

**THE GEOFFREY DYSON LECTURE 2007**  
**ROCK AND ROLL RHYTHMS IN SWIMMING**

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**ABSTRACT:** This paper is based on the ISBS 2007 lecture honouring Geoffrey Dyson, a pioneer in putting sports science into practice. Although the term 'rhythm' is frequently used by coaches, commentators, and participants in sports, it is rarely quantified. Using examples from analysis of butterfly and front crawl swimming the lecture described how rhythms may be quantified using Fourier analysis and presented 'typical' results from past and current research. In combination with kinematic and kinetic data analysis of rhythms can provide an insight into what constitutes 'good rhythm' in specific sports skills. Importantly, there is potential to use knowledge of rhythms and their development to improve coaching practice and sports performance.

**KEY WORDS:** butterfly, Dyson, Fourier, front crawl, freestyle, rhythms, swimming.

**INTRODUCTION:** Athletes and coaches frequently use the term 'rhythm'. Players 'get into a rhythm' when they are playing well. However, 'rhythm' is rarely defined or treated in sports biomechanics texts. It is interesting that the word 'rhythm' does appear in Geoffrey Dyson's landmark text 'The Mechanics of Athletics' .

'...the best method of clearing a hurdle or water jump is that which returns the athlete quickly to the track with a rhythm and effort akin to a running action' (Dyson, 1977, pp. 151)

But what do we mean by 'rhythm' in sports skills and how do we know when the rhythm is good? 'Good rhythm' may connote timing between joints or body segments appropriate to achieving the desired outcome, the existence of regular oscillations and wavelike motions, smoothness or a lack of 'jerkiness', and efficiency in achieving the desired outcome.

While participants, coaches and commentators may recognise rhythms as being appropriate or inappropriate via qualitative assessments, to investigate rhythm scientifically and objectively a method of measuring the rhythms in human motion is required. If rhythms are regarded as being represented by sinusoidal waveforms, an assumption that fits well with dynamical systems theories of movement control incorporating limit cycle oscillators (e.g. Kelso, 1995), then Fourier analysis is a powerful tool. Fourier analysis represents any time series data, for example a displacement/time signal, as a series of waveforms. Each waveform (harmonic) is an integer multiple of the fundamental frequency. For example, if the fundamental frequency is 1hz, then harmonics occur at 2hz, 3hz.... Fourier analysis determines the amplitude of each of those contributing waveforms and the 'phase', that is, the timing of when the peaks of the wave are attained.

The aim of this paper is to provide an overview of the characteristics of the rhythms in butterfly and front crawl swimming based on research that applies Fourier analysis in conjunction with analysis of kinematics and kinetics.

**METHODS:** In addition to Fourier analysis, other advanced analysis methods are required to address the role of rhythms and body waves and to be confident about the results. First,

motion of the body must be quantified in three dimensions. This is particularly the case for front crawl swimming in which the stroke is asymmetrical and comprised of rotations about several axes with continuously changing orientation. Second, because net forces acting on the swimmer's whole body cannot be determined directly, and no single point adequately reflects the swimmer's whole body centre of mass (WBCM), net forces must be derived indirectly from WBCM. Third, the segmental and whole body energy and the angular motion must be quantified. The net torques acting on the body are then derived from the whole body angular momentum.

At the Centre for Aquatics Research and Education (CARE) four underwater cameras and two above water cameras are zoomed to a calibrated space (figure 1). The coordinates of the markers of a triangulated calibration frame have been established accurately using surveying techniques. With the cameras being in the water rather than looking through windows, we obtain very good accuracy and reliability as reported by Psycharakis et al. (1995).

