THE ATTRACTOR METHOD – A TECHNIQUE TO QUANTIFY DIFFERENCES OF CYCLIC PROCESSES AND THEIR VARIABILITY

Manfred M. Vieten¹ and Randall L. Jensen²
Sports Science, University of Konstanz, Konstanz, Germany¹
School of Health & Human Performance, Northern Michigan University, Marquette, Michigan, United States of America²

The purpose of this study is to introduce a new, very sensitive method to quantify differences of movement pattern and the variability of cyclic processes. While the method is applicable to practically all cyclic processes, we restrict ourselves to an example of walking under various conditions.

KEY WORDS: cyclic motion, chaos theory, quantifying movement pattern

INTRODUCTION: Typical cyclic processes in sports biomechanics are running, bicycling, Nordic skiing etc. However, repeated movements such as a golf swing or a tennis serve might be treated similar to cyclic movements, if an adequate number of repetitions are available.

Conventional analysis tries to identify characteristic pattern from a few cycles. In human locomotion analysis of parameters such as stride length, frequency, sway, variation of the center of mass etc. are very common. In this way essential information on the dynamics of multiple strides are lost. Describing the dynamical system by its state vector on the other hand would give the full information of a system. For systems with a high degree of freedom – e.g. human locomotion - the dimensionality of the phase space, in which the state vector resides in, normally exceed today’s technologic capabilities. As a consequence, predicting the complete progression of such a complicated system is not feasible. However, a subspace of the phase space often holds enough information for answering interesting questions about a system. We will present an example of “walking on a treadmill” under the three situations “simple walking”, “walking while counting backwards by 3”, and “walking with additional weights at the ankles”. The questions we ask are: “Are the running pattern (attractors) different within the different situations?” “Is the variation in the running style altered by the circumstances?” We present the method and apply it to an example to illustrate the sensitivity of the method.

METHODS: The method is applicable to any data describing cyclic processes. However, for this presentation we use acceleration data from walking on the treadmill to illustrate the approach. The three dimensional acceleration data were measured at a sampling rate of 500 Hz with accelerometers fixed at the lateral sides of the right and the left foot below the ankle joints and filtered with a low pass filter (Vieten 2004) at a cutoff at 4.5 Hz. While walking, these two points are located at the ends of a long kinematic chain, reaching from one foot to the other, and having a high degree of freedom. Consequently, this phase sub-space contains a substantial portion of the information of the movement. Still, our six dimensional acceleration space is far away from being able to completely describe the movement. Therefore, a single point in this six dimensional space does not provide enough information to accurately predict the future configuration of the movement. But, this is not our intention anyway. We show that differences in walking under different conditions, even if very subtle, are quantifiable and therefore highlight changes in the configuration e.g. fatigue. We calculate the sub-attractors - the projection of the full attractor onto the six-dimensional subspace - of the movement and the deviations $D$ of the actual tracks from this attractor.
Methodology

\[ D_{aC}(t_j) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( \overline{A}_{aC}(t_j) - \overline{a}_{aC}(i \cdot \tau_j) \right)^2} \]  

(1.1)

Here is

\[ \overline{A}_{aC}(t_j) = \frac{1}{n} \sum_{i=1}^{n} \overline{a}_{aC}(i \cdot \tau_j) + \frac{1}{n} \sum_{i=1}^{n} \overline{b}_{aC}(t = i \cdot \tau_j) \text{ with } \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \overline{b}(i \cdot \tau_j) = 0 \]  

(1.2)

with \( \overline{a}_{aC} \) the acceleration with indices \( a = \) right or left foot and \( C = \) measured at the beginning or the end of the intervention or at different dates. By calculating the normalized distance between two attractors we establish the measure for the change of the movement (index \( x_i \) indicates the x, y, z directions of the acceleration).

\[ M = \frac{1}{v} \sqrt{\sum_{i=1}^{3} \left[ \left( A_{r,B,x_i} - A_{r,E,x_i} \right)^2 + \left( A_{l,B,x_i} - A_{l,E,x_i} \right)^2 \right]} \]  

(1.3)

A second parameter is a measure of the change in variability.

\[ D = \sqrt{\left( D_{r,B} - D_{r,E} \right)^2 + \left( D_{l,B} - D_{l,E} \right)^2} \]  

(1.4)

The brackets <...> stand for the average of the included expression. For this presentation we concentrate on our walking example and discuss the whole procedure using one subject walking at a speed of 1.39 m/s. The full results of 30 subjects as well as the detailed mathematics of these procedures can be found in (Vieten, Sehle, Jensen 2013). All calculations were done using StatFree 8.1 (Vieten 2006), which is freely available on the Internet.

RESULTS: The following graph shows the acceleration of the left foot in green. The red line symbolizes the attractor in the three dimensional subspace, where the gap indicates where the cutting of the traces took place.

Figure 1: Cyclic acceleration data and its attractor (left foot)
The calculations of the attractors are based on about 31-thousand 3D acceleration data points. This gives a very high statistical power and makes it possible to see subtle differences in the movement pattern.

![Figure 2: Comparison of two Attractors of one person](image)

- **a)** simple walking at two different days - \( D_1 = 1.3 \), \( D_2 = 1.3 \), \( M = 4.3 \), \( D = 0.2 \);
- **b)** simple walking (red) compared to walking while counting backwards by 3 (blue) - \( D_1 = 1.3 \), \( D_2 = 1.3 \), \( M = 5.1 \), \( D = 0.3 \);
- **c)** simple walking (red) compared to walking with additional weights of 2 kg at each ankle (blue) - \( D_1 = 1.3 \), \( D_2 = 1.4 \), \( M = 8.3 \), \( D = 0.3 \);

The approximation of the attractor is very precise. Figure 3 shows the approximated attractor and its 5-sigma error bars. This means the probability of having the real attractor outside the error bars is two in one million!

![Figure 3: 2D view of an attractor with a 5 sigma error margin](image)

**DISCUSSION:** There exist other well-established non-linear methods. The Lyapunov exponent and the Floquet multiplier are applicable for classical deterministic systems to analyze their stability. Those methods function well for non-noisy data and give deep insight into the working of dynamical systems, an ability that our method is lacking. However, their application using data obtained from experiments is not without problems. To be able to handle more or less noisy data, algorithms have been developed (for a comprehensive overview see Williams 1997), but the reliability of these algorithms is not assured (Bruijn et al. 2012). Our method on the other hand is specifically developed to cope with real live, noisy
data. The main outputs of our method are the two relative parameters $M$ - the velocity normalized mean distance between limit cycle attractors $- \ D$ - the difference in variability between the two movements, and the absolute variation $D$ - the deviation of the actual movement compared to its attractor. The method is easy to apply, gives stable results and is very sensitive. The method allows the comparison of cyclic movement in two different situations or at two different times. The absolute measure $D$ allows comparisons between different subjects. Using three-axial accelerometers at a sampling frequency of 500 Hz or higher gives a twofold advantage. First, the errors of the measures are very small because of the strong statistical outcome. Second, accelerometers are small, easy to handle, and inexpensive.

**CONCLUSION:** The above-described method constitutes a very sensitive approach to quantify differences of cyclic movements. It is possible to rate differences in movement pattern for individual subjects. We have used this method for analyzing walking (Vieten, Sehle, Jensen, 2013) to diagnose fatigue in multiples sclerosis and stroke patients (Sehle et al. 2014a & 2014b). We are in the process of evaluating movement pattern in triathlon, golfing and for patients before and after a physiotherapeutic treatment.

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