INERTIAL AND VIBRATIONAL CHARACTERISTICS OF SOFTBALL AND BASEBALL BATS: RESEARCH AND DESIGN IMPLICATIONS

Larry Noble, Kansas State University, Manhattan, USA

INTRODUCTION: The design of baseball and softball bats has been an ongoing process since the inception of the game in the early 19th century. Until the early 1970's, the only material to be used was wood and the potential for design improvements was limited because they involved changing the physical dimensions of the bat. Also, most of the changes were initiated by the player and were carried out by local craftsmen in wood shops. However, with the introduction of hollow-wall aluminum bats in the 1970's and other metal alloys as well as composite materials since then, the potential for applying relevant mechanical principles to improve the performance characteristics of bats has improved dramatically. In recent years a plethora of bats with innovative features have become available to the consumer. Some of these innovations are a result of the research and development efforts of the bat manufacturing industry, often in collaboration with scientists not directly affiliated with the industry, while others are a result of flawed ideas that are not completely thought through. Braham (1997) provides a recent, up-to-date summary of recent innovations by the leading bat manufacturers, but an evaluation of the claims of these products using relevant criteria is lacking. The purpose of this paper is to provide a scientific basis and a focus for examining and developing new bat design features. The paper will provide an overview of the factors that are relevant to the design of baseball and softball bats, including related theoretical and empirical studies.

Factors relevant to the design of baseball and softball bats

When developing or evaluating a design feature of a baseball or softball bat, the following factors must be considered: (1) the manner in which the bat is swung and forces are transmitted to the bat during the swing, (2) the constraints resulting from rules in the particular sport, and (3) the relevant properties of the bat. I will review each of these general factors in detail, making reference to related scientific literature.

Characteristics of batting relevant to bat design

Baseball and softball batting is a two-handed sidearm striking skill with the bat held near one end and swung as a physical pendulum. Power hitters attempt to impart maximum velocity to the impacted ball in the desired direction by generating maximum linear velocity of the impacting part of the bat to the ball. The motion of the bat is predominantly in the horizontal plane and the rotation axis ranges from .15 to.20 m off the knob end of the bat toward the hitter's body during the swing. During the swing, the maximum linear (COP) and rotational velocity of the bat is approximately 33 m/s and 36 r/s, respectively, for college females and 38 m/s and 44 r/s, respectively, for college males. Because the primary goal of the power swing is to maximize bat velocity on impact, it is somewhat surprising that maximum bat velocity has repeatedly been found at from .01 to .05 s prior to impact (Shapiro, 1974; McIntyre & Pfautsch, 1982; Messier & Owen, 1984; Spragg, 1986). While this finding has been reported in the scientific literature frequently, a plausible explanation of the reason has not been found. A review of the elastic properties of the bat (appearing later in this paper) and the characteristics of the swing may provide a tenable hypothesis. It is possible that, at the beginning of the swing, torque applied to the bat handle to rotate the bat toward the incoming ball and the inertia of the barrel end of the bat cause the bat to bend, with the barrel of the bat lagging behind. This bending mode, usually referred to as the diving board mode , would begin to release when the rotational acceleration of the bat begins to drop. It is conjectured that the elite hitter learns through trial and error to adopt a bat and swing that are matched such that the velocity of the impact point of the bat is maximized at impact. To accomplish this end, accelerating forces would be reduced quickly at either 1/4 or 1 3/4 of the period of oscillation of the diving board mode. For example, if the fundamental, diving board mode of a bat is 25 Hz, then the period of oscillation is 40 ms. For the hitter to take advantage of this elastic behavior, this bending mode would need to be „released“ at approximately 10 or 70 ms prior to impact. While this characteristic of the skilled golf swing has been empirically verified (Cochran & Stobbs, 1986), no empirical data in support of this hypothesis applied to softball or baseball bats have been found.

Rules most relevant to bat design

The rules regarding baseball bat characteristics are different from those regarding softball bats. Also, rules are different for different genders and different levels of play. For adult males, the maximum baseball and softball bat barrel size is 2.25 and 2.75 in (.057 and .070 m), respectively. The maximum bat length is 42 and 34 inches (1.067 and .864 m) for baseball and softball, respectively, while the maximum softball bat weight is 38 oz (10.569 N). There is no maximum baseball bat weight. All bats used in professional baseball must be made of wood. The most recent rule, which places an upper limit on the coefficient of restitution (Bat Performance Factor) for bats at different levels and types of play, is having a tremendous impact on the direction of bat design activity. Bat performance factor will be discussed in greater detail later in this paper.

Inertial and vibrational properties relevant to bat design

Several inertial and vibrational properties of the bat are relevant to its effective use: (1) mass, (2) moment of inertia, (3) coefficient of restitution, (4) location of node of the fundamental vibration node, and (5) center of percussion location.

