FIGURE SKATING JUMP OPTIMIZATION THROUGH COMPUTER SIMULATION

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Computer simulation of figure skating jumps can identify opportunities for improving skater performance by utilizing the kinematic description of jumping dynamics derived from cinematographic analysis as the starting point for computer-based experiments on a simulated skater. The freedom to explore alternatives in body position, speed, thrust and timing with the computer simulation will provide the coach with a tool for systematic assessments of jumping technique. In addition, the ability to use the computer to examine the very rapid movements during take-off will help both the skater and the coach to conceptualize the components of movements that may have been previously unclear or even undefined. Consequent benefits would include:

1. Clear description of proper jumping techniques.

2. Opportunities for achieving more efficient jumps--higher, longer, more revolutions--with concomittant freedom to increase concentration on artistic dimensions.

3. Reduced risk of injury.

4. Determinations of the suitability of specific jumps for particular body types, flexibility, strength and intellectual capacity.

The teaching of figure skating that would evolve from a more complete understanding of jumping dynamics could be characterized by more appropriate progression in the development of the skater. Teaching of jumps could be precise with little need for relearning. Possibilities for injury due to high stress levels could be pinpointed and adjustments in technique or improvements in boot design could be made to limit these stresses.

This paper describes the initial phase in the development of a computer-based process for the dynamic simulation of figure skating jumps. High speed cinematography is used to determine the kinematic data for each limb--e.g. velocity and acceleration histories, initial positions--necessary for solving the equations of motion by explicit numerical integration. As the computer simulation proceeds a computer graphic animation is displayed. Because the position of each body segment is calculated at each time step, the influence on jump trajectory due to changes in body position as well as variations in the timing and in the magnitude of the forces exerted by the skater can be examined by comparing the animated trajectories or plots of kinematic data, joint forces and the torques acting on each body segment.

As indicated in the next section, the numerical method for predicting the movements of the simulated skater is particularly simple. Each body segment is considered as a distinct element during the dynamic analysis with coupling to adjoining limbs accounted for by including the requisite joint forces, (see references Cundall (1983) and Cundall (1978)). While maintaining sufficient accuracy for predicting complex jumping dynamics, this model is easy to understand and use.

NUMERICAL MODEL

Many previous studies have successfully represented the human body as a set of connected rigid segments, e.g. Hanavan (1964), Huston et al (1971), Huston et al (1976), Hatze (1977) and Aleskinsky (1978). To introduce the modeling approach used in the simulation of figure skating jumps in three spatial discussions, a simplified six-segmented hominoid representative of two-dimensional symmetric motions in the sagittal plane will be considered. Corresponding bilateral segments can be considered to move as one segment. As pictured in Fig. 1, the respective body segments are:

1. feet
2. lower legs
3. upper legs
4. trunk (including head)
5. upper arms, and
6. lower arms and hands.

These connected segments are considered as a dynamic system in which the energy is transmitted by mechanical forces.

A simple dynamic system composed of three connected masses is diagrammed in Fig. 2. In this example the three masses are constrained to move in one dimension; they are connected to each other and to a rigid boundary by springs and dashpots, and mass 3 is driven by an external force F(t). The analysis of such a system is straightforward.

The biomechanical model used in the development of the computer simulations is analogous to the dynamic system indicated by Fig. 2. In the biomechanical model each of the body segments are connected to adjoining segments by springs

