JOINT MOMENTS AND PEDALLING RATES IN BICYCLING

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Abstract

Joint moments are of interest because they bear some relation to muscular effort and hence rider performance. The general objective of this study is to explore the relation between joint moments and cadence. Joint moments are computed by modelling the leg-bicycle system as a five-bar linkage constrained to plane motion. Using dynamometer pedal force data and potentiometer crank and pedal position data, system equations are solved on a computer to produce moments at the ankle, knee, and hip joints. Cadence and pedal forces are varied inversely to maintain constant power. Results indicate that average joint moments vary considerably with changes in cadence. Both hip and knee joints show an average moment which is minimum near 105 RPM for cruising cycling. It appears that an optimum RPM can be determined from a mechanical approach for any given power level and bicycle-rider geometry.

Introduction

The subject of optimal pedalling rates has long been of interest to cycling enthusiasts as well as professionals. 'Spinning' (pedalling at a high RPM) has been a technique used to improve performance by most accomplished cyclists. Exactly why pedalling at a high RPM leads to improved performance has been a question often posed. Associated questions are, "What is the optimum cadence and what factors influence this cadence?" The authors are aware of no previous biomechanical analysis directed towards answering these questions.

One objective of this study is to determine the relationship between cadence and cyclist performance from a mechanical viewpoint. A second objective is to determine the optimal cadence which leads to maximum performance. A final objective is to explore the sensitivity of variables, which the cyclist can control via his pedalling technique, on the optimal cadence.

Undertaking an analysis to satisfy the objectives requires defining a performance indicator. The performance indicator used herein is the sum of the average absolute hip and knee joint moments. The appropriateness of this indicator hinges on two assumptions. One is that muscular effort is inversely related to performance. The other is that joint moments are directly related to muscular effort. The work of Jorge and Hull [1], which indicates that there is little simultaneous activity of agonist/antagonist muscles in the major groups while cycling, supports the second assumption. The first assumption is certainly intuitively satisfactory under some conditions. Muscle physiology is not considered in this analysis, however, and may affect the validity of the first assumption to some degree.

The Model

The bicycle-rider system was modeled as a five-bar linkage in plane motion with the fifth bar fixed in space (see Fig. 1a). To completely constrain the model, the angular position, velocity, and acceleration of both the crank and the pedal relative to the pedal spindle are necessary inputs. With these inputs, and the basic relations for the kinematics of linkages, the entire system is kinematically defined. The linear and angular accelerations of each link become functions of the geometry and system inputs.

To examine the kinetics of the model, the equations of motion for each of the force links illustrated in Fig. 1b can be written by applying the following relations:

$$\Sigma F_x = \text{mass} * A_x \tag{1}$$

$$\Sigma F_y = \text{mass} * A_y \tag{2}$$

$$\Sigma \vec{M}_g = I_{cg} * d\vec{\omega}/dt \tag{3}$$

where A_x and A_y are components of the linear acceleration of the mass centers, F_x and F_y are force components, $\vec{\omega}$ is the angular velocity vector, I_{cg} is the moment of inertia about the mass center, and $\vec{M}_g$ represents moments about the mass center. With the measured pedal forces and the angular and linear accelerations, the equations of motion can be solved for the joint torques working from the foot to the shank and finally to the thigh. Thus, the joint moments are functions of both the input pedal forces and the kinematic constraints.