Mass and moment of inertia determine the amount of effort required to swing the bat. There is an inverse relationship between bat linear and angular acceleration and mass and moment of inertia, respectively, for a given linear and angular impulse (integral of force/torque and time). Thus, the more mass and/or moment of inertia, the more impulse required to produce a given change in bat speed or direction. In other words, greater mass and moment of inertia compromise the hitter’s ability to control the path of the bat as it moves toward the ball as well as to generate bat velocity during the swing. Theoretical models of the relationship between mass and impact parameters indicate that lighter bats than have been used by most skilled players would be more effective (Kirkpatrick, 1963; Adair, 1990). In a study which sought to empirically determine an individual’s ideal bat, this concept was supported (Bahill & Karnavas, 1991). Furthermore, bats now used by elite softball and baseball players are much lighter than they were 10 years ago.

Bat manufacturers and retailers do not provide moment of inertia measurements with their products; however, moment of inertia is a critical design parameter and is also used to develop bat selection guidelines.

When the ball and bat are impacted, during impact the bat behaves in some respects as a physical pendulum and in some respects as an elastic body. Taking both rigid-body and elastic properties into consideration, the best part of the bat on which to hit the ball is called the „sweet spot“. The „sweet spot“ is a general, nonscientific term, that means that the best overall results are obtained from impacts on this point. In other words, impacts on the sweet spot feel best to the hitter and results in imparting velocity to the ball are best. Or, in more precise terms, the sweet spot is the impact location where the transfer of energy from the bat to the ball is maximal while the transfer of energy to the hands is minimal. On closer examination, four parameters have been identified as having an effect on the „sweetness“ (liveliness) and location of the sweet spot: (1) center of percussion (COP), (2) node of the fundamental vibrational mode, (3) coefficient of restitution, and (4) the maximum „power“ point.

Center of percussion. When the ball hits the bat at the center of percussion (COP), there is no reaction impulse (shock) at the axis. The impact axis for bats has been shown to be the point under the first knuckle of the top hand (Plagenhoef, 1971; Noble, 1983). Thus, COP impacts are more comfortable than at other locations because there is no painful impact „shock“ that is experienced during impacts at other locations. The distance from the impact axis to the COP of a bat can be found from the following equation:

$COP_{dist} = T^2g/4\pi^2 = .2483877*T^2$ (units in meters)

where T is the period of one oscillation when the bat is suspended from the axis, and g is the gravitational constant in meters. The COP has been demonstrated to be the impact location producing the greatest post-impact velocity with stationary bats (Weyrich, Messier, Ruhmann, & Berry, 1989). Another empirical study involving 18 elite slow-pitch softball hitters reported a correlation of .58 between the perceived location of the sweet spot and the COP. Thus, COP location explained 33% of the variability in perceived sweet spot impact location (Noble, 1983). Brody developed a theoretical construct for determining the impact location of a swinging bat with a pitched ball that would result in greatest postimpact ball velocity (Brody, 1986). This location was not on the COP, but was a function of the relative velocity and mass of the ball and bat as well as the inertial properties of the bat.

In an early study (Bryant, Bryant, Chen, & Krahenbuhl, 1977) comparing the dynamic and performance characteristics of aluminum and wood bats, data were reported showing an impact area of several cm in length on hollow-wall aluminum bats where there was zero-order reaction impulse while reaction impulse on wood bats was a direct linear of function of distance from the COP. However, a later study by Noble and Eck (Noble & Eck, 1986) presented a theoretical construct and empirical data demonstrating that, assuming the bat is rigid during impact, reaction impulse is a direct linear function of the distance of the impact from the COP (Figure 1). Also, the slope of the regression line of impact reaction impulse on impact-COP distance is a direct linear function of the distance of the COP from the impact axis (Figure 2). In other words, the greater the distance of the COP from the hands, the smaller the reaction impulse resulting from an impact of a given distance from the COP. This study also demonstrated that the distance of the COP from the axis was:

COP = **I**/Mr

where **I** = moment of inertia about the axis, M = the total bat mass, and r = distance from the axis to the center of mass. Strategies were later presented for systematically displacing the location of the COP (Noble & Eck, 1985) by placing mass at various locations along the longitudinal axis of the bat. While these strategies were effective in displacing the COP to a more distal location on the bat, this „improvement" did not enjoy wide acceptance by the players because of the greater excitation of the fundamental vibrational mode resulting from COP impacts (Noble & Walker, 1994b).