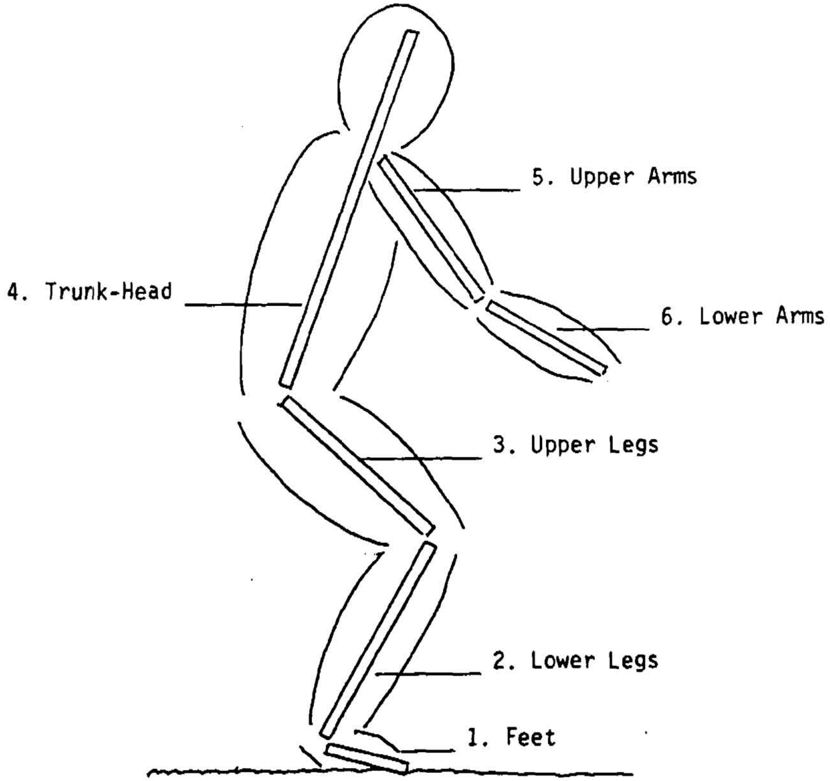

Figure 1. Six Segment Hominoid

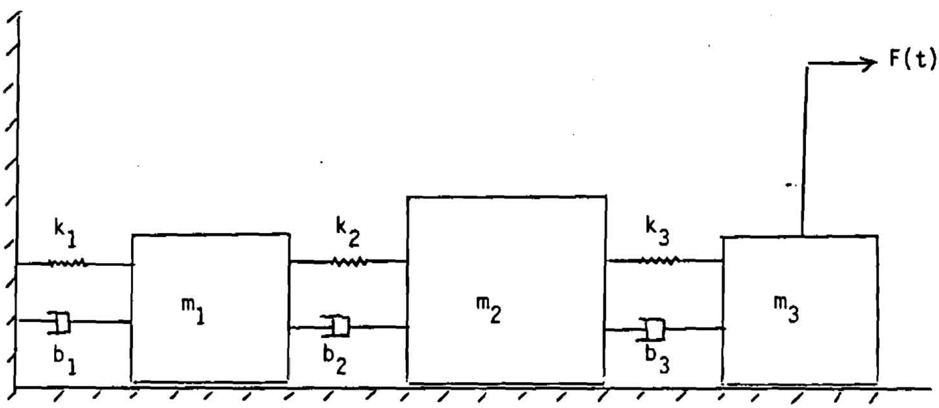

Figure 2. Simple Mechanical System

and dashpots. Forces are transmitted from segment to segment through the springs; rapid oscillations characteristic of a spring-mass system but not of a human body, are damped by the dashpots. The hominoid sketched in Fig. 1 is, however, free to move in two dimensions; therefore analysis of both linear and angular accelerations must be included. Another distinct difference between the two systems is that the force F(t) driving the mechanical system in Fig. 2 is external to the system while the active elements are internal for the biomechanical model. The equivalent center of mass torques that drive the hominoid are representations of the torques developed by the muscles.

Major goals of the computer modeling work have been (1) to develop a method for the simulation of complex three dimensional jumps that accurately predicts motions based on the equations of dynamics and (2) to utilize as simple an approach as possible so that the method can be widely understood and easily used. The distinct element method facilitates these goals because at each time increment the algorithms examine each body segment in turn with easily calculated joint forces linking the segments. By considering only the equations of motion for one segment at a time, the formulation is very concise. The similarity of the calculations for each segment lend themselves to efficient loop structures at each time step as well as for subsequent times. In the following paragraphs a two dimensional segment is analyzed to provide some indication of the method. Analysis in three dimensions is made slightly more complicated, and difficult to draw, by either time varying moments of inertia or accelerating coordinate systems.

Equations of Motion for Each Segment

Figure 3 is a schematic of the lower legs (segment 2) indicating the forces and torques acting on this representative body segment. The joint forces F_{a2} and F_{b2} act at each end of this limb. The equivalent muscle torque M_2 and the weight of the limb m_2*g are shown acting through the center of mass. By defining the location of the ankle joint with respect to the center of mass of the lower leg by the radius vector r_{a2}, the location of the knee joint by the radius vector r_{b2}, the moment of inertia of the lower leg about its center of mass as I_2, the position of the center of mass of the lower leg relative to a stationary coordinate system by p_2, and the angular orientation of the limb as measured counter-clockwise from the positive x-axis by θ_2, the equations of motion for this segment may be written as

$$m_2 * \ddot{\vec{P}}_2 = \vec{F}_{a2} + \vec{F}_{b2} - m_2 * \vec{g} \tag{1}$$

$$I_2 * \ddot{\vec{\theta}}_2 = \vec{r}_{a2} \times \vec{F}_{a2} + \vec{r}_{b2} \times \vec{F}_{b2} + \vec{M}_2 \tag{2}$$

where $\ddot{\vec{P}}_2 = \dfrac{d^2\vec{p}}{dt^2}$ and $\ddot{\vec{\theta}}_2 = \dfrac{d^2\theta_2}{dt^2}$.