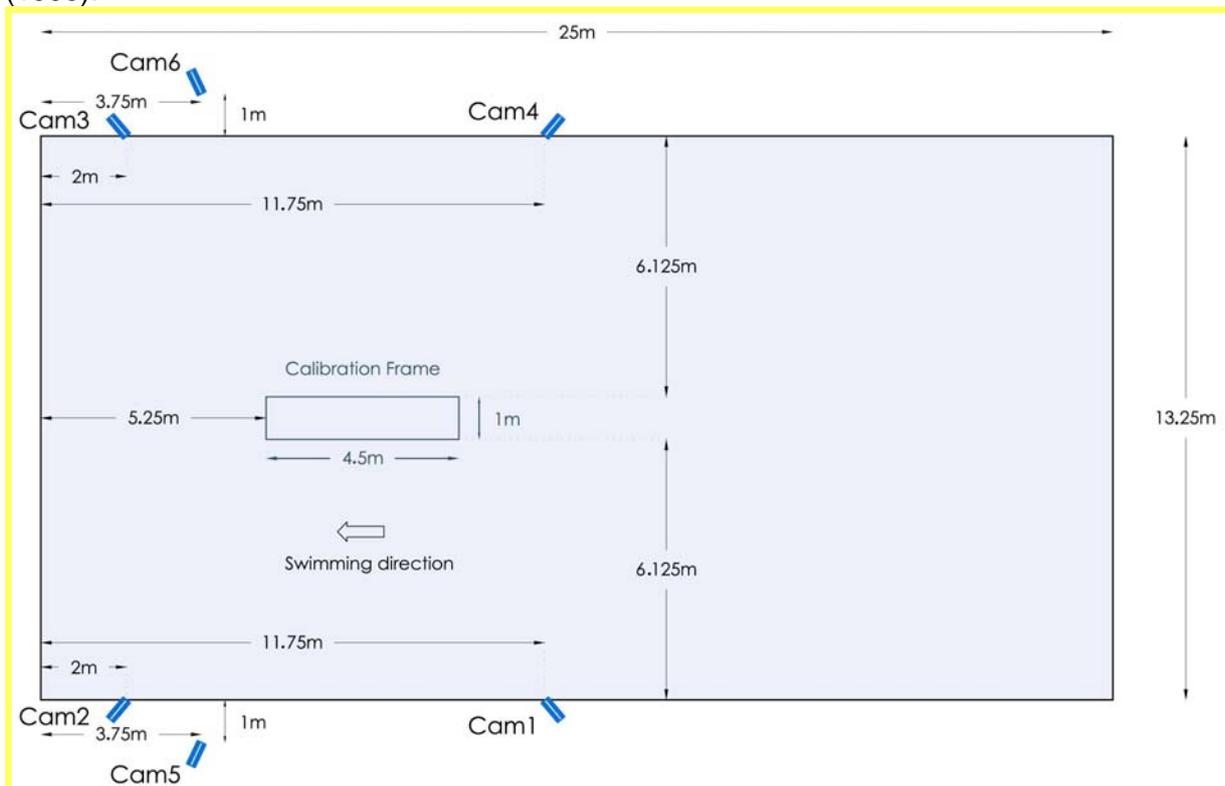


Figure 1: Positions of cameras and calibrated space for 3D data collection.

To maximise the accuracy of the derived kinematics, energies and kinetics, personalised segment masses, moments of inertia and segment centre of mass location data are obtained using the elliptical zone method of Jensen (1978). Digital photographs of individual swimmers with landmarks defining the segment endpoints are taken simultaneously from front and side views. A MATLAB program (Deffeyes and Sanders, 2005) is used to trace the outlines of each body segment. Each segment is then modelled as a series of ellipses from which the mass and its distribution are determined.

Local and transfer terms of segment angular momentum are determined by Dapena's method. These are then expressed with respect to axes of the external reference frame. Translational and rotational kinetic energies of segments are determined. In addition to the directionless scalar values, segmental energies are expressed 'by virtue' of translational and rotational motion with respect to the axes of the external reference frame. Segmental

translational kinetic energies are also expressed with respect to the external reference frame with its axes translating with the WBCM. This was necessary to investigate how energy, independent of segmental energy possessed by virtue of motion of the whole body, is transmitted within the body by waves.

**RHYTHMS IN BUTTERFLY SWIMMING:** Sanders et al. (1995) established that the motion of elite butterfly swimmers is characterised by sinusoidal vertical undulations with the fundamental frequency and 2nd harmonic accounting for more than 98% of the power of the vertical displacement-time signals for all of head, shoulder, hips, knees, and ankles. The undulations are sequenced so that a body wave with the fundamental frequency (H1) progresses along the entire body at a rate (relative to the swimmer's centre of mass) that is slightly faster than the swimmers speed. Among elite swimmers the rate of progression of H1 is related to swimming speed (males  $r=.88$ ; females  $r=.96$ ). A body wave of twice that frequency (H2) travels from the hip to ankle at a faster rate of approximately 2.5 metres per second. These findings are reinforced by data obtained recently from three skilled female butterfly swimmers. Both studies have indicated that the displacement-time signal is very well represented by the sum of two sinusoidal harmonics H1 and H2. There is very little power in the signal not taken up by those two waveforms.