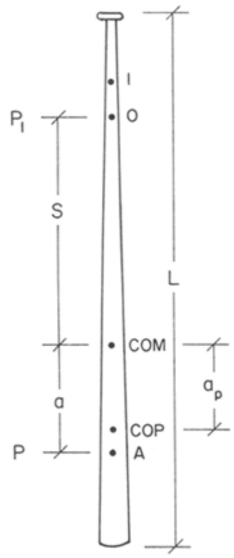

Figure 1. Mechanical system describing center of percussion (COP) location as a function of impact location. (Legend: P = impact impulse, P_1 = impact reaction impulse, A = impact location, a_p = distance from COM to COP, O = impact axis, M = mass, S = axis-COM distance, I_o = moment of inertia about impact axis). Note. From „Effects of Selected Softball Bat Loading Strategies on Impact Reaction Impulse" by L. Noble and J. Eck (1986). *Medicine and Science in Sports and Exercise* **18**, 51. Copyright 1986 by the American College of Sports Medicine. Adapted with permission.

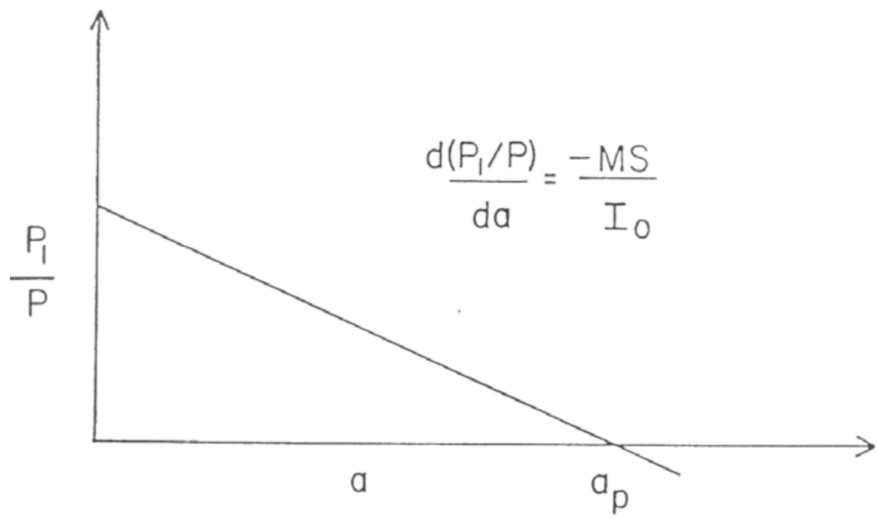

Figure 2. Theoretical relationship between impact reaction impulse and impact location. (Legend: P = impact impulse, P_1 = impact reaction impulse, a_p = COP location, a = distance from impact axis to impact, M = mass, S = axis-COM distance, I_o = moment of inertia about impact axis). Note. From „Effects of Selected Softball Bat Loading Strategies on Impact Reaction Impulse" by L. Noble and J. Eck (1986). *Medicine and Science in Sports and Exercise* 18, 51. Copyright 1986 by the American College of Sports Medicine. Adapted with permission.

Vibrational properties. The above discussion relates to the rigid body behavior of the bat during impact. The bat also exhibits important and relevant elastic properties during impact as well as during the swing because the bat is not completely rigid. The vibrational behavior of a bat approximates that of a uniform beam, described in detail in most engineering textbooks on vibrations (Thompson, 1993). If we assume, for simplicity, that a bat can be represented by a uniform rod rigidly suspended at the point of contact with the hands, then the various normal vibrational modes are only those for a rigid clamped rod. Figure 3a illustrates the approximate length (modeled after a uniform rod), node locations, and relative amplitude of these modes. The lowest frequency mode, commonly called the diving board mode, corresponds to a vibration where the axis is at a nodal point and the barrel end is at a maximum. This node has a wavelength (λ_0)of approximately 4H, where H is the distance from the axis to the barrel end of the bat. This fundamental mode only has one node and it is located at the clamped point. The next highest frequency mode has a node at the handle and another at 3/4H. If the ball strikes the bat at a node of a given vibrational mode, then that mode will not be excited. Since all modes have an anti-node at the