Beginning with a completely defined initial configuration, the state of the hominoid a small increment in time later is predicted by first computing the

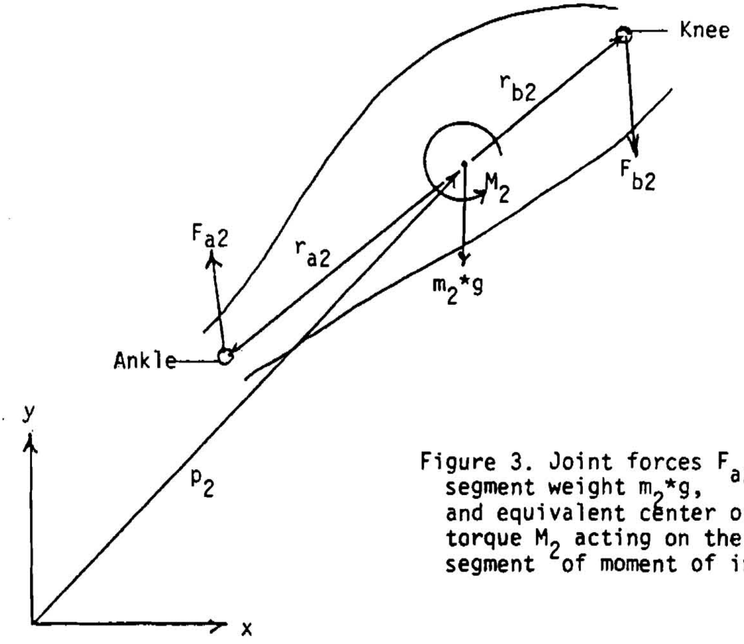

Figure 3. Joint forces F_{a2} and F_{b2}, segment weight m_2*g, and equivalent center of mass torque M_2 acting on the lower legs segment of moment of inertia I_2.

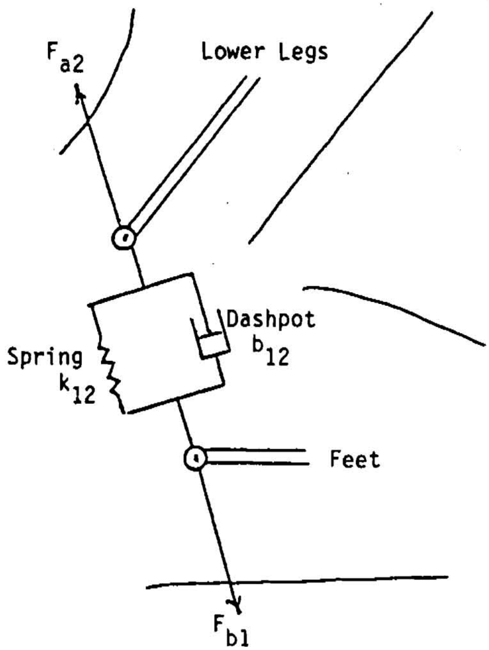

Figure 4. Schematic of model for transmitting joint forces.

joint forces and then using them in the integration of equations 1 and 2. The method for solving these equations so that the segments remain coupled at the joints while transmitting the forces associated with the particular motion under study is outlined in the following discussion.

Limb Coupling

In this model, joints are defined as the contact points or connections, between body segments. If one body segment is moved, due to an interaction with an external object or the generation of an internal torque simulating a muscle contraction, then a force may be transmitted through the joints to adjacent body segments thereby displacing them. A schematic of the simulated condition at the joint between the lower legs and the feet is shown in Fig. 4. To provide a representation of the method for modeling the coupling between segments, an ideal mechanical spring-dashpot pair is inserted in each joint. With no attempt at physiological rigor, it is reasonable to consider these elements as a first approximation to the complex biological system of bones, ligaments, tendons and muscles. Connection of segments by the spring-dashpot pairs facilitates the calculation of the approximate joint forces and associated segment torques because movement of one body segment relative to the adjoining segment results in well-defined changes in spring tension which are readily determined. Oscillations characteristic of a spring-mass mechanical system but inappropriate to this biomechanical model are rapidly attenuated by the damping force due to the dashpot.

