DETERMINATION OF TURNING PARAMETERS IN CARVED SKIING AND APPLICATION TO A NUMERICAL SKI-BINDING MODEL

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Skiing has regained in popularity after the introduction of the carving technique. The biomechanics of carving have been investigated in numerous studies. However, a comprehensive study of the behaviour of the ski/binding system taking into account the interactions between athlete, skiing equipment, and snow is still missing. In a first phase of the current study, the forces acting between skier and ski equipment and the evolution of the edging angle during carving were determined using video analysis and force measurements. Next, the passive snow resistance to a penetrating ski was determined using two specially designed tools. Finally, the determined quantities served as boundary conditions for a finite-element simulation of the ski/binding system in the carving situation. Calculated ski shapes were compared against measured turn radii and good agreement was found. The implemented model is intended to help in the development of improved ski equipment. As such, it can for example be used to study the effect of different skier's actions on the equipment behaviour.

KEY WORDS: alpine skiing, carving, finite element simulation.

INTRODUCTION: Skiing has become more popular after the introduction of the carving technique. The biomechanics of this technique have been investigated for example by Raschner et al. (2001). Lüthi et al. (2004) compared several commercially available measurement techniques, focusing on the precision of the testing devices. The snow resistance strength with regard to carving skis has been examined by Mössner et al. (2003) and Federolf et al. (2004). However, a comprehensive study of the behaviour of the ski/binding system taking fully into account the interactions between athlete, skiing equipment, and snow is still missing. In this study, the biomechanical turning parameters as well as the snow resistance strength were determined. This data provided the boundary conditions for a newly developed simulation tool for ski and binding in carved turns. The simulation is based on the finite element method (FEM) and calculates the ski's shape and thus its radius in the conditions of a carved turn. This radius was compared to measured radii of actual turns.

METHODS: Several carving turns defined by gates were performed by a high-level ski racer in November 2002 in the ski-dome of Neuss, Germany. The subject used test skis equipped with Kistler® force plates, which consist of three 3-component piezo force sensors. Two plates were mounted on each ski, one placed between the toe part of the binding and the ski, the other one between the heel part of the binding and the ski. The skier's motion was filmed with two digital high speed cameras with fixed camera perspectives. The cameras were placed in such a way that the recording of two full carving turns was optimal. The camera perspectives were calibrated by a 3 x 3 x 4 point calibration frame. The 3D-position of the calibration points were geodetically determined with a precision better than 1 mm.
From the video measurements, the skier's motion was determined using the video analysis software WINAnalyze®. The skier was modelled according to the anthropometric model of Hanavan (1964). This model consists of 15 body segments and 18 body points that need to be tracked in the video analysis. From the motion of the tracked points, the edging angle of the ski was derived using:

$$\cos\vartheta = \frac{(AK \times \vec{t}) \cdot \vec{n}}{\left|(AK \times \vec{t})\right|} \tag{1}$$

where $\vec{\tau}$ is the tangential and $\vec{n}$ the normal vector to the motion curve of the center of mass projected to the plane of the ski slope. AK is the vector defined by the position of the skier's ankle (A) and his knee (K). This calculation method is based on the assumption that the ski axis is parallel to the tangent of the skier's motion and that the cross product $AK \times \vec{\tau}$ is parallel to the ski's running surface and independent of the bending angle of the knee. Thus this method is only applicable for carved turns.
From the space-time data obtained by video analysis the acceleration of the skier's center of mass was derived using the piecewise linear regression technique. From the centre of mass' acceleration, $a_{c.m.}$, the total force $\vec{F}_{Ski->Skier}$ acting between skier and ski was calculated using the following equation of motion:

$$\vec{F}_{Ski->Skier} (2) \; m\vec{a}_{c.m.} - \vec{F}_G - \vec{F}_{cw} = -\vec{F}_{Skier->Ski}$$

where $\vec{F}_G$ and $\vec{F}_{cw}$ denote the gravitational force and the air resistance force on the skier, respectively. The interaction between ski and snow was investigated by measuring the resistance pressure on a penetrating piston with two field portable devices. One operated at constant deformation rate, the other one imprinted the snow with various impact velocities but was decelerated during the penetration process due to the snows penetration resistance. Both devices operated at varying edging angles. Thus, it was possible to characterize the resistance pressure over a broad range of deformation rates and edging angles for various snow conditions.
The obtained edging angle, the forces acting on the ski and the snow resistance pressure were implemented as boundary conditions in a finite element model of the skiing equipment. Two actual ski models of the manufacturer Stöckli were implemented, taking each material layer, the geometry and the camber of the ski into account. The binding model represented a binding of the manufacturer Fritschi in a simplified way, focusing on the mechanical functions of the components. The ski was modelled by shell elements, the binding by volume elements. Assuming the following quasi-static equation allowed to calculate the deformation of the ski/binding system during a carved turn:

$$\vec{F}_{Skier->Ski} + \vec{F}_{Snow->Ski} + \vec{F}_{G,ski} + \vec{F}_{Inertia,Ski} = 0 \qquad (3)$$

where $\vec{F}_{G,ski}$ and $\vec{F}_{Inertia,Ski}$ denote the gravitational force and the inertia force acting on the ski. With edging angle and external forces as input, the model can compute the penetration depth into the snow and the pressure distribution at the ski snow interface. At the same time the ski's shape is determined. To calculate the turn radius of the ski, the position of the lower ski edge was projected onto the plane of the snow surface and a circle was fitted to its tail using three points between the position of the ski boot and the ski end.

