The Long Jump and Triple Jump are the two horizontal jumping events of Track and Field. They have in common 1) the primary goal of maximizing the horizontal distance jumped; 2) a sprint-like approach on a runway (often the same one) to a take-off marker; 3) an attempt to achieve a desired flight phase trajectory; and 4) similar training techniques used by the athletes. With the exception of these few common factors the events are considerably different and, as one would expect, the principal difference lies in the fact that the triple jump consists of three successive jumps from alternate legs whereas the long jump consists of a single jump from a single leg. Due to this difference, the execution of the triple jump is considerably more complex than the execution of the long jump, and for the same reason, the biomechanical analysis of the respective jumps is also more complex for the triple jump. This paper briefly summarizes some biomechanical analysis techniques that can be used to study the Long and Triple Jumps. Generally the techniques have been applied to study the various phases of the long jump and it is clear how the techniques will apply to the triple jump even though very few biomechanical analyses of the triple jump are reported in the literature. Also presented are preliminary experimental data concerning the triple jump and a procedure to synthesize the jump at an elementary mechanics level.

GENERAL ORDER OF ANALYSIS

A biomechanical analysis of the long and triple jumps often occurs in the following order:

(A) Observe the activity and collect qualitative displacement data by studying films or video sequences.

(b) Collect quantitative displacement data from film or video sequences and possibly calculate velocities, accelerations and mass center displacement.

(C) Use an elementary mathematical model of the jumper to calculate the effects of various initial velocities on the resulting mass center trajectory during the flight phase(s).

(D) Collect force plate data that can be used by itself or in conjunction with (A), (b), and (C).

(E) Develop and use more complex mathematical models to simulate the activity or to determine parameters not directly measurable (i.e., joint forces and moments, angular momenta, etc.).
Item (A) above merely means that one observes the event to obtain an understanding of how the jump takes place. Sometimes this observation is embodied in (B) when appropriate cine or video data collection methods are used. Item (C) is described in texts in kinesiology and biomechanics and represents an elementary application of mechanics to study the jump. Items (D) and (E) require advanced electronic experimental equipment and/or advanced applications of mechanics principles.

Applications in the Long Jump

It is assumed that the reader is familiar with the basic idea of how long jumping takes place and an effort will not be made here to describe the many details and variations of the approach, take-off, and landing. For such a discussion please see Bush, 1978; Doherty, 1976; Dyson, 1977; Ecker, 1976, and Wilt, 1972. The general discussion here will focus on the support and flight phases.

Observation of the jump shows that it consists of two main parts -- (1) the Support Phase where the athlete develops the vertical component of the take-off velocity and (2) the Flight Phase where the athlete attempts to orient the body segments properly to produce an acceptable landing. Figure 1 shows a sketch of these main parts of the jump, using the trajectory of the jumper's mass center as the designator of the motion. In the figure, the distance \( l_1 \), spanning locations of the mass center at \( A \) and \( B \), represents the support phase. The distance \( l_2 \), spanning locations of the mass center from \( B \) to \( C \), represents the flight phase. A mechanics analysis consists of a study of these two phases. However, the actual recording of long jump performance is not based on a measurement of the distance traveled by the mass center, but by the distance measured from a designated "scratch" line to the body part (usually the feet) touching the landing area closest to the "scratch" line. Thus, as suggested by Hay (1978), the actual measured performance consists of three segments, \( a_1 \), \( a_2 \), and \( L_1 \), as shown in Fig. 1. Here \( a_1 \) is the distance from the "scratch" line to the location of the mass center at take-off (location \( B \)); \( L_1 \) is the horizontal distance traveled by the mass center during the flight phase (\( B \) to \( C \)); and \( a_2 \) is the distance from the location of the mass center at landing (location \( C \)) to the position of the feet. Thus, the measured distance \( M_1 \) is given as

\[
M_1 = a_1 + a_2 + L_1 \quad (1)
\]

An estimation of \( a_1 \) and \( a_2 \) can be obtained conveniently from measurements taken of film records of the athlete's jumps and, compared to the magnitude of \( L_1 \), they are quite small.