Consider the partial view of a sample situation shown schematically in Fig. 5. Initially the simulated feet and lower legs are stationary, Fig. 5a. At time t, assume that torques equivalent to the effect of additional muscle contractions M_1 and M_2 begin to twist the two limbs, Fig. 5b. The spring-damper system is compressed and generates equal and opposite forces F_{b1} and F_{a2} at the ankle joint which push on the feet and lower legs, respectively. The forces will accelerate the limbs to lift the lower legs up while increasing the downward force pressing the feet to the floor, Fig. 5c.

Evaluation of the joint forces is accomplished by determining the force in the spring and the damping force due to the dashpot. The change in the spring force is due to the change in separation δ_{12} (t + Δt) between the end of the lower leg segment (point a2) and the end of the feet segment (point b1):

$$d\vec{F}_{spring}\,(t + \Delta t) = k_{12} * \vec{\delta}_{12}\,(t + \Delta t) \qquad (3)$$

The change in separation $\vec{\delta}_{12}$ (t + Δt) is determined by multiplying the relative velocity between segment end points as viewed from point a2

$$\vec{U}_{12}\,(t) = \vec{U}_{b1}\,(t) - \vec{U}_{a2}\,(t) \qquad (4)$$

by the time increment Δt.

$$\vec{\delta}_{12}\,(t + \Delta t) = \vec{U}_{12}\,(t) * \Delta t \qquad (5)$$

This incremental change in spring force $d\vec{F}_{spring}$ corresponding to movements occurring between time t and time t + Δt must be added to the spring force that existed at time t

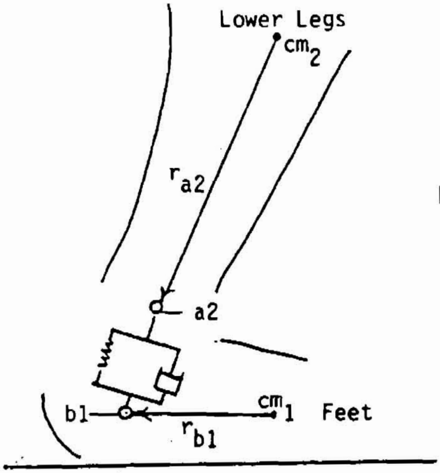

Figure 5a. Feet and lower legs at rest at time t.

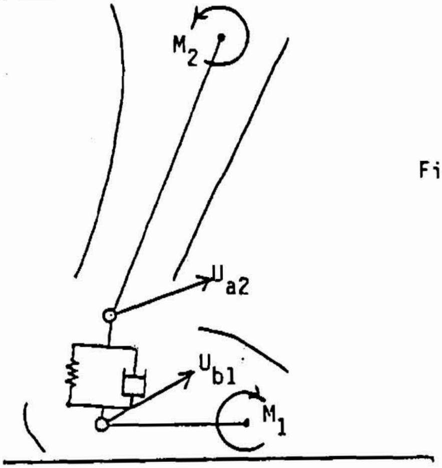

Figure 5b. Feet and lower legs in motion.

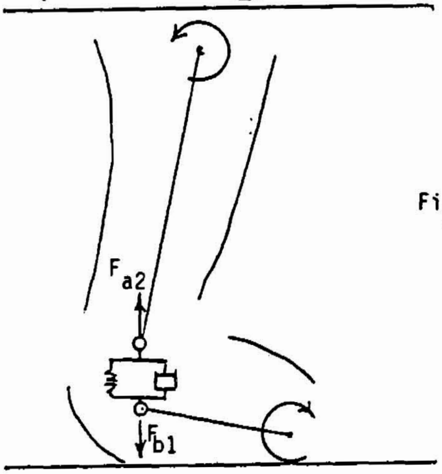

Figure 5c. New position at time t + Δt.

$$\vec{F}_{spring}\ (t + \Delta t) = \vec{F}_{spring}\ (t) + d\vec{F}_{spring}. \tag{6}$$