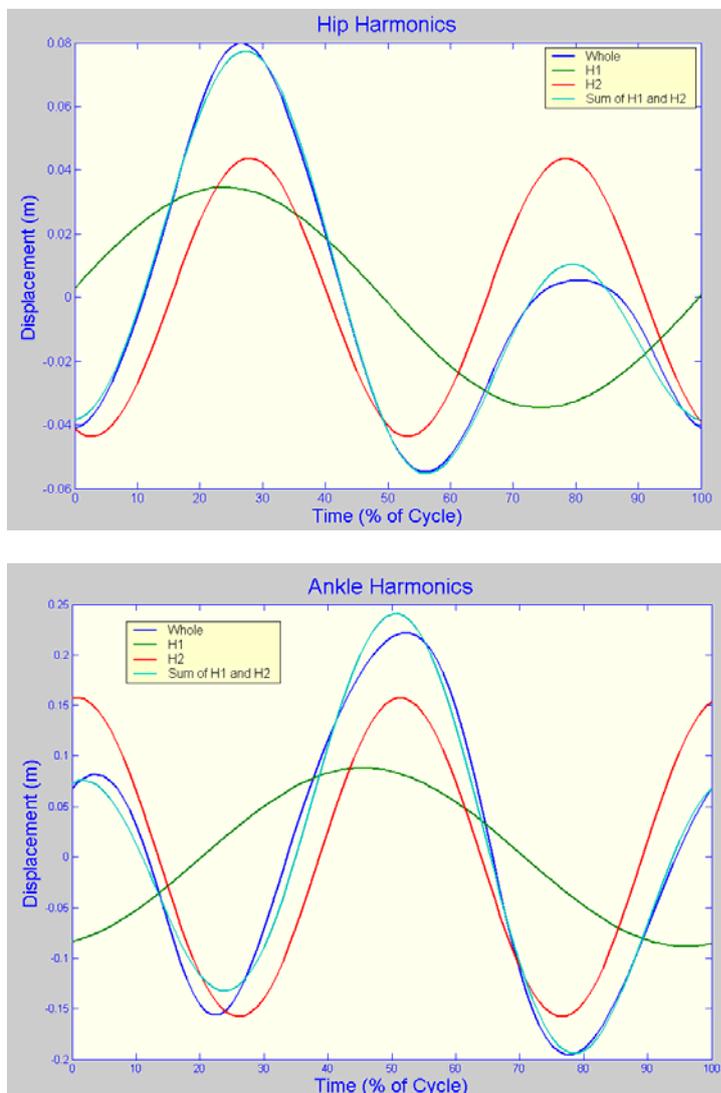


Figure 2: Typical butterfly vertical displacement-time signals, H1, H2, and sum of H1 and H2 waveforms for hip (upper) and ankle (lower).

The different frequencies and the phase relationship between them enables each body part to have a period of large amplitude of vertical motion and a period of smaller amplitude of vertical motion because the signal is reinforced at one part of the cycle and attenuated at another. This yields a strong beat and a weaker beat (figure 2). The sequencing of the peaks of the waves at each body landmark is such that a wave comprised of contributions of H1 and H2 travels along the body. The amplitudes of the composite wave, and therefore the vertical motions of the body parts change progressively along the body.

**How travelling waves might assist performance:** Travelling waves may assist performance in two ways. First, the basic function of waves is to transmit energy from one place to another. In the case of swimming the goal is to transmit motion in the form of mechanical kinetic energy to sites where work can be done on the water to produce propulsion. Second, the mechanism of working on the water in human swimming remains unclear (McCabe and Sanders, 2005). Broadly, propulsion can be achieved by accelerating a mass of water backwards in accordance with Newton's Laws or by creating a pressure difference across a propelling limb. Propulsion from vortices is well established as a mechanism of propulsion in fish (Triantifillou et al., 2002). Propulsive force is generated when the vortex is pushed away, that is, 'shed'. The change in momentum of the mass of water corresponds to a change in momentum of the swimmer in the opposite direction in accordance with the conservation of momentum principle. The vortex is shed near the time of direction change of the end of the oscillating segmental chain. Many swimming scientists believe that propulsion from vortices is a dominant source of propulsion in human swimming. However, its relative contribution remains unknown.

