A COMPUTER SIMULATION FOR LEG STIFFNESS IN MAXIMAL SPRINTING

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When humans and other mammals run, the body's complex system of muscle, tendon and ligament behave like spring. A computer simulation for terrestrial running at maximal speed is presented in this study, based on a leg with the properties of non-linear spring. At maximal speed, the horizontal speed has been considered constant while the vertical component was considered zero at landing. Three non-linear springs have been considered for quadriceps, hamstring, and gastrocnemius muscles with different stiffness coefficients. The goal of present study was to determine the relative importance of changes to the leg spring stiffness and the angle swept by the leg spring when the sprinter alters the landing angles (from braking to take-off) at maximal speed. Results also showed that at maximal speed the stiffness coefficients presented non-linear behaviour.

KEY WORDS: computer simulation, non-linear leg stiffness, maximal sprinting

INTRODUCTION: When sprinters run, they bounce along the ground using musculoskeletal springs to alternately store and return elastic energy (Cavagna et al., 1964, 1977). Muscles, tendons and ligaments can all behave like springs, storing elastic energy when they are stretched and retuning it when they recoil (Alexander, 1988). During running, this complex system of musculoskeletal behaves like springs (the leg springs). The idea of considering leg as a simple spring-mass model, consisting of a single linear leg spring and a mass equivalent to the sprinter's mass, has been shown to describe and predict the mechanics of running remarkably well (Alexander, 1992; Alexander and Veron, 1975; Blickhan, 1989; Blickhan and Full, 1993, Cavagna et al., 1988; McMahon and Cheng, 1990; Shahbazi, 2004). But in fact sprinters run faster, the vertical excursion of the centre of mass during the stance phase decreases, and the time that each foot is on the ground decreases. In the spring-mass model, these changes could be the result of an increased stiffness of the leg spring or an increased angle swept by the leg spring during the ground contact phase. By increasing the angle swept by the leg spring, the vertical excursion of the centre of mass and the ground contact time can be reduced changing the stiffness of the leg spring.
Experimental evidence shows that it is possible for humans to alter the stiffness of their leg springs. When humans hop in place and vary their hopping frequency, the stiffness of the leg spring can be changed by as much as twofold to accommodate different hopping frequencies (Farley et al., 1991). Finally, when sprinters run with increased knee flexion (Groucho running), the stiffness of the leg spring appears to decrease (McMahon et al., 1987). These studies showed that it is possible to change the stiffness of the leg springs during bouncing movements.
The aim of present paper was to determine the relative importance of changes to the leg springs stiffness and the angle swept by the leg spring in adjusting the behaviour of the spring-mass system when sprinters alter their leg landing angle at a given sprinting speed.

MECHANICAL MODELING: As is depicted on Figures 1 and 2, three main muscle groups have been considered and simulated to three springs with different stiffness coefficients. The friction coefficients were considered as equal for all muscles; $C_1 = C_2 = C_3$, and the vertical displacements of knee and hip were also considered equal such that; $h_B = h_K + h_H = 2h$. According to Shahbazi (2004) the change in muscle length can be expressed in terms of vertical displacement such that:

$$l_1 = l_3 = h/\sin\alpha_1 \quad \text{and} \quad l_2 = h/\sin\alpha_2 \tag{1}$$

According to Figure 2, the force equilibrium on X and Y axis can be written as

$$(F_2 + F_R)\sin(\pi - \alpha_2) = Mg + F_{13}\sin\alpha_1 \tag{2}$$

$$-(F_2 + F_R)\cos\alpha_2 = F_{13}\cos\alpha_1 \quad (3)$$

On the other hand we can write for the spring simulated the following equations:

$$K_2 h + C\dot{X}\sin\alpha_2 + F_R\sin\alpha_2 = Mg + (K_1 - K_3)h \quad (4)$$

$$-K_2 h\cot\alpha_2 - C\dot{X}\cos\alpha_2 - F_R\cos\alpha_2 = (K_1 - K_3)h\cot\alpha_1 \quad (5)$$

$$\frac{1}{2}(K_1 + K_3)l_1^{\,2} + \frac{1}{2}K_2 l_2^{\,2} = Mgh_B + \frac{1}{2}M\dot{Y}^2 \quad (6)$$