The distance \( L_1 \), representing the dominant part of the jump, lends itself to an elementary mechanics analysis by way of the ballistics equations (Dyson, 1978; Ramey, 1976). With reference to Fig. 1, let \( v_x \) and \( v_y \) be the horizontal and vertical velocities respectively of the jumper at take-off. Then the ballistics analysis shows that

\[
L_1 = \frac{v_x(v_y + \sqrt{v_y^2 + 2gy_0})}{g} \quad (2)
\]

Here \( g \) is the local gravitational constant and \( y_0 \) is the difference in elevation between the mass center at take-off and landing.
FIGURE 1
Long Jump
Equation 2 illustrates the relationship that exists between the take-off velocities, and the distance the mass center moves. Dyson (1978) uses this equation to show an upper bound of expected performance for long jumps in the range of \( L = 37.5 \) ft (the current world record is 29'-2")

Ramey (1972) uses this equation in a differentiated form to illustrate the influence that changes in these velocities have on the changes in \( L \).

The ballistics approach discussed above is an important first step in the analysis of the jump. It is seen to combine the first three elements of the analysis process mentioned previously. Nevertheless, because of the significance of the take-off velocities, as illustrated in Eq. 2, it becomes important to study how the velocities are acquired. This study requires that one consider the forces associated with support phase, since virtually all the vertical velocity is developed during this phase and a portion of the horizontal velocity acquired during the approach on the runway is lost. A force platform is usually used to record the support phase force histories.

There has been considerable work done on the development of force platforms that can be used in jumping studies (Ramey, 1975; International Society of Biomechanics, 1975-). A typical record is shown in Fig. 2. These force records can be used to study the changes in velocity during the support phase by using the impulse-momentum equations of mechanics.

It is shown by Ramey (1973) that the impulse-momentum equation yields the following relationship for the change in velocity during the support phase

\[
\Delta \mathbf{V} = \int_{t_a}^{t_b} \mathbf{F} \, dt \\
\Delta V = \frac{\int_{t_a}^{t_b} F \, dt}{m}
\]

(3)

where \( \Delta \mathbf{V} \) is the change in the velocity vector during the support phase, \( \mathbf{F} \) is the resultant force vector acting on the athlete during the support phase (obtained from the force platform), \( m \) is the mass of the athlete, \( dt \) is a differential time segment, and \( t_a \) and \( t_b \) are the times at the beginning and end of the support phase.

Define \( \Delta \mathbf{V} = \mathbf{v} - \mathbf{v}' \), where \( \mathbf{v} \) is the velocity vector at take-off and \( \mathbf{v}' \) is the velocity vector at the beginning of the support phase. Then upon substitution for \( \Delta \mathbf{V} \) in Eq. 3 one determines the take-off velocity vector as

\[
\mathbf{v} = \frac{\int_{t_a}^{t_b} \mathbf{F} \, dt}{m} + \mathbf{v}'
\]

(4)

Equation 4 shows how the take-off velocity is related to the force history and that a knowledge of the initial velocity \( \mathbf{v}' \) must be known. One can approximate \( \mathbf{v}' \) by measuring its horizontal and vertical components, \( v'_x \) and \( v'_y \). That is, \( v'_x \) can be obtained from measurements taken as the jumper passes through a timing gate placed immediately before the take-off area or, alternatively it can be approximated by
FIGURE 2
Typical Force Records for the Long Jump
calculations of the mass center motion just prior to reaching the take-off region. The vertical component, \( v'_y \), generally must be determined using the latter procedure.*

It should be noted that \( v' \) in Eq. 4 can be either positive or negative and thus increase or diminish the effect of the first term on the right side of the equation. Generally for long jumping, the horizontal component of \( v' \) will be positive and the first term on the right side of Eq. 4 will be negative. On the other hand, the vertical component of \( v' \) will usually be negative while the first term on the right side of Eq. 6 will be positive. In actual athletic performances, the slight lowering of the jumper's center of mass observed on the penultimate stride tends to reduce the magnitude of the negatively signed vertical component of \( v' \) and thus not diminish, as greatly, the effect of the impulse represented by the first term on the right of Eq. 4.