**RESULTS OF THE RECENT RESEARCH IN BUTTERFLY SWIMMING:** Figure 3 shows the energy possessed by virtue of translation in the desired direction for one of the three female swimmers involved in the recent study.

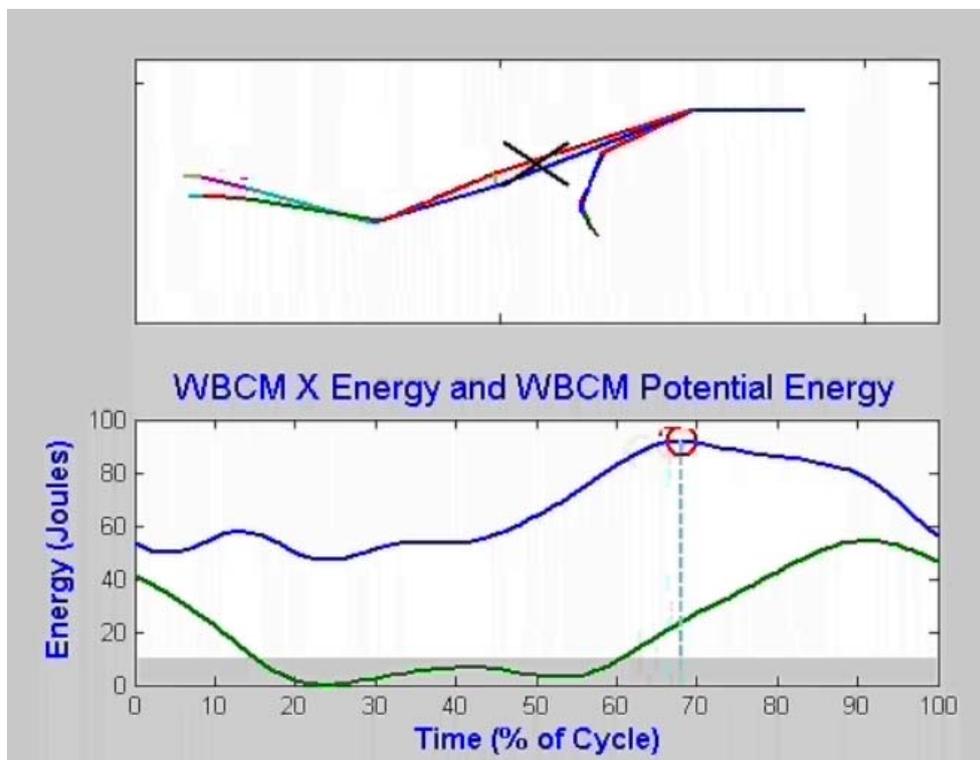


Figure 3: Typical energy possessed by virtue of translation in the desired direction (upper) and gravitational potential energy (lower) in butterfly swimming.

The energy fluctuates considerably from the minimum to the maximum. In fact, half the energy is dissipated to the water due to the action of resistive forces during the cycle. There is a large increase in energy during the arm pull in combination with the downbeat of the second kick. The fact that there is so much fluctuation in the energy of primary interest suggests that this is very uneconomical - like driving a car in the city as opposed to driving at a more constant speed along a flat country road. And yet Michael Phelps' world record in 200m fly stands at 1:52.09. At the same swimming competition, Melbourne in March 2007, he set a world record for 200m front crawl of 1:43:86. This was only 8.23 seconds quicker than the butterfly time. Thus, elite butterfly swimming is only about 8% slower than elite front crawl despite having only one arm stroke per cycle.