RESULTS AND DISCUSSION: The edging angles of the right and the left ski determined by video analysis are shown in Figure 1. For the outer ski, maximum edging angles between 60° to 74° were measured. On average, the edging angle of the inner ski was about 15° lower. These results fit to the observations in carved turns published by Raschner et al. (2001).
The total force FSki'Skier was determined from the results of the video analysis using Equation 2. Comparison to direct force measurement of FSkier'Ski obtained from the sensor plates by Kistler® showed good agreement in the overall time behaviour and the absolute magnitude of forces. Figure 2 displays the results of Kistler® force plate measurements of the normal force component on the left and right ski, which amounts to about 90% of the total force acting between ski and skier (Lüthi et al., 2004). The force distribution between left and right ski differs from the data published by Raschner et al. (2001) who found a near-balanced force distribution for carved turns. Figure 2 shows a distinct shift of weight onto the outer ski during both analysed turns. This is a first indication that only the outer ski in the examined turns actually carved.

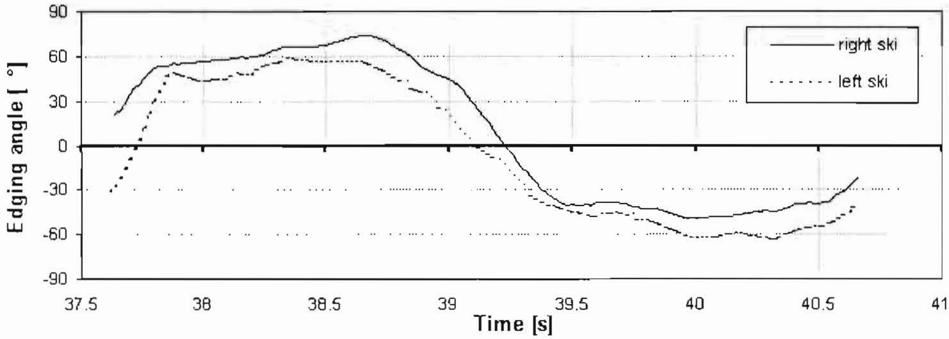


Figure 1: Edging angle of the left and right skis in two succeeding carved turns.

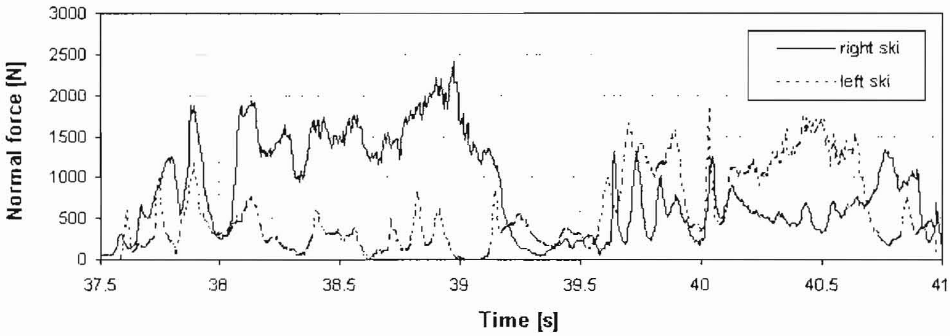


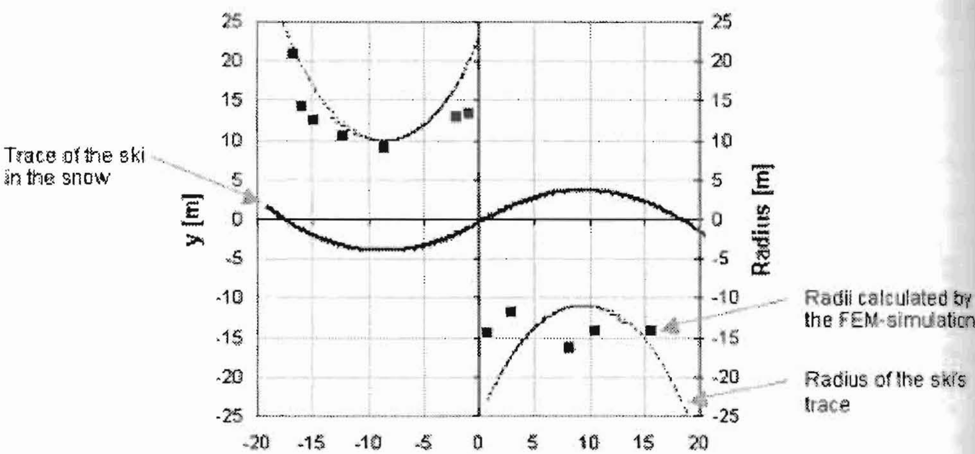
Figure 2: Force component normal to the ski surface measured by Kistler® force sensors.