The real advantage of the approach embodied in the use of Eq. 4 is that the force history is explicitly displayed. Ramey (1972) suggests how Eq. 4 can be manipulated to quantitatively assess the influence of changes in the take-off velocities as determined by changes in the forces during the support phase.

It is worth noting that although the force plate records provide useful information, the use of this data in further calculations requires them to be combined with displacement data from a cine analysis. This corresponds to item (D) of the general analysis process described earlier.

To this point, the discussion has centered on the mechanics associated with the motion of the jumper's mass center and the forces associated with the support phase. However, another part of the biomechanics analysis concerns itself with the effects produced by movements of the limbs during the flight phase. It is well known that during the flight phase of the jump, the athlete positions the limbs to produce a desired reorientation of the whole body and to prepare for a suitable landing. The mathematical description of this reorientation phenomenon follows the conservation of angular momentum principal of mechanics and has been used in several studies. For this type of analysis the human body is modelled as a collection of hinge and ball and socket connected rigid bodies. Figure 3 is an example used in a three dimensional analysis of the flight phase of the long jump (Ramey, 1981). The conservation of angular momentum equation is summarized in Eq. 5.

\[
H_o = \sum_{i=1}^{n} (I_i \cdot \omega_i + r_i \times m_i \times r_i)
\]

Here \( H_o \) is the angular momentum vector taken about the system mass center and remains constant during the flight phase; \( n \) is the number of body segments used in the model; \( I_i, \omega_i, r_i, m_i, \) and \( r_i \) are the inertia dyadic, angular velocity vector, position vector with respect to an inertial reference frame, mass, and linear velocity vector respectively of body segment \( i \).

Equation 5 can be used to determine the angular momentum \( H_o \) associated with various long jumping techniques (sail, hitchkick, hang, somersault, etc.) by measuring the quantities on the right side and performing the requisite calculation. The measurements are based on data reduced from a cine analysis of the jump(s) and

*When one is studying standing long jumps, Eq. 4 is simplified by the fact that \( v' = 0 \).
FIGURE 3

A Model Used in a 3-D Simulation
FIGURE 4

TRIPLE JUMP
body segment mass distribution either assumed or determined approximately in situ. Hay (1977) and Bedi (1977) describe some of these methods.

The application of Eq. 5 in the analysis is clearly no longer an elementary process. The equation must be expanded using the advanced concepts of analytical dynamics, which is usually the case when the body is modeled as a collection of rigid bodies. Equation 5 has been used to produce a simulation of the flight phase of the long jump, including the somersault long jump (Ramey, 1981). Hatze (1981) takes the concepts of analytical mechanics further and provides a simulation of the support phase of the long jump. This latter application is considerably more complex than any of the others.

The two simulation procedures just mentioned indicate the considerable complexity associated with modeling the long jump. Such approaches will eventually lead to a better understanding of the event, however, at this writing much must yet be done.

**Triple Jump**

Compared to the work done on the biomechanical analysis of the long jump, virtually nothing of that nature has been done for the triple jump. Most of the work falls in category A of the general approach described earlier and is typically illustrated by Bush, 1978; Doherty, 1976; Dyson, 1977; Ecker, 1976, and Wilt, 1972.

As one attempts to proceed with the mechanics analysis of the triple jump, as has been done for the long jump, the similarities and differences between the two jumps become clear. Figure 4 shows a sketch of the support and flight phases of the triple jump (this figure is the counterpart to Fig. 1 for the long jump). The obvious difference is that there are three support and flight phases in the triple jump compared to just one support and flight phase in the long jump. However each support-flight phase pair of the triple jump essentially resembles the support-flight phase pair of the long jump. The resemblance just noted is unfortunate because many novice coaches and jumpers attempt to extrapolate the knowledge and experience from the long jump to the three phases of the triple jump. Such an extrapolation does not work. In doing a biomechanical analysis one can take advantage of the resemblance, but care must be exercised to account for the differences. In the following paragraphs we shall discuss, in some detail, how one could proceed with the ballistics and impulse momentum type analysis in the triple jump as was described for the long jump. The complexity will become clear.