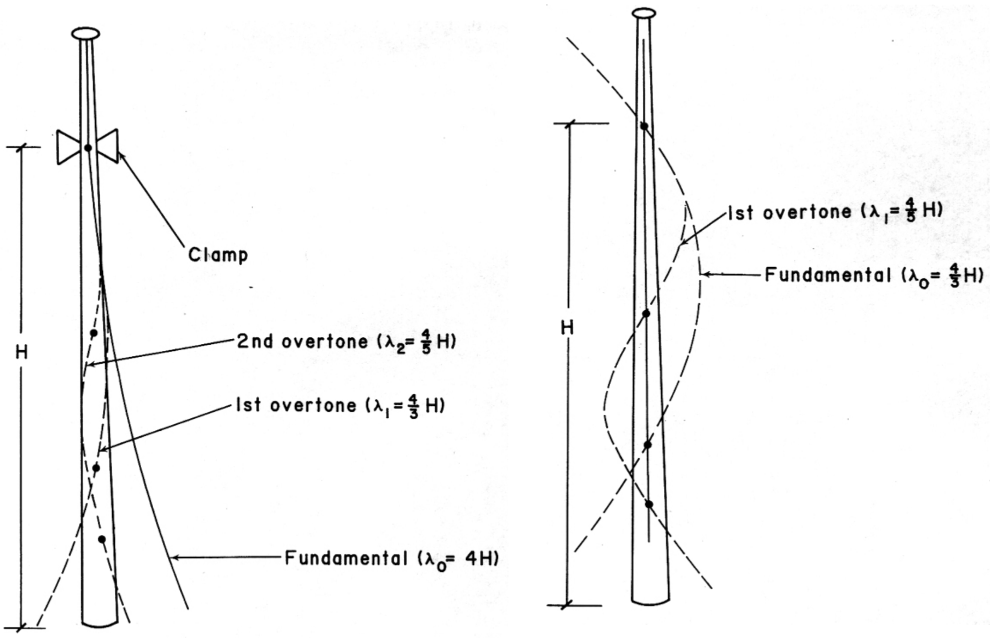

Figure 3. Vibrational modes of bat as (a) a uniformed rod clamped at the axis, and (b) as a free-free rod. (λ_i = approximate wavelength of *i*th harmonic modeled as a uniform rod). Note: Part a from „Effects of Selected Softball Bat Loading Strategies on Impact Reaction Impulse“ by L. Noble and J. Eck (1986). *Medicine and Science in Sports and Exercise* **18**, 51. Copyright 1986 by the American College of Sports Medicine. Part b from „Baseball Bat Inertial and vibrational Characteristics and Discomfort Following Ball-Bat Impacts“ by L. Noble and H. Walker (1994). *Journal of Applied Biomechanics* **10**, 134. Copyright 1994 by Human Kinetics, Inc. Adapted with permission.

unclamped, or barrel end of the bat, all modes of vibration can be excited when striking the bat at the barrel end. The frequency of the diving board mode has been reported to be 27 Hz for an aluminum softball bat and 18 Hz for a wood softball bat (Brody, 1990) and that of the first overtone mode was 317 and 209 Hz, respectively. Both of these bats were 34 inches (.864 m) in length. Shorter bats and bats with greater strength/mass ratios will have higher fundamental frequencies. It is likely that this mode is excited during the swing, as has been demonstrated during the swinging of golf clubs (Cochran & Stobbs, 1986); however, this low-frequency mode is not excited by the impacting ball (Brody, 1990; Noble & Walker, 1994a; Noble & Walker, 1994b).

During impact, the vibration behavior of the bat corresponds to that of a free, non-supported bat whether irregardless of the firmness of the grip. Figure 3b illustrates the lowest (fundamental) and first harmonic modes, approximate wavelengths and node locations of the free, unsupported bat. The approximate locations of the two nodes for the first mode are 29% of the bat length from each end. The next highest mode has three nodes with approximate locations as depicted in Figure 3b. The number of nodes for each successively higher mode increases by one at each step. Also, the amplitude associated with each mode decreases as the frequency increases and increases as the distance from the node increases.

The node locations and frequency of the fundamental and first harmonic modes can be measured by, first, supporting the bat in a horizontal position by threads attached to the ceiling. A vibration exciter and velocity sensor are horizontally oriented and placed as indicated in Figure 4. Specific placement of the exciter and sensor is not critical as long as they are not placed on one of the nodes. A resister is put in series with the exciter coil, and the voltage displayed on the horizontal axis of an oscilloscope. This voltage is proportional to the current through the resister and exciter coil which is proportional to the force applied by the exciter. The output of the velocity sensor is displayed on the vertical axis of the oscilloscope. This display is generally an ellipse whose axes are oriented at an angle determined by the gain setting of the oscilloscope, the phase relationship of the two signals, and the location of the velocity sensor. The input is gradually increased in frequency until resonance is achieved. At resonance, the exciting force and velocity are in phase and the ellipse is reduced to a straight line. The straight line changes slope as the velocity sensor is moved along the long axis of the bat. The slope changes from positive to negative as the sensor passes over a node and is horizontal when the sensor is located at a node.