The damping force is the damping constant assumed for that joint multiplied by the relative velocity

$$\vec{F}_{damping}\ (t + \Delta t) = b_{12} * \vec{U}_{12}\ (t) \tag{7}$$

Summing these forces

$$\vec{F}_{b1}\ (t + \Delta t) = \vec{F}_{spring}\ (t + \Delta t) + \vec{F}_{damping}\ (t + \Delta t) \tag{8}$$

and because the spring and the dashpot are assumed weightless

$$\vec{F}_{a2}\ (t + \Delta t) = - \vec{F}_{b1}\ (t + \Delta t) \tag{9}$$

To compute these forces one can begin by determining the segment end point velocities from the kinematic relationships

$$\vec{U}_{a2}\ (t) = \vec{U}_{cm_2}\ (t) + \vec{W}_2\ (t)\ X\ \vec{r}_{a2} \tag{10}$$

$$\vec{U}_{b1}\ (t) = U_{cm_1} + \vec{W}_1\ X\ \vec{r}_{a1} \tag{11}$$

where $\vec{W}_1 = \vec{\dot{\theta}}_1$ and $\vec{W}_2 = \vec{\dot{\theta}}_2$. Then after calculating the relative velocity U_{12} and displacement δ_{12}, equations 8 and 9 give the joint forces which are used to solve equations 1 and 2 to obtain the updated linear and angular velocities and the predicted linear and angular displacements of the segments.

APPLICATION TO FIGURE SKATING

This computer simulation system is being developed to utilize initial kinematic descriptions of figure skating jumps obtained from cinematographic analysis as the starting point for subsequent computer-based experiments to optimize jumping technique. Once the computer model has processed the initial data, predicted changes in jump trajectory due to the effect of specific adjustments in such variables as initial velocity, body position, thrust or timing can be displayed for the investigator as an animation of the resultant motion or as detailed plots of position. Complete kinematic data as well as force and torque histories may also be retrieved.

General discussion of figure skating jumps includes the skater beginning to maneuver by skating in the direction of the anticipated movement, jumping into the air, executing a series of rotations (0 to four), and landing. Every skater and coach is well versed in the importance of maximizing time spent in the air. This requires a large angle of take-off, i.e. nearly vertical, and a force sufficient to project that skater's center of mass as high as possible. These simple terms require a more practical explanation with detail about how to achieve the desired results when translated by the coach or skater for a specific jump.

With a given skater in mind the height of the center of gravity on take-off is narrowly defined depending upon body position; however, the vertical component of velocity at take-off is influenced by several factors. The velocity of the total body entering the jump combined with the vertical impulse produced during the take-off phase determine the vertical velocity imparted to the body. This impulse involves the forces produced by the muscle action of the thigh, leg and foot, and to some extent the part to whole transfer of momentum of the arms and free leg. Obviously the timing of these take-off actions is critical. Focus for both execution by the skater and error correction by the coach should be directed toward

1. the horizontal velocity entering the jump,
2. the position of the body on take-off,
3. the time required for execution of the take-off,
4. the position of the free leg relative to the vertical axis of the body,
5. the point at which the free leg and arms stop their movement thus transferring momentum to the total body mass, and
6. the hip position throughout the movement.

It is difficult for the coach to observe, and certainly for the skater to focus upon, the factors listed in the preceding paragraph during any one particular jump because of the number of parameters and because of the short time period in which they occur. Systematic variation by a skater of any particular element of the jump, to see the effect of the changes, is nearly impossible because of the patterns of movement that the body has learned during years of training. It would be very helpful if the skater could develop sound technique from the beginning. Coupling high-speed cinematography with the dynamic computer model will provide a computer simulation amenable to systematic parameter variations thereby providing additional opportunities to examine and perfect a skill through a better understanding of what does or might occur. Thus it is possible for the coach and student to observe a world class skater executing a complex jump and focus on key body positions as well as the temporal factors of the jump. It is even possible to vary components of the jump, e.g. the position of the free-leg relative to the vertical axis of the body on take-off, and visualize the effect this adjustment would have on performance. But one is not limited to studying advanced techniques; the whole sequence of training can be analyzed from the simplest to the most complex. Biomechanically sound progressions in skill development can be discerned so that skaters can advance efficiently.

Systematic variation of the different components of particular jumps accomplished with the computer simulated skater would provide a clear understanding of the components which make significant alterations in performance as opposed to components which have a minor impact. This could be accomplished away from the ice. A coach and/or an advanced skater could sit before the computer screen and explore the effect of a variety of changes in techniques to delineate the changes worth trying during practice.

Finally, it is possible to visually contrast different skaters both through an examination of differences in position, velocity and timing and through an exploration of the different forces that are exerted. For example, world class skaters could be compared with national contenders to provide insights into the subtle differences. Feedback could make the national level skater a stronger contender.

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