The mean snow resistance pressure to the penetrating plate can generally be approximated by a partially linear function of the penetration depth d (Federolf et al., 2004). This function exhibits a strong hysteresis due to the non-elastic deformation processes in the snow. Therefore, it was split into two parts referring to the loading and the unloading processes

$$p_{load}(d) = \begin{cases} A \cdot d + B & d > 0 \\ 0 & d \leq 0 \end{cases} \tag{4}$$

$$p_{unload}(d) = \begin{cases} C \cdot (d - d_{max}) + p_{max} & d > d_o \\ 0 & d \leq d_o \end{cases} \tag{5}$$

For the snow in the ski-dome of Neuss, where the test runs were conducted, the snow specific coefficients were determined as A = 6.5 KPa/mm for edging angles below 40°, A = 0.5 KPa/mm for edging angles above 40°, B = 65 Kpa, and C = 15 MPa/mm. The pressure on the withdrawing plate P_{unload} depends on the maximum penetration depth d_{max} and the maximum pressure P_{max}. After the penetration, the snow is able to expand over the distance d_{max} - do and thus release remaining elastic stresses. The loading forces and moments acting on the binding, the edging angle, and the snow resistance function provided the input data necessary to calculate the actual turn radius of the ski. This radius was then compared to the radius determined for the actual trace of the ski remaining in the snow. Figure 3 exhibits the trace of the ski in the snow, which was interpolated from several geodetically measured points and projected to the plane of the snow surface (thick line). The x-axis was chosen parallel to the fall line of the slope, the y axis is perpendicular to the x-axis within the plane of the snow surface. For better clarity only the right ski was included in the graph. The evolution of the trace's radius as a function of the x is indicated by the thin broken line. For several selected points of time the ski radius was calculated using the FEM simulation program and added to the graph (black squares). Good agreement with the measurement data was obtained for the first of the two selected turns. The radii of the second turn did not correlate. For this turn the right ski is the inner ski and obviously does not carve in the turn (as already mentioned when discussing the force data). Deviations are also visible in the transition phase from the first to the second turn. In this transition phase, in which the skier rises and shifts his weight on the other ski, the skis are not carving but gliding forward.



CONCLUSION: The parameters which determine the deformation of a ski in the case of a carved turn could be identified and were measured during several test runs. Together with the determined snow resistance strength, they were implemented as boundary conditions to a FEM simulation tool. For the turn phases in which the ski actually carved, a good correlation between the numerically calculated ski radius and the actually performed turn radius was found. The presented FEM tool is suitable to study the impact of changing external conditions, e.g. snow conditions or a different weight of the skier. The effects of different ski or binding designs or body actions of the skier - if they change the forces or moments acting between ski binding and ski boot - can also be investigated.

REFERENCES:

Federolf P., Fauve M., Szabó D., Lüthi A., Rhyner H., Schneebeli M., W. Ammann & Dual J. (2004). Mechanical Properties of Snow during Rapid Impacts. Proc. of the Snow Engineering Conference 2004, Davos, Switzerland. p. 209-214.

Hanavan, E.P. (1964). A mathematical model of the human body. AMRL Technical Report (TR-64-102), Wright-Patterson Air Force Base, OH.

Mössner, M., Nachbauer, W., Innerhofer, N. & Schretter, H. (2003) Mechanical Properties of Snow on Ski Slopes. 15th International Congress on Ski Trauma and Skiing Safety (ICSS), St. Moritz / Pontresina, Switzerland.

Lüthi A., Federolf P., Fauve M., Oberhofer K. Rhyner H. Ammann W., Stricker G., Schiefermüller C., Eitzlmair E., Schwameder H., & Müller E. (2004). Determination of Forces in Carving using three independent methods. 3rd International Congress on Science and Skiing, Aspen, Co, USA. abstract book. p. 3-4.

Raschner C., Schiefermüller C., Zallinger G. Hofer E., Müller E. & Brunner F. (2001). Carving Turns versus Traditional Parallel Turns - a Comparative Biomechanical Analysis. Müller, E. et al. (eds.), Science and Skiing II, Kovac, Hamburg, p. 203 - 217.