The fundamental frequency (free condition) of 34-inch (.864m) aluminum softball bats is usually within the range of 180-360 Hz and can be varied throughout this range by changing the shape of the bat and strategic redistribution of mass (Noble & Walker, 1993). The easiest way to change the fundamental frequency is to modify the diameter of the bat in the taper region, which is near the antinode of the

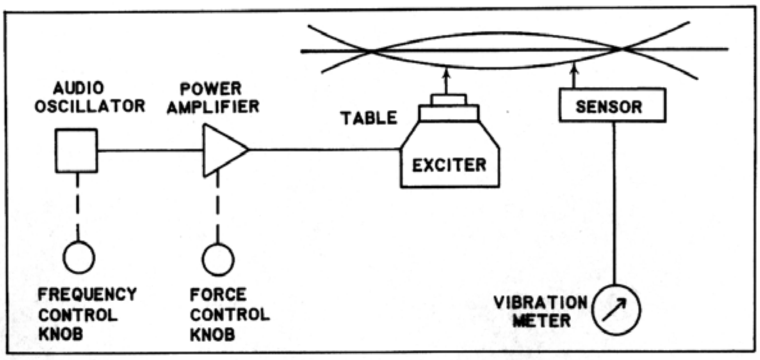

Figure 4. Vibration measurement system. From „Baseball Bat Inertial and vibrational Characteristics and Discomfort Following Ball-Bat Impacts" by L. Noble, H. Walker (1994). *Journal of Applied Biomechanics* **10**, 134. Copyright 1994 by Human Kinetics, Inc. Adapted with permission.

fundamental mode. Node locations of the fundamental mode can also be strategically displaced as much as .06 mm using similar strategies. The distal node of the fundamental mode has been identified as one of the determinants of the sweet spot. Impacts on the node do not excite the low-frequency, fundamental mode, resulting in a higher-frequency sound and smaller, higher frequency vibrations of the bat handle. Thus, it is reasonable to expect that nodal impacts would be more comfortable for the hitter. Fortunately, in most bats the location of this node and the COP are less than .02 m apart, approximately 25 per cent of the bat length from the barrel end. When using bats with the node and COP close together, impacts at both locations are more comfortable than at other locations (Noble & Walker, 1994a) with no significant difference between the two. However, strategies developed to strategically displace the COP toward the barrel end were found to significantly increase the distance (.06 m) between the node and COP. Impacts at the node on these bats were found to be more comfortable than impacts at any other location, including the COP (Noble & Walker, 1994b). This is consistent with results from a study investigating the sweet spot location of tennis racquets indicating the node as the predominant predictor of sweet spot location (Hatze, 1994). Furthermore, the node-COP distance has been found to be one of the most powerful predictors of player's perception of bat performance and preference (Noble & Dzewaltowski, 1994).

No empirical studies have been found investigating the effects of impact location relative to the location of the distal node of the fundamental mode on post-impact ball velocity. However, an excellent theoretical presentation of this effect has been developed, but unfortunately it remains unpublished (Van Zandt, 1991). This paper applied the theory of the elastic behavior of an irregularly shaped, cylindrically symmetric object to a wood baseball bat (frequency of diving board mode and first harmonic - 27 Hz and 137 Hz, respectively). The node-COP distance was .01 m. A set of equations was developed and used to find the first 20 normal modes of vibration and calculate each mode's effect on the recoil of the ball. Figure 5 illustrates estimates of the post-impact ball velocity as a function of impact point along the length of the bat. The impact ball and bat velocities were 40 m/sec and 16 m/sec, respectively. The solid curve shows the expected result, the dashed curve shows the effect of suppressing all modes above the fundamental for the free condition (137 Hz), and the starred curve shows the performance expected for an infinitely rigid club having otherwise identical properties. Two observations relative to the effect of impact location and post-impact ball velocity are notable: (1) post-impact velocity is significantly lower for impacts not on the node, especially as the impact location moves toward the handle of the bat; and (2) the higher-frequency modes serve to increase post-impact ball velocity. For a collision only .10 m toward the bat handle from the node, post-impact ball velocity is expected to decrease by 5%. The effect of this loss in ball velocity would cost a distance in ball flight of 10%, or 42 ft (12.8 m). The higher elastic modes play an important role in bat performance, restoring approximately 50% of the loss in ball velocity. This degradation in bat performance would be less with stiffer bats with higher natural frequencies, such as those made from composites, aluminum, and other metal alloys. The loss in bat performance would be eliminated if the fundamental frequency of the bat was „tuned" to the ball-bat contact, or dwell, time. To obtain impact frequency tuning, the fundamental frequency should equal the reciprocal of twice the dwell time. For example, the dwell time of a softball and bat collision (velocity = 31 m/s) has been measured at .0035 sec (Plagenhoef, 1971). For this case, the frequency-matched bat would have a fundamental frequency of 143 Hz. This procedure would be difficult to effectively apply with precision because the dwell time is a function of collision velocity, which is not entirely under the control of the hitter, as well as the hardness of the ball.