Figure 3 also shows there is a large change in potential energy. Raising the WBCM is associated with raising the upper body necessary to facilitate arm clearance and clean entry as well as breathing. This requires additional work to be done by the swimmer and therefore an additional demand on the physiological system to provide energy. But is it possible that the potential energy can be used to aid swimming speed? Perhaps there is a rhythmical exchange between potential energy and useful kinetic energy as in a pendulum or bouncing trampolinist? Unfortunately, rather than high potential energy occurring at times of low kinetic energy, the period of high potential energy is only slightly out of phase with the period of high translational energy in the desired direction.

However, the possibility of transfer of energy via a series of rotations remains. In fact, when the upper body is raised, the centre of buoyancy is well out of alignment with the centre of mass and thereby produces a torque producing rotational energy in the trunk. Could this rotational energy be transmitted by a wave along the body to contribute to a propulsive kick? Evidence for this would be a cephalo-caudal sequencing of energy gains and losses culminating in large energy gains and losses related to propulsive phases of the stroke.

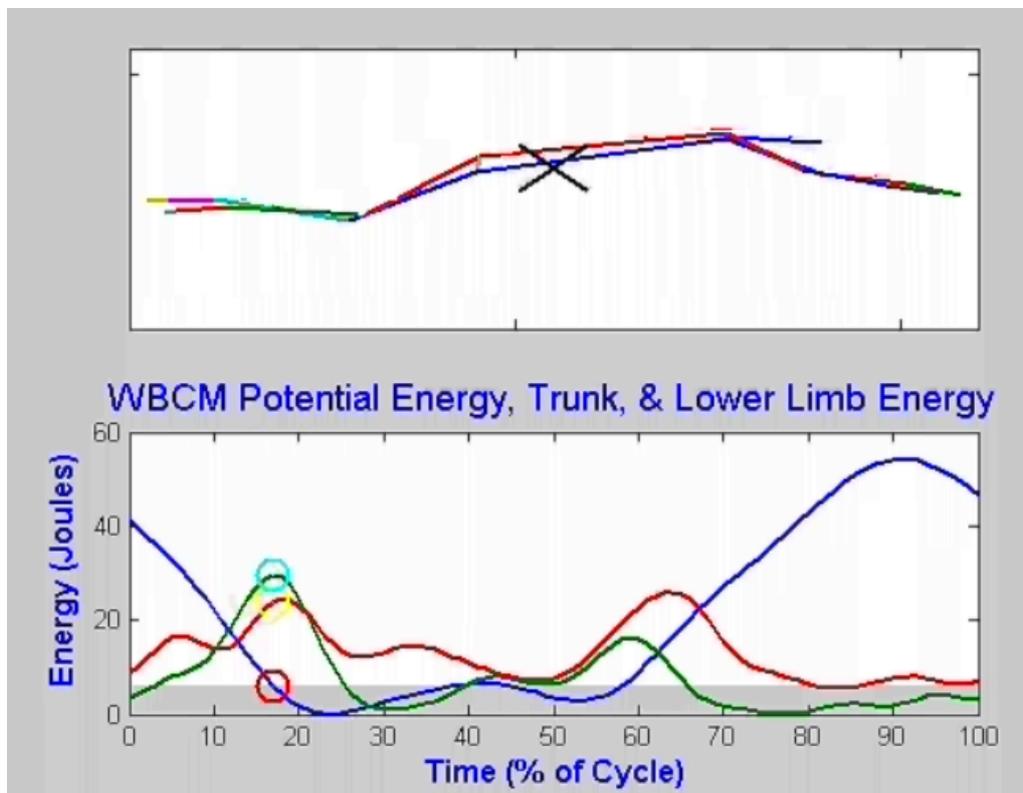


Figure 4: Typical potential (blue), trunk (green), and total lower limb energy (red) in butterfly swimming.

Figure 4 shows potential, trunk, and total lower limb energy. Note that these energies do not include the energy possessed by virtue of whole body translation. Both the trunk and lower limb energies are high together. This doesn't fit well with the idea that the travelling wave transmits energy from the trunk to the lower limbs. However, the timing of energy peaks of the thighs and shanks (figure 5) indicates that there is a sequencing of energy acquisition compatible with the idea of cephalo-caudal energy transmission by a travelling wave.