Refer to Fig. 4. Here observe that the measured distance, \( M_T \), of a triple jump is given as

\[
M_T = a_1 + a_2 + s_2 + s_3 + \sum_{i=1}^{3} L_i
\]

where \( a_1 \) is the distance from the "scratch" line to the center of mass at take-off; \( a_2 \) is the distance from the center of mass to the location of the body segment touching the landing region closest to the "scratch" line; \( s_2 \) and \( s_3 \) are the distances traveled by the mass center during Support Phases 2 and 3; and, \( L_i \) (i=1,2,3) is the distance traveled by the mass center during Flight Phase i. Thus we note that \( a_1 \) and \( a_2 \) in the triple jump analysis correspond to \( a_1 \) and \( a_2 \) in the long jump analysis; however, there is no counterpart to \( s_2 \) and \( s_3 \) in the long jump analysis.
One may proceed to use the ballistics and impulse-momentum equations to calculate the distance jumped just as was done in the long jump. First determine the distance travelled during the flight phases, $L_i$, using the ballistics equation. The generalization of Eq. 2 yields

$$L_i = \frac{(v_x)_i}{(v_y)_i + \sqrt{(v_y)_i^2 + 2gy_i}}$$

Here $(v_x)_i$ and $(v_y)_i$ denote the horizontal and vertical components of the mass center take-off velocity at the beginning of Flight Phase $i$ and $y_i$ denotes the difference in elevation of the mass center between the beginning and end of Flight Phase $i$.

As described in the long jump discussion, the take-off velocities are functions of the initial velocity at the beginning of the support phase and the forces developed during the support phase. Thus, using the impulse-momentum equations in component form expressions for the velocities are obtained,

$$\begin{align*}
(v_x)_i &= \int \frac{F_x}{m} dt + (v_x')_i \\
(v_y)_i &= \int \frac{F_y}{m} dt + (v_y')_i
\end{align*}$$

Here $(v_x')_i$ and $(v_y')_i$ are the horizontal and vertical components of the mass center velocity at the beginning of Support Phase $i$ and $(F_x)$ and $(F_y)$ are the time varying horizontal and vertical components of the force vector acting during the support phase. The integration indicated in Eq. 8 is to be taken from the time of the beginning of the particular support phase to its end.

Substitution of Eq. 8 into Eq. 7 shows the interaction of the variables for a particular flight phase. That is,

$$L_i = \frac{\left[ \int \frac{F_x}{m} dt + (v_x')_i \right] \left[ \left( \int \frac{F_y}{m} dt + (v_y')_i \right) \sqrt{\left( \int \frac{F_y}{m} dt + (v_y')_i \right) + 2gy_i} \right]}{g}$$

It should be noted in Eq. 9 that several terms are functions of events that happen during previous flight and support phases. First consider the term $(v_x')_i (i=1,2,3)$. For the first flight phase, $(v_x')_1$ is determined from cine analysis at a timing gate as mentioned in the long jump discussion. For Flight Phases 2 and 3 however, the horizontal velocity at the beginning of the support phase is equal to the horizontal velocity at the end of the previous flight phase which in turn is equal to the horizontal velocity at take-off of the previous flight phase (since air resistance is neglected). Thus,

$$\begin{align*}
(v_x')_i &= (v_x')_i-1
\end{align*}$$

Next consider the vertical component $(v_y')_i (i=1,2,3)$. For the first phase $(v_y')_1$ is obtained from cine analysis as described in the long jump discussion. For the other
FIGURE 5
Triple Jump Experiment with One Force Platform
phases, \((v_y')_i\) \((i=2,3)\) is the vertical velocity at the end of the previous flight phase. Then on using the ballistics equation in this instance one finds

\[
(v_y')_i = -\sqrt{(v_y')_{i-1}^2 + 2gy_{i-1}} \quad (i=2,3)
\]

If Eq. 8 is substituted into Eqs. 10 and 11, and the resulting relationships are then substituted into Eq. 9, one can determine how \(L_i\) of the second and third flight phases depend on the forces and velocities of the previous and current support and flight phases (not shown here).