Input Data

Kinematic inputs were derived from both crank and pedal angles which were measured using continuous rotation potentiometers (see Ref. [2]). Crank velocity was purposely held constant using a paceometer and thus crank angular acceleration was zero. The determination of pedal angular velocity and acceleration was more difficult. Differentiating the pedal

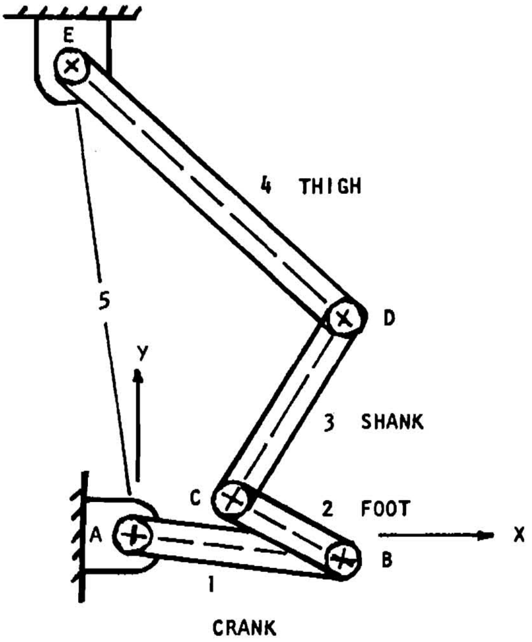

Fig. 1a Five-bar linkage model of the rider-bicycle system

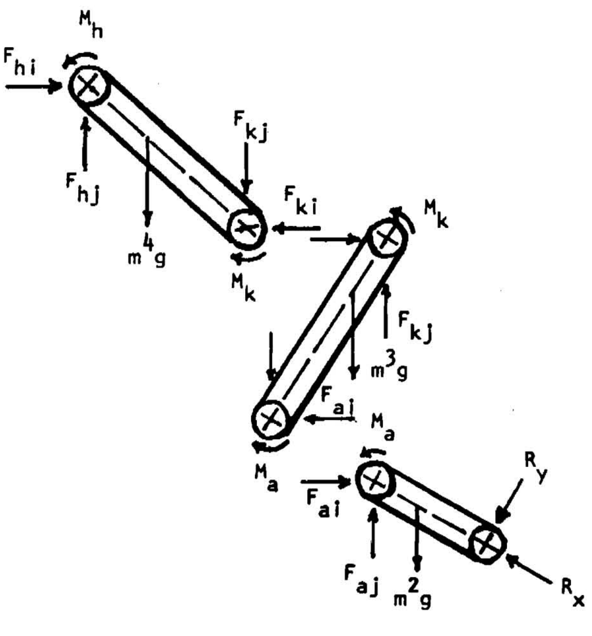

Fig. 1b Free-body diagram of the leg

angle data proved too inaccurate due to the error amplification inherent in differentiating. Both polynomial and Fourier least squares curve fitting of the pedal position data were attempted to no avail. Unfortunately, the derivatives of polynomials are not periodic and periodic velocity and acceleration were imperative. Derivatives of Fourier series with more than one harmonic are not nearly smooth enough to be accurate representations of the actual pedal angular velocities and accelerations. Finally, what proved successful was a least squares sine function fit to the pedal angle data where the sine function has the form:

$$\text{Pedal angle} = A_1 \sin(\theta) + A_2 \cos(\theta) + A_3 \quad (4$$

where θ is the crank angle and A_1, A_2, and A_3 are constants to be determined. Figure 2 shows typical pedal angle data and the corresponding fitted curve.

The force input data was obtained from a six-load component dynamometer as described in Ref. [2]. For this study, only normal and tangential pedal forces were required. Typical force input data can be seen in Fig. 3.

Analysis and Results

Equations of motion were solved on a computer to determine the joint moment time histories. The solution of equations proceeded in the four following stages:

1. Kinematic and quasi-static joint moment time histories were computed at various RPMs keeping power constant.
2. Average absolute joint moments were computed as a function of RPM at constant power.
3. Sensitivity of joint moments to variance in pedal angle profiles was studied.
4. Variance in optimal cadence for significantly diverse pedal profiles was studied.

The first area of analysis examined both the kinematic and the quasi-static contribution to the total joint moment time histories. For clarification, kinematic moments are moments to accelerate the leg segments only, whereas the quasi-static moments result from pedal forces. By setting the pedal forces equal to zero, the kinematic joint moments could be examined. Similarly, by setting acceleration terms equal to zero, quasi-static moments were produced.