Coefficient of restitution. The coefficient of restitution (COR) of two colliding objects, such as the ball and bat, is the ratio of the difference between their velocities immediately after impact compared to the difference between their velocities prior to impact. This ratio has been shown to be a function of collision velocity as well as temperature. For simplicity, the COR of balls and bats are evaluated separately. Ball COR is usually determined by impacting the balls with a wooden wall backed by concrete. Rules for different levels of play for softball and baseball have been established setting an upper limit for the COR, usually determined by impacting the balls with a steel plate at 60 mph (26.4 m/s). The COR is now often stamped on the cover of the ball. For most adult softball competition, the maximum COR is .55. The COR of bats has been shown to be a weak function of impact location along the barrel of the bat where the diameter is constant. Thus, it does not play a significant role in determining the location of the sweet spot. If a given ball impacts with a ball with these conditions held constant, the bat with the higher coefficient of restitution will produce the greatest post-impact ball velocity. Improving the COR of bats has been the primary focus of the research and development efforts of the major bat manufacturers during the past decade. The COR has been significantly improved through the use of materials of higher strength/mass ratios and strategic manipulation of the wall thickness of the barrel of the bat. These dramatic increases in COR have caused great concern on the part of coaches and officials associated with all levels of play. This concern relates to the safety of the participants and to the changes in the nature and integrity of the games that those associated with the game identify with. An outgrowth of this concern is the adoption of a standard for evaluating the

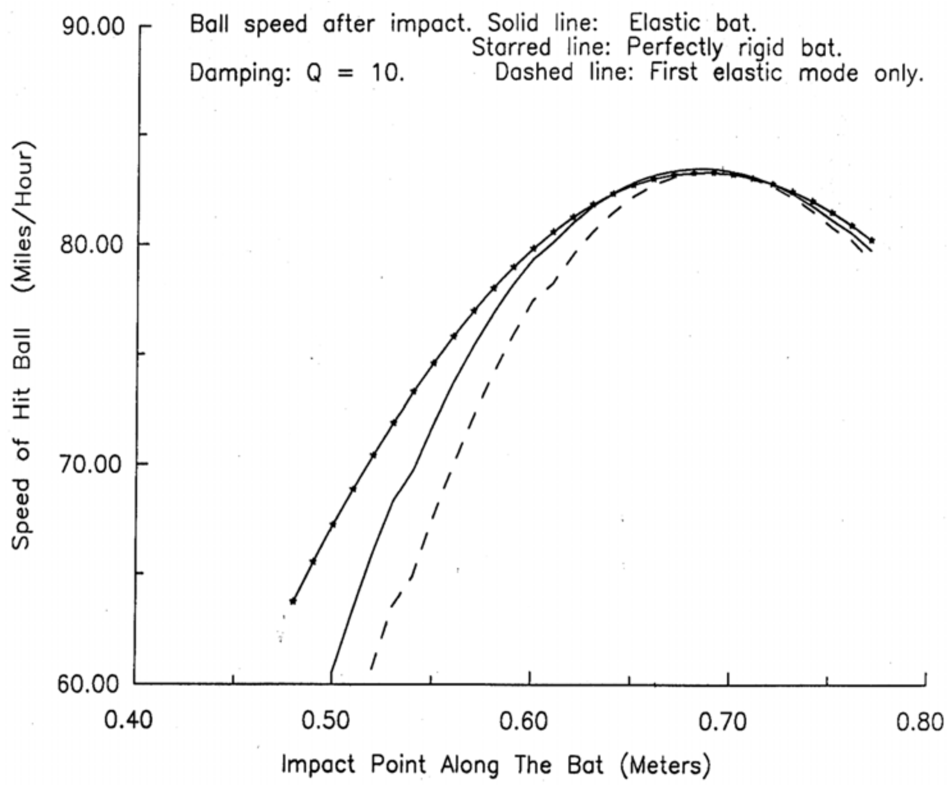

Figure 5. Theoretical estimates of the degradation of post-impact ball velocity at different impact locations due to bat vibrations. From *The Dynamical Theory of the Baseball Bat*, by L. L. Van Zandt (1991). Unpublished manuscript. W. Lafayette, IN: Purdue University.

„liveliness" (COR) of bats and implementing rules placing an upper limit on this aspect of bat performance. A standard method of testing to measure the COR of bats has recently been adopted by most of the baseball and rules committees in the USA (ASTM, 1995). This procedure involves impacting a ball with a known COR with a stationary bat with a fixed axis (free to rotate) at the bat's COP at a ball speed of 88 ft/sec (26.8 m/s). The COR is calculated by the standard method of comparing the difference between the velocity of the bat and ball before impact to that following impact. A Bat Performance Factor (BPF) is then calculated from the ratio of bat and ball COR to the ball COR. For example, if the predetermined ball COR is .5 and the measured COR of the ball-bat collision is .55, then the BPF of the bat would be 1.1. The BPF of most wood bats is between .9 and 1.0 while that of the latest aluminum alloy bats is typically above 1.1. Maximum bat performance standards have now been adopted by most levels of play for both baseball and softball. For example, the maximum BPF allowed for most adult slow pitch softball is 1.2 and that established for collegiate baseball is 1.15. The adoption of this standard has changed the recent trend in bat design from focusing on increasing COR to improving other performance characteristics that comply with the rules.

