THE CALCULATION ON THE ROTATIONAL KINEMATICS ABOUT THE LONGITUDINAL AXIS OF JAVELIN BY THREE METHODS

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The motion of javelin is complicated after it is released. It is necessary to use some skills to calculate the rotation kinematics about the longitudinal axis of javelin. In this study, the Euler angle, the Cardan angle and the screw axis methods were used to calculate these kinematic variables. Two synchronized Redlake high-speed cameras (250 Hz) and a javelin with three fixed reflective markers were used in the experiment. Generally the results by three methods were close, and all of the three methods are suitable for the calculation on these variables. In the Euler angle and Cardan angle methods, the order of rotation sequence needs to be taken account, however no similar problem occurred in screw method. The results suggested that the longitudinal axis should be set as second axis in either Euler angle or Cardan angle methods to avoid Gimbal Lock (singularity) problem.

KEY WORDS: Javelin, Euler angle, Cardan angle, Screw axis, Longitudinal axis

INTRODUCTION: The rotation movement about the longitudinal axis of javelin is like a bullet that being shouted off from rifle gun, which rotation could aid the object to move through the air more stably. However, previous researches rarely had further discussions or calculations on the rotation kinematics about javelin axis. The motion of javelin is complicated after javelin is released. It includes translation, rotation about the three axes of global coordinate frame, and vibration. Moreover, the longitudinal axis direction of javelin doesn't coincide with the global system axes. So the angular variables about the longitudinal axis can't be directly calculated just by some simple methods (i.e. $v = \omega \times r$ neither by the projection angle on global axis). Such a calculation needs some mathematical, physics and computer programming knowledge to be accomplished. Three of the most often used methods to calculate the angular variables in biomechanics are the Euler angle method, Cardan angle method and Screw axis method (Robertson, Caldwell, Hamill, Kamen & Whittlesey, 2004). Each method has its advantage and limitations. The purpose of this study was to examine the accuracy of these methods on the angular variables about javelin axis.

METHODS: An athlete (not expert in javelin, personal record: 40 m) was recruited as the subject, height: 178 cm, weight: 78 kg. Two synchronized Redlake high-speed cameras (sampling rate: 250 Hz, shutter speed: 1250 Hz) were setup on two sides of the runway to record the data. In order to access the local coordinate system, some "anatomic" landmarks need to be set on javelin. A total of three non co-linear markers were used. First one (point A) is located on the tip portion. In order to setup another two markers, a wing-liked device made of #14 Zn-wired was fixed on the tail portion. The wing weights 7.39 g. The position of the wing is not unchangeable, and it could be adjusted to a proper site for individuals. B and C points were fixed on two sides of the wing (Fig. 1). No matter whether the tip landed first, the position and shape of the wing was checked and adjusted after each throwing.

The position of the wing was determined by the following three principles:

1. No injury to athlete
2. No interference with athlete
3. As close to grip as possible (in order to reduce the error from vibration)

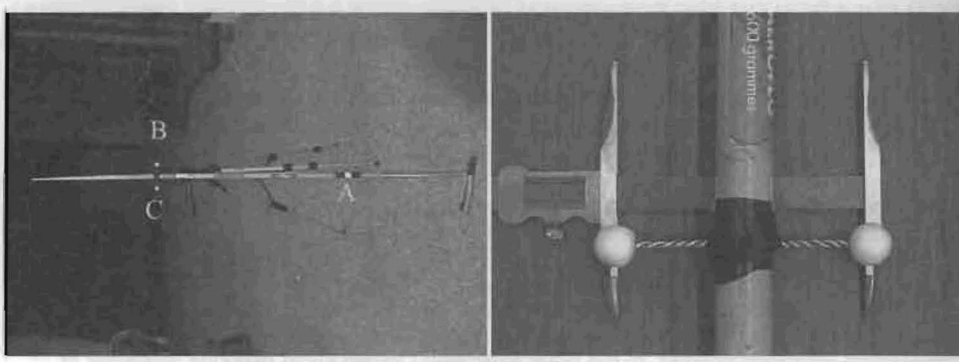

Figure 1 The overview of javelin with three landmarks (left) and magnifying photo of the wing with two marks (right).

The video clips were digitized by Kwon-3D motion analysis system, and the raw data was smoothed by Butterworth 4^{th}-order zero lag with 6 Hz cutoff frequency. Position vectors of the marks defined the local coordinate system. P point was set as the middle of B and C. Vector PA (the direction of javelin longitudinal axis) was defined as Z-axis. X-axis was defined as $PC \times PA$. Y-axis was defined as $Z \times X$. Two neighbor film frames defined the rotation matrix. In Euler and Cardan angle methods, any possible order of rotation such as Z-axis was set as first, second or third axis was analyzed for comparing the results. The screw method used the revised formulation from Spoor and Veldpaus (1980). Matlab 6.5 was used in computer programming.