Figures 5a and 5b show the time histories of ankle kinematic and quasi-static moment profiles respectively at three different cadences all at a constant power of 98 Watts per leg. This cadence range spans the typical pedalling speeds for the utility cyclist (63 RPM), the average tourer (80 RPM), and the long distance competitor (100 RPM) (Ref. [3]). Important insights to note in this figure include first the relative insignificance of the kinematic moments at the ankle joint compared to the quasi-static.

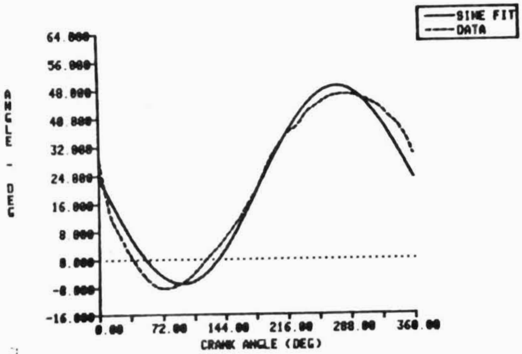

Fig. 2. Sine fit to pedal angle data (D3)

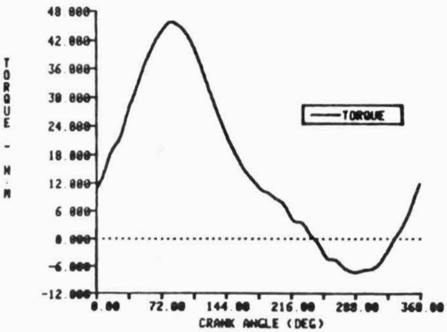

Fig. 4 Crank torque for D3, reference case

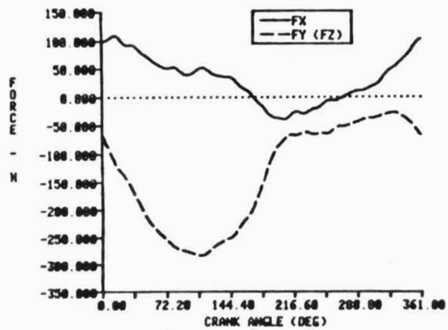

Fig. 3 Pedal forces for rider A, case 3 (D3 - 63 RPM)

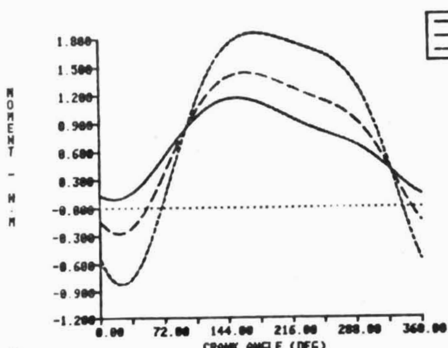

Fig. 5a Kinematic ankle moments (D3 - 98.3 Watts)

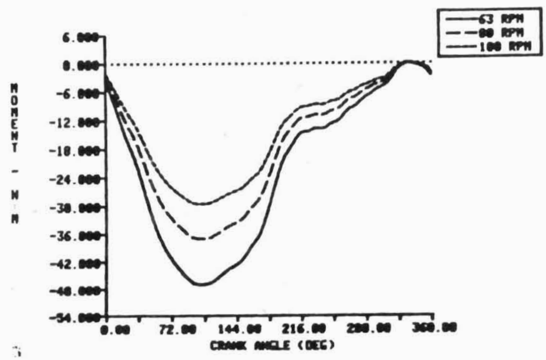

Fig. 5b Quasi-static ankle moments (De - 98.3 Watts)

This is expected because the moment of inertia for the foot is small. Far more ankle torque is required to provide power to the pedals than to angularly accelerate the foot.