Maximum „power“ point. The impact point along the longitudinal axis of the bat that will result in the greatest post-impact ball velocity, or maximum power point, is another important performance characteristic of the bat. Brody (1986) developed a theoretical framework and estimates of the maximum power point for a bat that is swung and impacts with a pitched ball, behaving as a free-free body during impact. Estimates of the location of the distance from the maximum power point to the bat COM (PPTDIST) were found to increase as the mass and moment of inertia of the bat increases and as the ratio of bat/ball mass increases. Furthermore, PPTDIST increases as the ratio of bat angular velocity to ball linear velocity increases. Thus, PPTDIST was estimated to be less for baseball than for fastpitch softball. PPTDIST was calculated for an aluminum softball bat and ball with varying bat/ball velocity and mass ratios and found the location of the maximum power point to be located between the COM and COP. No empirical data relative to these estimates was reported. Weyrich, Messier, Ruhmann (1989) provided empirical data on the effects of impact location on postimpact ball velocity. Bats were not swung, but held stationary by either clamps or strings mounted to the ceiling while balls were propelled against the bat at 27 m/s. Impacts on the COP produced greater postimpact ball velocities than impacts on the COM and impacts near the barrel end. While these results are apparently contradictory to the theoretical work of Brody(1986), these bats were not swung, as Brody’s estimates assume, and no impacts were studied in the area between the COM and COP. The empirical data provided by Noble (1983) in an earlier study does, however, appear to conflict with Brody’s estimates. In this study, the location of 52 self-perceived „sweet spot“ impacts of 12 elite male softball players and 5 highly skilled college male baseball players was examined using cinematographic methods (200 f/s). The „sweet spot“ location, the point where the results were best, coincided very closely with the COP location. Furthermore, sweet spot location relative to the COP did not appear to be different for softball and baseball impacts. If it can be assumed that these elite players’ perception of sweet spot is synonymous with the maximum power point, then these results do not seem to support Brody’s 1986) theoretical estimates of maximum power point location. However, because COP location only accounted for 33 per cent of the variability in sweet spot location in the empirical study (Noble, 1983), other characteristics of the bat, ball, hitter-bat interface and of the swing probably affected these results. Further empirical research is necessary to clarify this issue.

CONCLUSIONS: Several important factors relevant to the design of baseball and softball bats were identified.: (1) how is the bat swung and how forces are transmitted to the bat during the swing, (2) what are the constraints resulting from rules in the particular sport, and (3) what mechanical properties are relevant. The most important mechanical properties of the bat are: (1) mass, (2) moment of inertia, (3) coefficient of restitution (COR), (4) vibrational properties, (5) center of percussion location (COP), and (6) maximum power point location. While mass and moment of inertia determine the magnitude of the efforts to swing and control the bat during the swing, COR, COP, and vibrational properties largely determine the behavior of the bat during impact. Obviously, all of these factors play a vital role in the overall effectiveness of the bat in imparting maximum energy to the ball while imparting minimal energy to the hitter. Correlating studies related to the swing characteristics and vibrational properties of the bat provides a hypothesis for the surprising and recurring finding of maximum bat velocity occurring prior to contact, rather than at contact. Bat design objectives during the past few years has involved: (1) interior loading strategies to displace the COP toward the barrel end, (2) changing the weight distribution and shape of the bat to displace the COP toward the barrel end while keeping the distance between the distal node of the fundamental vibrational mode and COP small, and (3) increasing the COR. Implementing these strategies has involved the adoption of improved metal alloys and composite materials having a greater strength/mass ratio. Because of recent rule changes setting an upper limit on COR, research and design activity will undoubtedly change direction and focus. One of the potentially fruitful areas of research and design activity appears to develop bats that are „frequency tuned". A frequency tuned bat would be designed to take advantage of the elastic properties of the bat **during the swing** as well as **during the impact**. **Swing tuning** involves coupling the diving board mode (clamped-free condition)with characteristics of the swing so this mode is excited and released at the most opportune time. **Impact tuning** involves coupling the mode of the fundamental node to the ball-bat dwell time.

REFERENCES:

American Society for Testing Materials. (1995). *Standard Test Method for Measuring Bat Performance Factor.* Proposed ASTM Standard (Revision 5.3, 9/12/95).

