (1973) and many others based their studies on Hill's model. Senator (1982) added the effects of air resistance by a term quadratic in speed. Vaughan (1983) used a modification of these approaches by introducing a 0.7-power law. However, the justification for the empirical law was not established by reference to the physical processes of running. Recently, utilizing the rotational equation of motion for the leg and experimental data for stride frequency, Holmlund and von Hertzen (1997) have shown that the internal and external resistive forces may well be approximated by a combination of linear and quadratic terms in running speed. They have also derived an expression for the internal resistive force in terms of physiological quantities.
The aim of this paper is to compare the validity of four sprint models, differing in the form of the resistive law, by fitting the physiological model parameters into Olympic 100 m data and analyzing the predictive power of each model. The parameter values obtained are compared with those in the existing literature. Finally, conclusions are drawn.

METHODS: The different models may be classified according to the resistive force law as linear (L), linear-quadratic (LQ), quadratic (Q) and Vaughan (Va) models. The equation of motion for all these models can be written in the form

$$\frac{dv}{dt} + k_r v^{\alpha} + k_D [v - v_w] \left| v - v_w \right| = f \, , \tag{1}$$

where v denotes the runner's velocity, f the propulsive force per unit mass, v_w wind velocity, α the exponent of the resistive law, t time, and k_r and k_D the internal and external resistance coefficients, respectively. The exponent α takes the values 1, 1, 2 and 0.7 for the L-, LQ-, Q- and Va-models, respectively. For parameter estimation, the numerical solution of equation (1) or the analytic solutions given by Holmlund and von Hertzen (1997) were fit, in the sense of least squares, into the data of the men's 100 m final at the 1988 Olympic Games in Seoul. This data comprises the running times at ten timing stations at 10, 20,...,100 m, the start reaction times and the wind velocity during the run (Brüggemann and Glad, 1990). Firstly, the reaction times as obtained from starting block force data were subtracted from the measured running times. Secondly, since the wind velocity during the final was +1.10 ms^{-1}, the wind corrected data was calculated. To do this, we fitted the solution of equation (1) with v_w = 0 into the data to get a first approximation for the parameters f and k_r. Then we estimated the effect of the wind on the running times at the timing stations and repeated the procedure until convergence for the values of f and k_r was reached. When estimating the parameter values for each model, we used wind correction by the very same model, although the variation in the correction from model to model was very small, typically ± 0.01 s in the range 50 - 100 m. Although the value of k_D differs slightly from runner to runner, the effect of this is of minor importance (of second order), since the air drag itself is small compared to the other forces present in the running action. For this reason, and to restrict the number of the system parameters to two, the constant k_D was set to the value 0.0033 m^{-1} in all calculations (Ward-Smith, 1985), except for the L-model, which has $k_D = 0$.

RESULTS AND DISCUSSION: The estimated values of the physiological parameters for the six best finalists are presented in Table 1. These finalists were: Ben Johnson (BJ), Carl Lewis (CL), Linford Christie (LC), Calvin Smith (CS), Dennis Mitchell (DM) and Robson Silva (RS).

Table 1. *Men's parameters evaluated from the Olympic data accounting for the reaction times and wind velocity.*

	Vaughan		Linear		Linear-Quadratic		Quadratic	
	f	k_r	f	k_r	f	k_r	f	$k_r + k_D$
	(ms^{-2})	($m^{0.3}s^{-1.3}$)	(ms^{-2})	(s^{-1})	(ms^{-2})	(s^{-1})	(ms^{-2})	(m^{-1})
BJ	12.66	2.18	10.40	0.883	10.20	0.836	7.56	0.0560
CL	11.36	1.95	9.39	0.800	9.22	0.754	6.87	0.0516
LC	11.30	1.95	9.33	0.799	9.16	0.753	6.82	0.0518
CS	12.16	2.12	10.00	0.864	9.82	0.818	7.27	0.0559
DM	12.07	2.11	9.92	0.862	9.75	0.817	7.22	0.0562
RS	11.90	2.10	9.78	0.858	9.61	0.813	7.11	0.0564

After finding out the parameter values, the distance-time relationship for each runner and model could be determined. To measure the predictive power of each model, the quadratic residual errors (QRE) were calculated. The QRE was defined as the sum of the squared time errors (original-calculated) over the ten timing stations. The calculated QRE-values for Va-, L-, LQ- and Q-models were 0.022, 0.026, 0.027 and 0.068 s^2, respectively. Consequently, the Va-model gives the best fit, the L- and LQ-models give a good and almost equal fit, and the Q-model clearly gives the poorest fit.