Notice also that, by increasing RPM, the absolute average kinematic moments increase, whereas the average quasi-static moments decrease. This result is of major significance to this analysis because as RPM increases the necessary pedal force to maintain constant average power decreases. This is seen in the definition of power as:

$$P = \int_{0}^{2\pi} \omega \cdot d\tau \tag{5}$$

with ω being the angular velocity and $d\tau$ being the instantaneous crank torque. Also, as cadence increases, link segment accelerations increase and thus kinematic joint moments increase to provide these accelerations.

A final observation for Fig. 5b is that the quasi-static ankle profile indicates maximum moment from about 80° to 140°. This coincides with the area of maximum power production as can be seen in the torque curve of Fig. 4. Also, the moment produced in the backstroke is of much smaller magnitude than that produced in the power stroke but in the same direction. This accounts for the small negative torque produced in that portion of the cycle.

Examine now the kinematic knee moments in Fig. 6a. Similar to the ankle, the kinematic moment increases with RPM, whereas the quasi-static moment decreases. In this case, however, the kinematic knee moment is not insignificant. The knee must accelerate the mass of both the shank and the foot and this combined inertia is significant enough to cause the kinematic knee moment to be a major contributor to the total moment.

Of further interest is the fact that the knee's quasi-static demand (see Fig. 6b) is greatest at the top (TDC) and bottom (BDC) of the crank cycle. At these points in the pedal cycle, tangential pedal forces are developed (see Fig. 3) that only the knee joint is in a position geometrically to produce. Similarly, near 140° and 290°, the pedal force vector extends through the knee joint, thus producing zero knee joint moment. Unlike the ankle, the knee produces substantial positive and negative quasi-static moments.

An interesting conclusion results from examining the total knee moment (Fig. 8a) in some detail. The kinematic knee moment is mostly negative with the peak value occurring at 90°. The quasi-static knee moment, on the other hand, shifts polarity with the positive peak observed at 45° and a negative peak observed at 180°. Because the kinematic and quasi-static peak moments are not in phase, combining the two yields a total knee moment where neither magnitude nor shape varies markedly from the quasi-static moment. Accordingly, it appears that pedalling rate (i.e. cadence) does not have a profound affect on the total knee moment.

The quasi-static hip moment illustrated in Fig. 7b exhibits similar behavior to the ankle and knee joints in that this moment decreases with increasing RPM. The largest magnitude for the quasi-static hip moment is seen in the 150°-180° region of the crank cycle. The large quasi-static moment is due to an extended leg producing a rearward resultant pedal load.

Further, Fig. 7b illustrates that the hip provides minimal positive quasi-static moment in the backstroke, indicating that the pedal is not being 'pulled up'. This agrees well with experience where only a small contribution to total torque (or power) is provided in the backstroke.

Of the three joints, the kinematic hip moment illustrated in Fig. 7a is the most significant. At RPMs of 90 and greater the kinematic moment is at least as great as the quasi-static moment. This is due to the requirement of the hip joint to accelerate the entire leg. Therefore, inertial loading can in no way be ignored about the hip joint. Similar to the quasi-static hip moment, the largest magnitude for the kinematic hip moment is seen in the 150°-180° area of the crank cycle. At this point, the kinematic moments must change an extending hip joint to a flexing joint where the link is experiencing maximum deceleration.

Superimposing the quasi-static and kinematic hip moments leads to an interesting result. Because the two moments are approximately 180° out of phase, addition yields a total hip moment (Fig. 8b) which is substantially lower than either of the two contributors. The results in Fig. 8b illustrate the dramatic effect of cadence on the total hip moment.

The second area of analysis concerns the absolute average joint moment versus RPM while maintaining constant power. A reference case was simulated using actual crank angle, pedal angle, and pedal force time histories as input. Varying pedal RPM was accompanied by inversely scaling the pedal force profile to maintain constant power. The result was a plot of absolute average joint moments versus RPM at a power of 98 Watts for one leg (see Fig. 9). As can be seen, there is a specific cadence for each joint that produces a minimum joint moment average. Notice that both the knee and hip minima are in the same range of RPM.