Adair, R. K. (1990). The Physics of Baseball. New York: Harper and Row Publishers.

Bahill, T., Karnavas, W. (1991). The Ideal Baseball Bat. *The New Scientist* **6**, 26-31.

Braham, J. (1997). Sports Technology: Keep your Eye on the Bat. *Machine Design* **July 10**, 56-66.

Brody, H. (1986). The Sweet Spot of a Baseball Bat. *American Journal of Physics* **54**, 640-643.

Brody, H. (1990). Models of Baseball Bats. *American Journal of Physics* **54**, 756-758.

Bryant, F. O., Bryant, L. N., Chen, S. S., Krahenbuhl, G. S. (1977). Dynamic and Performance Characteristics of Baseball Bats. *Research Quarterly* **48**, 505-510.

Cochran, A., Stobbs, J. (1986). The Search for the Perfect Swing. Grass Valley, CA.: The Booklegger.

Hatze, H. (1994). Impact Probability Distribution, Sweet Spot, and the Concept of an Effective Power Region in Tennis Rackets. *Journal of Applied Biomechanics* **10**, 43-50.

Kirkpatrick, P. (1963). Batting the Ball. *American Journal of Physics* **31**, 606-613.

McIntyre, D. R., Pfautsch, E. W. (1982). A Kinematic Analysis of the Baseball Batting Swings Involved in Opposite-Field and Same-Field Hitting. *Research*

Quarterly for Exercise and Sport **53**, 206-213.
Messier, S. P., Owen, M. G. (1984). Bat Dynamics of Female Fast Pitch Softball Batters. *Research Quarterly for Exercise and Sport* **55**, 141-145.
Noble, L. (1983). Empirical Determination of the Center of Percussion Axis of Softball and Baseball Bats. In D. A. Winter, R. W. Norman, R. P. Wells, K. C. Hayes, A. E. Patla, (Eds.), *International Series on Biomechanics, Volume 5B* (pp. 516-520). Champaign, Ill.: Human Kinetics Publishers.
Noble, L., Dzewaltowski, D. (1994). A Field Test to Determine the Attributes of Aluminum Softball Bats that Influence Perception and Preference: A Pilot Study. Unpublished research report submitted to Kansas State University Research Foundation, Manhattan, KS.
Noble, L., Eck, J. (1985, July). Bat Loading Strategies. In *Biomechanics in Sports III: Proceedings of the 3rd Symposium of the International Society of Biomechanics in Sport*s (pp. 58-71). Greeley, CO.
Noble, L., Eck, J. (1986). Effects of Selected Softball Bat Loading Strategies on Impact Reaction Impulse. *Medicine and Science in Sports and Exercise* **18**, 50-59.
Noble, L., Walker, H. (1993). Effects of Knob end Loading and Barrel Length on Selected Mechanical Characteristics of Aluminum Softball Bats. In *Biomechanics in Sports XI: Proceedings of the 11th Symposium of the International Society of Biomechanics in Sport*s (pp. 210-213). Amherst, MA.
Noble, L., Walker, H. (1994a). Baseball Bat Inertial and Vibrational Characteristics and Discomfort Following Ball-Bat Impacts. *Journal of Applied Biomechanics* **10**, 132-144.
Noble, L., Walker, H. (1994b). Effects of Impact Location on Softball Bat Vibrations and Discomfort. In *Biomechanics in Sports XII: Proceedings of the 12th Symposium of the International Society of Biomechanics in Sport*s (pp. 220-223). Siofolk, Hungary.
Plagenhoef, S. (1971). Patterns of Human Motion: A Cinematographic Analysis. Englewood Cliffs, N.J.: Prentice-Hall.
Shapiro, R. (1974). Three-Dimensional Kinetic Analysis of the Baseball Swing. Unpublished Doctoral Dissertation. Urbana-Champaign, Ill.: University of Illinois.
Spragg, C. (1986). *A Comparison of Selected Mechanical Factors in Male Baseball and Female Fastpitch Softball Batting.* Unpublished Master's Thesis. Manhattan: Kansas State University.
Thomson, W. T. (1993) Theory of Vibrations with Applications. Englewood Cliffs, N.J.: Prentice Hall.
Van Zandt, L. L. (1991). The Dynamical Theory of the Baseball Bat. Unpublished Manuscript from Department of Physics Colloquium Presentation, February 10, W. Lafayette, IN: Purdue University.
Weyrich, A. S., Messier, S. P., Ruhmann, B. S., Berry, M. J. (1989). Effects of Bat Composition, Grip Firmness, and Impact Location on Postimpact Ball Velocity. *Medicine and Science in Sports and Exercise* **21**, 199-205.