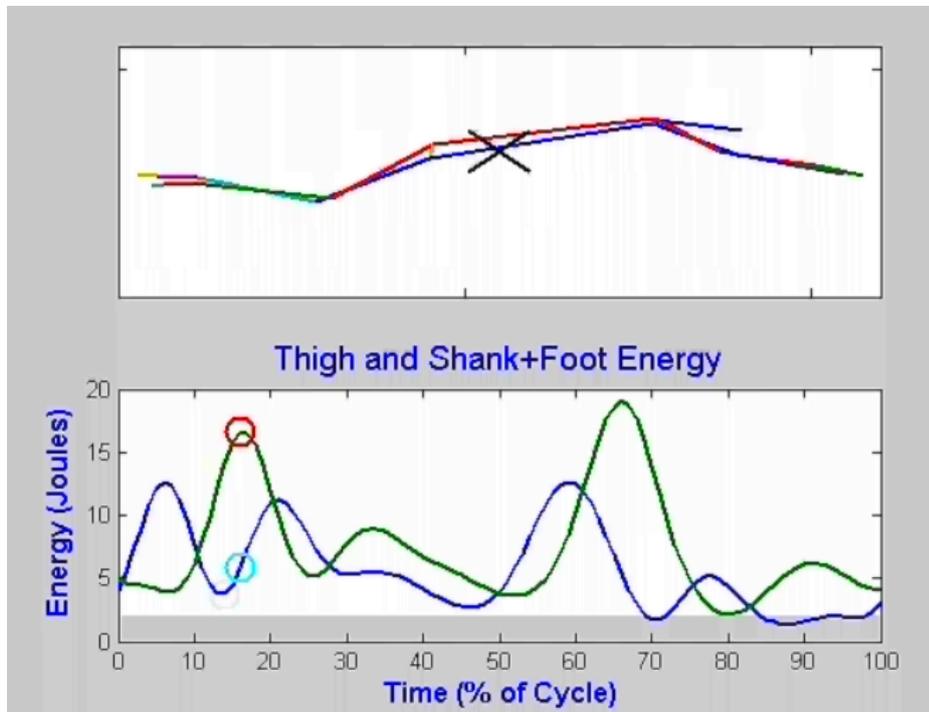


Figure 5: Typical thigh (blue), and shank (green) energy in butterfly swimming.

**Perspectives regarding how travelling waves might assist performance:** The data provide insights from both a hydrodynamic perspective and an energetics perspective. In the hydrodynamics perspective we are interested in how the body is moving relative to the water, that is, the external reference frame. In the energetics perspective we are interested in how the wave might transmit energy by virtue of the motions of the body parts relative to each other. In this case it doesn't matter how the whole body is moving vertically but how the body segments oscillate vertically with respect to a suitable reference point of the internal reference, appropriately the WBCM.

The two analyses yield different results. Typical results for one swimmer are presented in Table 1. When expressed relative to the WBCM (energy perspective) there is less vertical undulation of the shoulders and more vertical undulation of the hips than when expressed relative to the external reference frame (hydrodynamic perspective). The shoulder and hip undulations are almost exactly out of phase (159 degrees) and the wave velocity is slower.

From the hydro perspective the H1 wave from trunk to shoulders to hips of these swimmers was not very different from their swimming speed. Thus, the wave would not be very propulsive nor very resistive. From the energy perspective the out-of-phase oscillations of the hip and ankle together with the in phase energies of the trunk and lower limbs suggest energy transmission from trunk to lower limbs by a pendulum mechanism rather than a wave mechanism. The wave at the hip with respect to the WBCM is substantial and likely to transmit much energy into the lower half of the body. The phase of the H1 contribution is such that it reinforces the H2 wave at one part of the cycle and attenuates it at another.

**Table 1. Comparison of Shoulder and Hip Oscillations from Hydrodynamic and Energy Perspectives (Typical results).**

	Shoulder (cm)	Hip (cm)	Phase Diff. (deg.)	Wave Vel. (m/s)
Hydro Perspective	0.313	0.080	114.3	1.450
Energy Perspective	0.245	0.104	159.3	1.043
p (t-test)	0.007	0.039	0.008	0.007

If the upper body is acting like a pendulum by rotating about some instantaneous axis of rotation then the amplitude of hip undulations, and consequently the magnitude of energy transfer, depends on the position of that axis of rotation relative to the hips. The further the axis from the hips, the greater the hip undulation. Thus we can hypothesise that the pendulum mechanism is more effective in transmitting energy and gives an advantage to swimmers who are 'top heavy'. That is with mass distribution biased towards the upper body rather than the lower body. It has been recognised for many years that strong broad shoulders and slender hips offer an advantage from a hydrodynamic perspective. In butterfly, it may also assist in energy transfer and economy through the pendulum mechanism.