Explaining the results indicated in Fig. 9 is straightforward. At low RPM, the pedal power must come from a higher torque and thus higher quasi-static joint moments. As cadence is increases, the required torque and thus quasi-static joint moment decrease. In an contrasting way, at low cadence, kinematic moments are small because link accelerations are small. As RPM increases, the kinematic joint moments necessary for increased link accelerations increase. Accordingly, at either low RPM (< 80) or high RPM (> 120) the total joint moments are high. In the middle of this range, in this case about 105 RPM for both the knee and hip joints, the joint moment average is minimum. At this RPM the rider achieves the minimum joint moment possible for this given power.

Further interpretations of the results in Fig. 9 can be seen in a semi-quantitative way from Fig. 10 where the moment due to power generation (quasi-static) plots as hyperbolic because:

$$\text{power} = \text{torque} * \text{ang. vel.}$$

and thus

$$\text{torque} = \text{power} \,/\, \text{ang. vel.}$$

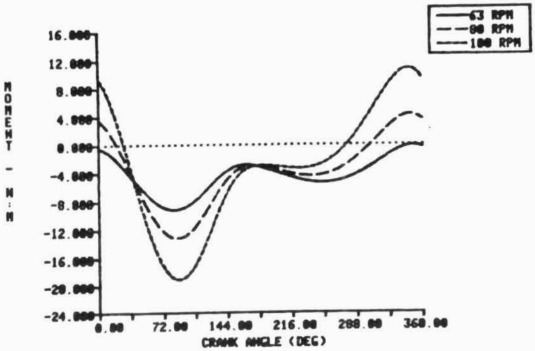

Fig. 6a Kinematic knee moments (D3 - 98.3 Watts)

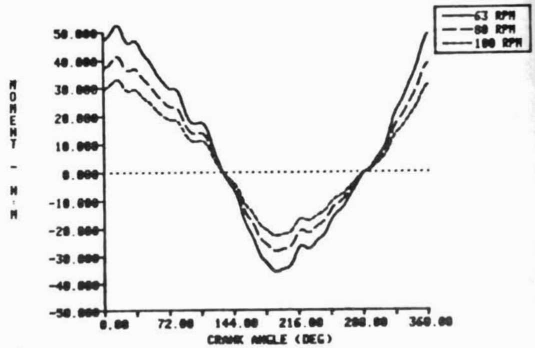

Fig. 6b Quasi-static knee moments (D3 - 98.3 Watts)

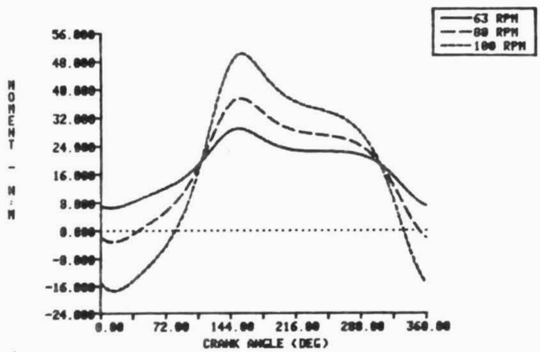

Fig. 7a Kinematic hip moments (De - 98.3 Watts)

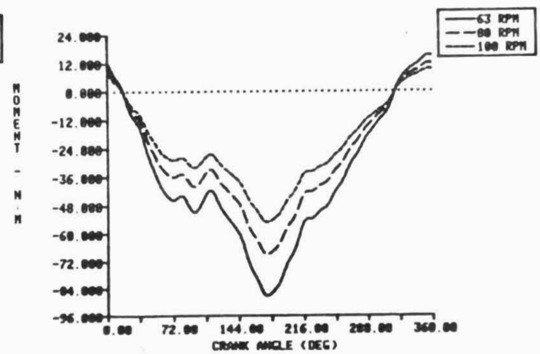

Fig. 7b Quasi-static hip moments (D3 - 98.3 Watts)