Taking (1) into account, (6) will become:

$$\frac{1}{2}(K_1 + K_3)h^2/\sin^2\alpha_1 + \frac{1}{2}K_2 h^2/\sin^2\alpha_2 = Mgh_B + \frac{1}{2}M\dot{Y}^2 \quad (7)$$

Finally we will be dealing with following three equations:

$$(K_1 - K_2 - K_3)h - C\dot{X}\sin\alpha_2 - F_R\sin\alpha_2 + Mg = 0 \quad (8)$$

$$(K_1 - K_3)h\cot\alpha_1 + K_2 h\cot\alpha_2 + C\dot{X}\cos\alpha_2 + F_R\cos\alpha_2 = 0 \quad (9)$$

$$(K_1 + K_3)h^2/\sin^2\alpha_1 + K_2 h^2/\sin^2\alpha_2 = 2Mgh_B + M\dot{Y}^2 \quad (10)$$

In order to solve the above equations for C, K_1, K_2, and K_3, we should establish a fourth equation. The total vertical force applied to each leg at landing is

$$\sum F = M\ddot{Y} \quad (11)$$

Therefore

$$K_2 h + C\dot{Y} + F_R - Mg = M\ddot{Y} \quad (12)$$

Rearranging (12) we can get a differential equation like

$$M\ddot{Y} - C\dot{Y} - \frac{K_2}{2}Y + Mg - F_R = 0 \quad (13)$$

The solution to this equation is

$$Y = Ae^{-\varpi t} + (Mg - F_R)/(K/2) \quad (14)$$

Inserting into the differential equation, we can get for C

$$C = \frac{K_2}{2\varpi} - M\varpi \quad (15)$$

Combining the equations (8), (9), (10), and(15) We can get for the muscles' stiffness coefficients the following equations:

$$K_2 = \frac{2\varpi Mg\cos\alpha_1 - 2\varpi\sin\alpha(F_R - M\varpi\dot{X})}{2h\varpi\sin\alpha_1(\cot\alpha_1 + \cot\alpha_2) + \dot{X}\sin\alpha}$$

$$K_1 = \frac{K_2}{2}\left(1 - \frac{\sin^2\alpha_1}{\sin^2\alpha_2}\right) + \frac{\sin\alpha_2}{2h}(C\dot{X} + F_R) - \frac{Mg}{2h}(1 - 4\sin^2\alpha_1) + \frac{M\dot{Y}\sin^2\alpha_1}{2h^2} \quad (16)$$

$$K_3 = K_1 + \frac{Mg}{h}\frac{\cot\alpha_2}{(\cot\alpha_1 + \cot\alpha_2)} \quad (17)$$

The whole leg stiffness coefficient can easily be obtained according to the mechanical rules (Holiday et al., 2002) :

$$K = 2\sin\left(\frac{\pi - \alpha}{2}\right)\frac{K_2(K_1 - K_3)}{K_2 + K_1 - K_3} \quad (18)$$

RESULTS AND DISCUSSION: Results for a typical simulation of running are on Figures 3, 4, 5, and 6. The curves show results from the model when the input parameters were chosen to present a sprinter of average size (mass = 73.4 Kg, leg length = 1 m) sprinting with his maximum speed at about 10.5 m/s. At this speed, the trajectory is rather flat and the CG (the centre of gravity) is about 6.2 cm below its position at stance. The general pattern of vertical force application to the ground and of vertical displacement of CG was similar at all strides during the maximal speed. The muscle force reaches its maximum at this point, where the muscle springs are under maximum deformation.

We systematically changed the entire range of parameters space. Results for the required muscle spring stiffness are shown in Figures 3, 4, and 5. In Figure 3 the different muscle stiffness are plotted versus ϖ, while leaving the angle of landing unchanged. In Figure 4 the different muscle stiffness are plotted versus ϖ, while leaving the frequency unchanged. In Figure 5 the different muscles' stiffness are plotted versus α_1 and α_2.

As can be seen, the nonlinear behaviour of the muscles' stiffness remarkably presented. The following procedure was used to obtain the best simulation of experimental situation. Firstly, the change in vertical displacement, which causes the landing angle α in Figure 3 and Figure 5 and secondly, the change in stride length, which causes the stride rate ϖ, in Figure 4. The muscle forces are presented on Figure 6.

Since the discrete damped springs model presented in this paper make several predictions that are testable by experiment, it is useful to review the plausibility of the simulation in light of comparisons with published experimental results.