**Propulsion:** Figure 6 shows the whole body acceleration profile and thereby reflects the net forces acting on the typical swimmer. In the first part of the cycle there are large fluctuations in net force associated with the first kick. Recall that the first kick had the smaller amplitude of vertical motion of the feet resulting from the phase relationship of H1 and H2. The arm action combined with the culmination of the strong upbeat produces a high force during the second half of the cycle. The hands contribute to one sustained period of acceleration but not to the three other periods of acceleration. However, the rapid acceleration of the upper segments during exit and recovery may help the feet to shed a vortex due to the inertial reaction effect.

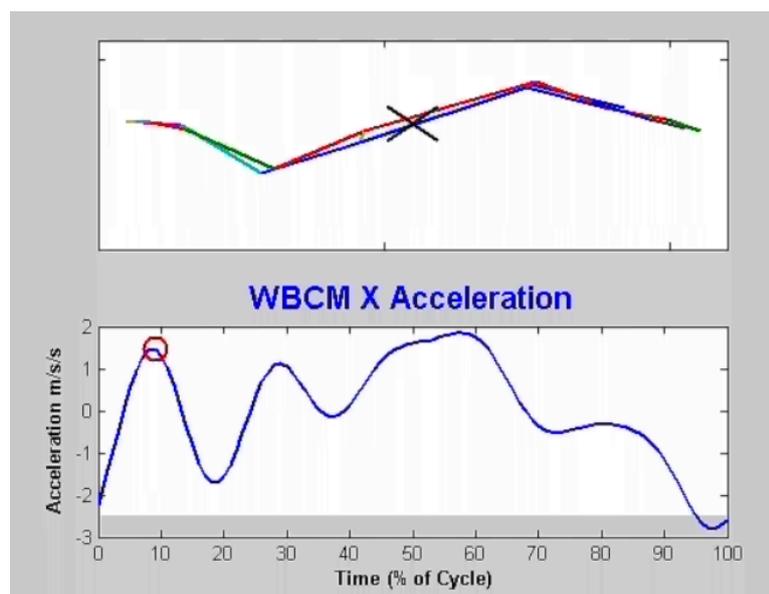


Figure 6: Typical acceleration of the WBCM in the desired direction in butterfly.

The velocity vectors of the centre of the feet indicated that there was no backward component of foot velocity at any stage during the cycle. This was due to the strong effect of whole body motion in the forward direction. In combination with the fact that the peak accelerations did not coincide with periods of fast vertical foot motion this provided indirect evidence for propulsion by shedding of vortices rather than other mechanisms. The greatest accelerations of the body during the pull did occur during a period of rapid backward hand motion. However, because the peak acceleration at this time was also influenced by the kick, it is not possible to attribute a likely propulsion mechanism to this period of propulsion.

**The role of the outswEEP:** The outswEEP of the hands coincided with the reversal of vertical motion of the shoulders from downward to upward. Thus, it appears that the outswEEP plays an important role in maintaining the sinusoidal rhythm. The downbeat of the feet at this time would tend to drive the shoulders too deep if the outswEEP wasn't counteracting that. Once the WBCM is moving upwards the upbeat of the kick combined with the insweep/backswEEP of the hands takes over the role of assisting the trunk to rotate backwards.

**Effect of wave phase on performance:** Recall that the H1 phase was such that as it reached the ankle it attenuated the upbeat of the first kick, reinforced the upbeat of the second kick, and reinforced the downbeat of the second kick (figure 2). Does that offer any advantage to the swimmer? Why not have two kicks of equal magnitude? While kicks of equal magnitude would seem appropriate for gaining propulsion from the kick, the possible effects on rotation of the body must be considered.

Maximum angular momentum in the clockwise direction coincides with the conclusion of the first downbeat and the entry of the shoulders into the water. Maximum angular momentum in the anti-clockwise direction coincides with the middle of the arm pull and the cessation of the strong upbeat between the first and second kicks. The clockwise rotation of the trunk is delayed as much of this angular momentum is taken up by the rotation of the arms.