RESULTS AND DISCUSSION:

Table 1 The angular displacement about the longitudinal axis after release. Unit: deg

Frame after release	Euler (Y-Z-Y)	Cardan (Y-Z-X)	Screw
1^{st}	7.2193	7.2185	7.2197
2^{nd}	7.6749	7.6738	7.6750
3^{rd}	8.0121	8.0122	8.0128
4^{th}	8.3871	8.3868	8.3879
5^{th}	8.6048	8.6046	8.6055

The results by Euler angle (Y-Z-Y) were similar to that of Cardan angle in the Y-Z-X sequence and screw method (Table 1). The order of rotation is needed to be taken account in Euler and Cardan angle, because different rotation sequence may result in different outcomes. The javelin longitudinal axis was defined as Z-axis of local coordinate system in this study. Because the two of the three rotation axes are the same in Euler angle, it has four kinds of sequence: Z-Y-Z, Y-Z-Y, Z-X-Z and X-Z-X. However, if the Z-Y-Z or Z-X-Z sequence is adopted, first axis coincided with the third one, it may be produce some error. It could be explained that because the direction of longitudinal axis of javelin didn't change much during neighbor frames. When a rotation occurs about the first and the third axes at the same time, it is not clear whether the rotation occurred about the first or the third. Therefore, when the directions of axes are defined, one should be very careful to avoid this singularity problem.
The similar problem about rotation sequence also occurred in Cardan angle. For avoiding Gimbal Lock, Z-axis is also recommended being set as second axis (floating axis). The screw method needn't consider the rotation sequence. In the screw method, no matter which axis of the local system was used to define the longitudinal axis, the results were the same.
The Angular velocity and angular acceleration:

Table 2 The angular velocity about the longitudinal axis after release. Unit: deg/s

Frame after release	Euler (Y-Z-Y)	Cardan (Y-Z-X)	Screw
1st	1835.6	1864.2	1864.5
2nd	1944.7	1962.1	1962.3
3rd	2029.5	2052	2052.2
4th	2112.3	2129.2	2129.5
5th	2170.4	2174.8	2174.9

Table 3 The angular acceleration about the longitudinal axis after release. Unit: deg/s^2

Frame after release	Euler (Y-Z-Y)	Cardan (Y-Z-X)	Screw
1st	24718	27189	27160
2nd	20284	22951	22927
3rd	19824	21870	21887
4th	6076.1	14905	10543
5th	2728.8	10569	3094.7

The calculation on angular velocity by the Euler angle and the Cardan angle methods are more complicated than by screw axis. In the Euler angle and the Cardan angle methods, it is needed that some rotation matrix were multiplied by the first order derivative of angular displacement to calculate the angular velocity by (Fowles and Cassiday, 1998). Table 2 showed that the angular velocity calculated by the Cardan angle method is more close to that by the Screw method than by the Euler angle method. Similar situation occurred in the results of angular acceleration (Table 3). But generally speaking, there is not much difference between the three methods in these variables.

Figure 2 showed that the angular velocity reached its peak (about 1864 deg/s ≈ 5.1 rps) sooner after javelin was released, and then significantly decreased during the follow through. The angular acceleration was significantly increased during the acceleration phase and reached its peak (about 27100 deg/s^2) soon before javelin was released, and then rapidly decreased during the follow through. It could be explained that the applied force disappeared after release and the wind caused resistance on the wing, which resulted in significant decrease of the angular velocity.

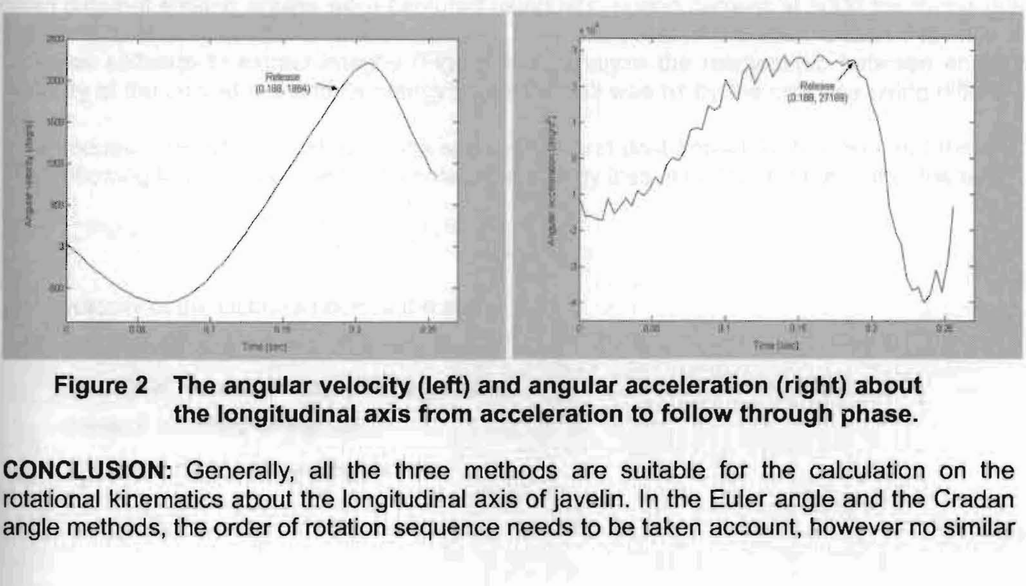

Figure 2 The angular velocity (left) and angular acceleration (right) about the longitudinal axis from acceleration to follow through phase.

CONCLUSION: Generally, all the three methods are suitable for the calculation on the rotational kinematics about the longitudinal axis of javelin. In the Euler angle and the Cradan angle methods, the order of rotation sequence needs to be taken account, however no similar problem occurred in screw method. The results suggested that the longitudinal axis should be set as second axis in either Euler angle or Cardan angle methods to avoid singularity or Gimbal Lock problem. In addition, a more convenient or precise device could be developed for loading the extra marks on javelin in the future researches.

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