CONCLUSION: A theoretical modelling for leg discrete muscles has been developed. For each muscle the stiffness coefficient and force variations versus stride rate and leg landing angle have been presented. As the model make several predictions, it would be useful to review the plausibility of the simulation in light of comparisons with published experimental results.

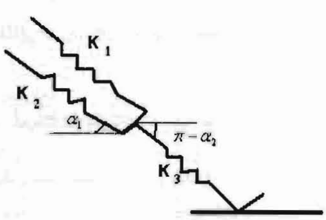

Figure 1 Leg's discrete muscle-spring Simulation.

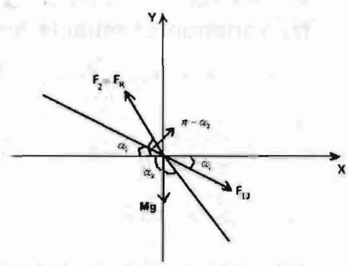

Figure 2 Leg's muscle and ground reaction forces.

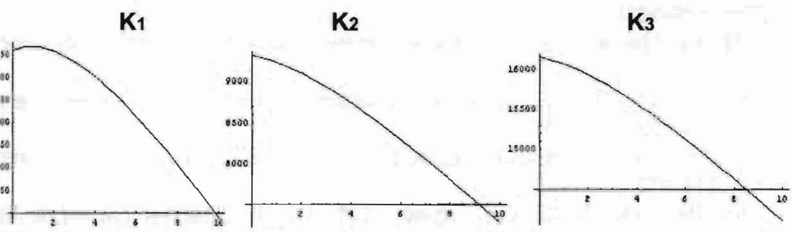

Figure 3 The variation of leg stiffness coefficients with different ϖ and constant angle α.

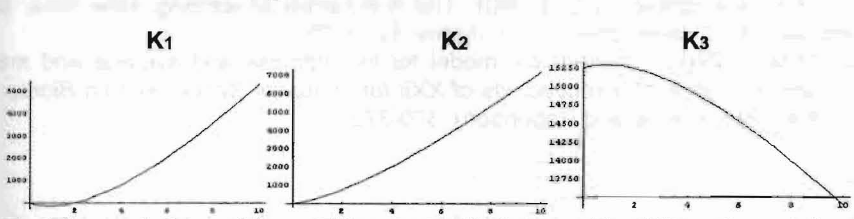

Figure 4 The variation of leg stiffness coefficients with different α and constant stride rate (ϖ). K1, K2, and K3 denote quadriceps, hamstring and gastrocnemius muscles' stiffness coefficients respectively.

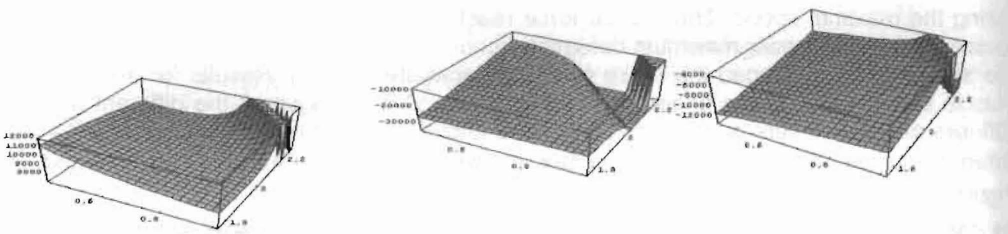

Figure 5 The variation of muscle stiffness coefficients; K_1, K_2 and K_3, which correspond to quadriceps, hamstring and gastrocnemius muscle group versus α_1 and α_2.

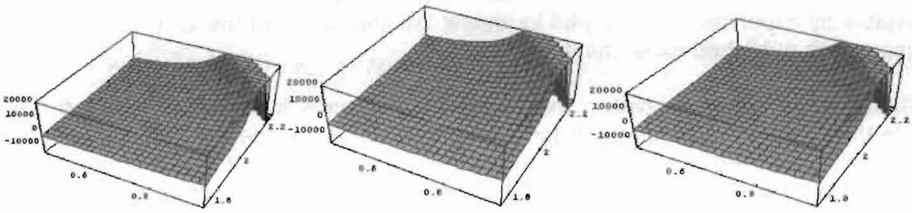

Figure 6 The variation of muscle forces F_1, F_2, and F_3 which correspond to quadriceps, hamstring, and gastrocnemius muscles group versus α_1 and α_2.

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