Before one can complete the calculation of the measured distance, \(M\) in Eq. 6, expressions for the horizontal distances travelled during the last two support phases, \(s_k\) and \(s_k\) must be determined. For simplicity determine \(s_k\) \((k=2,3)\) as the product of the average horizontal velocity during the support phase and the time duration of the support phase, \(\delta_k\). Thus,

\[
s_k = [(v_x')_k + (v_x')_k] \frac{\delta_k}{2} \quad (k=2,3)
\]

Substitution of Eq. 8 into Eq. 12 yields

\[
s_k = [(v_x')_k + \frac{\int v_x'}{2m} dt] \delta_k \quad (k=2,3)
\]

Here again one observes that due to the first term on the right side of Eq. 13 the distance travelled during the \(k\)-th support phase will be a function of events that occurred in the previous phases.

The preceding discussion concerning the expansion of Eq. 6 to calculate the measured distance shows that a rather complicated expression results. Additionally it is seen that the expression requires a knowledge of the force histories occurring during each support phase. To the author's knowledge, such force records have not been described in the triple jumping literature. The fact that the force histories are not cataloged is not surprising when one recognizes that three force platforms would be needed to obtain the data. Additionally, the force platforms would have to be arranged to suit the required flight phases of the test subject. This then requires mobile placements of the force platforms if several subjects are being used who have different jumping abilities.

In an effort to obtain a measure of the forces associated with the triple jump the author conducted some preliminary experiments using a single force platform. In this instance the force platform was set flush with an approach runway and the subject, an experienced collegiate triple jumper, was directed to take three jumps from the force platform. First the subject was required to make a normal approach and execute the first support phase from the force platform. The first flight phase associated with this jump was to be done in the usual fashion which required that the landing area be modified to accept a vigorous one-legged landing without causing injury to the subject. Next the subject was directed to make a normal approach but to execute the first support and flight phases on the runway. These first phases were to be adjusted such that the end of the first flight phase would occur on the force platform. Then in the usual fashion the subject was to complete the second support phase on the force platform. Once again the second flight phase would be continued in the normal fashion. The previous modification to the landing area was
FIGURE 6
EXPERIMENTAL TRIPLE JUMP FORCE - TIME RECORDS
kept in place so the subject could function as much as normal and yet not get injured. Finally, the subject was directed to initiate the first and second support and flight phases on the runway. These were to be adjusted such that the end of the second flight phase would occur on the force platform. The subject would then continue in the normal fashion to execute the third support phase on the force platform. The third flight phase in this case was executed in the usual fashion with a landing occurring in a standard pit. Figure 5 illustrates the procedure just described.

It is clear that the method described for obtaining the force records is merely an approximation of an actual jump since the three phases were not part of a single continuous jump. Nevertheless it is about all one can do with only one force platform. In these experiments the subject was able to maintain flight phases of approximately 14 feet each.

Figure 6 shows the vertical and horizontal force histories for the three support phases of the experiment. It is seen that the vertical forces have very similar shapes and only differ in magnitude and duration. Additionally these records are not substantially different from those one would observe for the support phase of the long jump. For the vertical components, the force record oscillates about a mean value. The oscillation is attributed to the excitation of the natural frequency of the horizontal transducers of the force platform and is not truly a measure of the force. Stiffer transducers are required in the force platform in order to provide a more realistic view of the horizontal force. Nevertheless, a measure of the time variation of the horizontal force is shown by the dotted line in the figures. The dotted line merely approximates the trend indicated by the actual signal. It is seen that just as with the vertical component of the force, the horizontal components are quite similar and do not differ greatly from those observed in the long jump. Naturally their magnitude and duration vary a little but this is as expected since each phase is taught to be executed differently.

If the force records were not of such an approximate nature further analysis could be done using some of the ideas illustrated in the previous discussion. Additionally one could proceed further to develop simulation models as have been done with the long jump. Such additional analysis will await future study.

CONCLUDING REMARKS

The intent of this paper was to summarize some of the biomechanical analysis concepts that are being applied to the long and triple jumps. It is clear that more study has been devoted to the long jump, probably because of its popularity compared to the popularity of the triple jump, because it is not as complex, and because it includes the fundamentals associated with nearly all vigorous jumps. As better analytical and experimental methods become available for studying the long jump it is expected that our knowledge of the biomechanics of the triple jump will be expanded.

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