It can be seen from Table 1 that the value of f is highest for the Va-model and lowest for the Q-model. The values of f and k_r for the L- and LQ-models are relatively close to each other, although the values of the L-model seem to lie systematically somewhat above those of the LQ-model. Due to its poor fit to the data, the parameter values provided by the Q-model should be regarded as unreliable. Inspection of Table 1 also reveals that BJ and CS have high values of f and k_r contrary to CL and LC with lower values. This means that BJ and CS can be characterized as strong but resistive while CL and LC are weaker but less resistive (with reference to the body mass). It was also found that BJ fits to the models much poorer than the other runners. The evident explanation lies in the well known fact that BJ's speed attains its maximum relatively early around 50-70 m (Van Coppenolle *et al*., 1989) and then decreases, whereas all the models predict an increasing velocity profile which asymptotically levels off towards a constant value. The average residual time error per station and runner for the Va-, L- and LQ-models is about 0.02 s, which means a good fit throughout the run. As an example, the measured and calculated times for CS using Va-model are given in Table 2.

Table 2. *Comparison of measured and calculated times* (Va-*model*) *for* CS*.*

distance (m)	10	20	30	40	50	60	70	80	90	100
measured time (s)	1.92	2.95	3.90	4.79	5.65	6.50	7.36	8.23	9.10	9.99
calculated time (s)	1.93	2.95	3.88	4.77	5.65	6.52	7.38	8.24	9.10	9.97

It is of interest to compare the values of the parameters of Table 1 with the corresponding values in existing literature. Furusawa *et al*. (1927) have reported for the L-model the values f = 8.6 ms^{-2} and k_r = 1.0 s^{-1} (mean value), Keller (1973) gives f = 12.2 ms^{-2} and k_r = 1.12 s^{-1}, Vaughan and Matravers (1977) have reported f = 10.26 ms^{-2} and k_r = 0.963 s^{-1}, whereas Woodside (1991) ends up with f = 14.4 ms^{-2} and k_r = 1.35 s^{-1}. It is quite evident that the f-values by Woodside and Keller are too high since recent starting block force data for international top level sprinters (Van Coppenolle *et al*., 1989) reveals initial acceleration values in the range of 12.5 – 12.9 ms^{-2} for the push off from the blocks. This must set an upper limit for the propulsive force per unit mass, since the vigorous two-leg push in a leaning position with continuous ground contact evokes a strong propulsive reaction. Although the values by Vaughan and Matravers are quite close to our values for the L-model, they must be considered as too high, since their data is for national level sprinters only. The explanation lies in the fact that their f-value is determined as the average of the initial slopes of ten velocity-time recordings. Consequently, their value is strongly affected by the push off from the blocks, leading to an evident overestimation.

Our mean values for the Q-model f = 7.14 ms^{-2} and $k_r + k_D$ = 0.055 m^{-1} are much lower than the corresponding values 10.3 ms^{-2} and 0.095 m^{-1} reported by Vaughan and Matravers (1977). It should be noted that their data is collected only at three points; the start, 27.5 m and the location of maximum velocity, whereas the data used by us is measured at ten positions with 10 m intervals. It is quite evident that the start weighted data of Vaughan and Matravers leads to larger values of the parameter f.

Vaughan (1983) has obtained for the Va-model typically the values f = 10.5 ms^{-2} and k_r = 2.0 m$^{0.3}$s$^{-1.3}$. The k_r -value is quite close to ours, while the f-value is lower, as it should, due to the difference in the level of the runners behind the data.

Vaughan and Matravers (1977) have also studied the square root, hyperbolic and exponential models. They found that the square root model gives the best fit. However, later Vaughan (1983) found that his $\alpha = 0.7$ exponent law gives a still better fit to the data. As far as we know, the LQ-model has not been used earlier.

CONCLUSIONS: Our results show that the Va-model best fits the real Olympic 100 m data. The L- and LQ-models give a good and almost equal fit, and the Q-model clearly yields the poorest fit. It is interesting to note that the L-model was already suggested by Furusawa, Hill and Parkinson (1927). There is, however, a noteworthy difference between the interpretation of the origin of the linear term in Hill's theory and in the LQ-model presented by Holmlund and von Hertzen (1997): Hill and colleagues invoked the concept of the viscosity of the muscles, while Holmlund and von Hertzen arrived at the linear term by writing the rotational equation of the leg combined to the observed fact that the stride frequency can, to a good approximation, be considered as constant. It must be noted that already Fenn (1930) strongly criticized the viscosity concept and proposed that the resistive force could be the result of tension of antagonistic muscles and other kinesiological factors. It is evident that the resistive force stems mainly from the rotational inertia of the leg, whereas the energy losses occur in the antagonistic muscles during the decelerating phases of the back and forth motion of the legs.

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