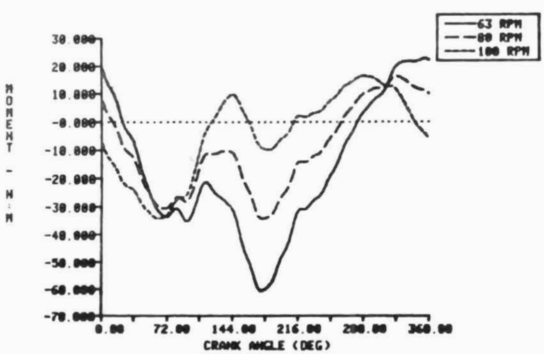

Fig. 8b Total hip moments (D3 - 98.3 Watts)

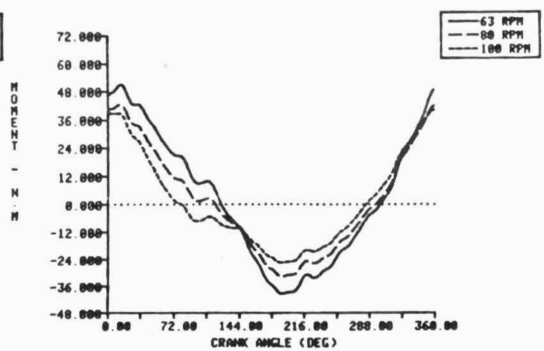

Fig. 8a Total knee moments (D3 - 98.3 Watts)

The moment due to the inertial terms (kinematic) plots as parabolic because accelerations are functions of the square of the angular velocity. Summing these effects produces an overall moment curve which includes a minimum point or trough.

It is interesting to consider how increasing the power demand affects the optimal cadence. Increased power demand shifts the hyperbole in Fig. 10 to the right and thus shifts the minimum moment in the same direction. Hence, higher power implies a higher optimal cadence. The kinematic contribution to moment averages is only a function of geometry because the geometry 'acts' upon the crank angular velocity to produce the various link accelerations. Therefore, without radical geometry changes, the kinematic effects are rather constant.

The third area of analysis investigated the sensitivity of pedal angle profile changes on kinematic moments. Even though the studies of Davis and Hull [4] show little change in pedal angle profile with increased loading, it was thought that changing cadence could cause a rider to vary his/her pedalling technique. In the preceding analyses, the pedal angle profile has been kept constant with changing RPM. This assumption is examined here.

The actual pedal angle profile (reference case) was modeled as a sine wave as mentioned previously. This curve (Fig. 2) has certain amplitude, offset, and phase shift. To determine how variance of the pedal profile affects kinematic moments, both the offset and amplitude were varied plus and minus 10 percent independently and kinematic moment plots were produced (see Figs. 11a and 11b). The phase angle change was not examined because previous research [4] shows that pedal angle phase is quite constant among a variety of cyclists. Because the kinematics have little effect upon the total ankle moment in the RPM range being considered (60 RPM - 160 RPM), only the knee and hip joints are of interest. Examining Figs. 11a and 11b indicates that varying both amplitude and offset changes the kinematic moment profiles only slightly. The consequent change in absolute average moment values would not be significant relative to the average moment changes due to RPM variance. This is not surprising because of the small contribution that the ankle angle makes to the kinematics of the entire system. A change of 20 percent to the knee or hip joint amplitudes or offsets would make a much more significant change in joint moment profiles.

A final area of analysis investigated the sensitivity of the optimum joint cadence to changes in the pedal force profiles. The pedal force profiles in Fig. 12a were chosen because they are significantly different from those in Fig. 3. The force profiles in Fig. 12a were scaled to provide the same average power as the force profiles in Fig. 3, namely 98 Watts.

Comparing Fig. 9 to Fig. 12b shows qualitative similarities. The shape of the curves for all three joints are the same in both figures. The hip has the most rapidly changing slope followed by the knee. This behavior is caused by the relative effects of the kinematic moments at these joints as discussed earlier. The difference between the two figures is primarily in the magnitude and location of the minimum average moments of the hip and knee. The reference case in Fig. 9 has its minimum hip moment of 12

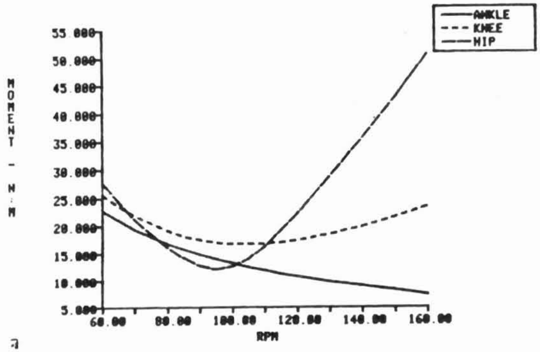

Fig. 9 Absolute average joint moments vs. cadence (D3 - 98 Watts)

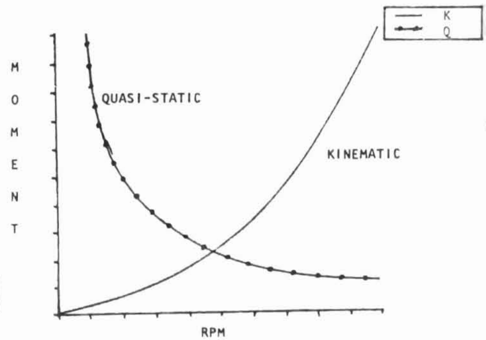

Fig. 10 Kinematic and quasi-static moment trends

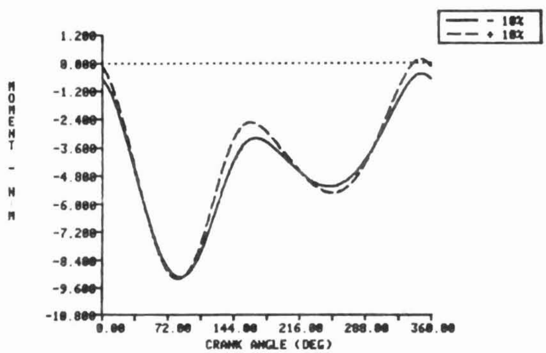

Fig. 11a Kinematic knee moments with pedal amplitude change

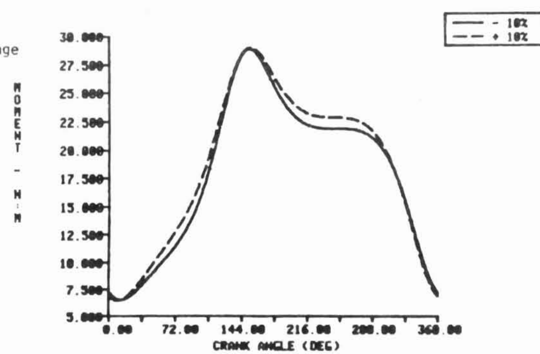

Fig. 11b Kinematic hip moments with pedal offset change

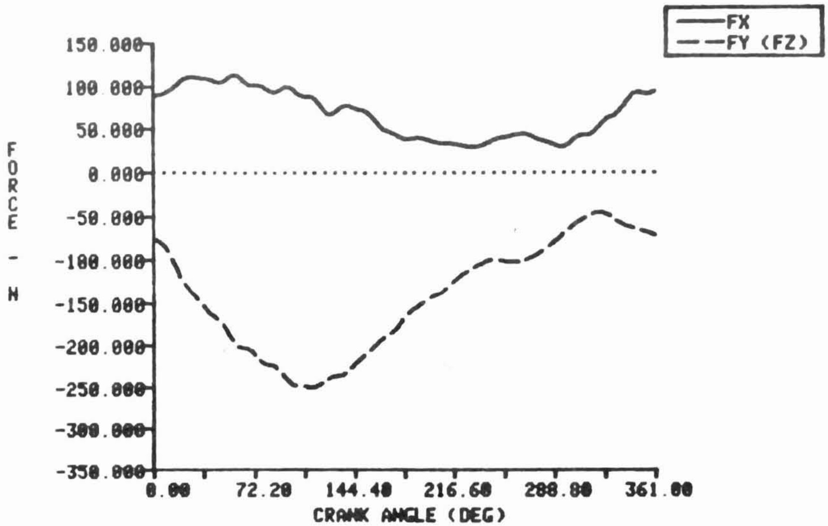

Fig. 12a Pedal forces for rider A, case 1 (D1 - 80 RPM)

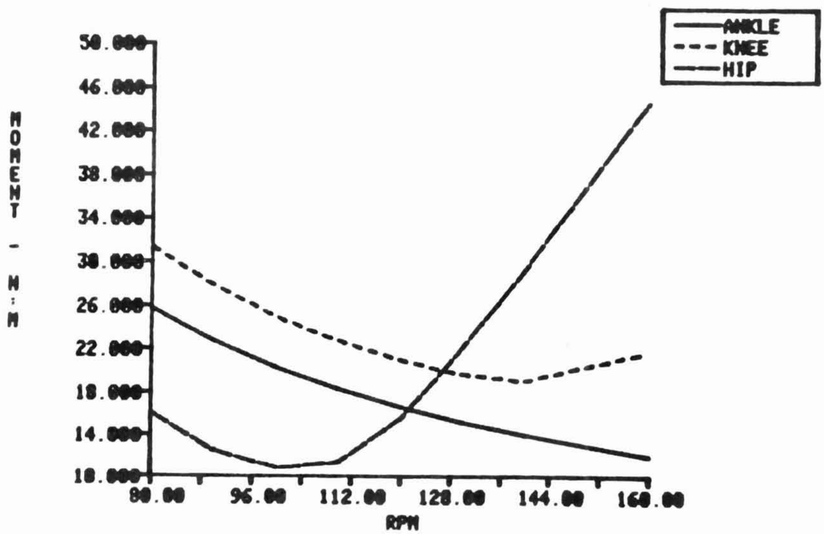

Fig. 12b Absolute average joint moments vs cadence (D1 - 98 Watts)

N-m at 95 RPM, whereas the sensitivity case in Fig. 12b has a minimum of 11 N-m at 100 RPM. The knee averages are quite different with the reference case indicating 17 N-m at 105 RPM and the sensitivity case showing 20 N-m at 140 RPM. These results occur because the moment curves in Fig. 12b are shifted to the right relative to the reference case. This shift indicates relative inefficiency in the pedal force profiles of the sensitivity case. Inefficiency results from the absense of negative shear loads in the backstroke [4].

What is most important concerning average moment profiles, however, is the overall optimum cadence. From inspection of Fig. 9, the optimal cadence for the reference case is in the 90 to 105 RPM range. Similarly, in the sensitivity case, the best RPM would be between 100 and 115 RPM as indicated by Fig. 12b. Therefore, even with quite different pedal force profiles, optimal RPM is in the same approximate range, which suggests that optimum RPM is not particularly sensitive to the pedal force profiles.

Concluding Remarks

The analysis presented herein has focussed on conventional bicycle pedalling where variables are limited to cadence and pedal loading magnitudes. It would be of interest to extend the analysis in two directions. Comparison of Figs. 9 and 12b indicates that, although the optimal RPM is not particularly sensitive to pedal force profiles, the joint moments are strongly related to pedal force profiles. Accordingly, one direction for further study would be to determine the pedal force profile which minimizes joint moments. A second direction of study might go beyond the realm of conventional bicycling by exploring the motion cycle which minimizes joint moments.

Acknowledgement

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