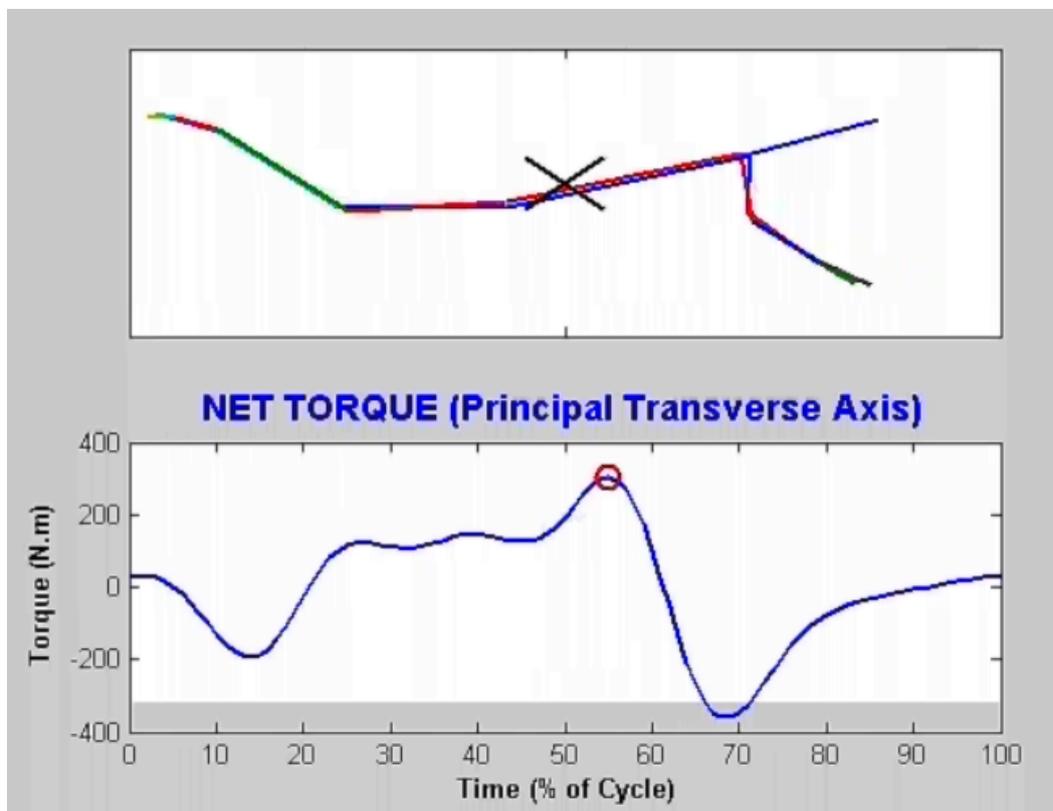


Figure 7: Typical net torque about the transverse principal axis in butterfly swimming.

Figure 7 shows the torques acting about the transverse principal axis. This first period of negative torque is due to the combined effect of the downbeat of the first kick and the buoyancy force which at this time is acting behind the centre of mass due to much of the upper body mass being out of the water.

The upbeat between the first and second kicks combines with the outswEEP of the hands to produce a sustained anticlockwise torque preceding a strong anticlockwise torque associated with the backward motion of the hands. The strong downbeat of the second kick then produces a strong clockwise torque in combination with the torque of the buoyancy force acting behind the WBCM at this time.

Simulation of the composite waveform moving from hip to ankle (H1+H2) in which the phase of H1 was changed relative to the phase of H2 indicated that when the phase is changed either the upbeat between the first and second kick or the downbeat of the second kick are weakened. We would expect this to reduce the torque tending to rotate the body backwards and forwards, thereby reducing the ROM of trunk rotation and the energy put into the system for transfer along the body. Thus, it may be speculated that this disturbance to the rhythm would reduce the economy. The recent studies indicate that the phase of H1 relative to H2 adopted by the swimmers produces a strong upbeat and strong downbeat and adds rotational energy to the trunk which is then transferred to the feet thereby optimising economy.

**RESULTS OF RECENT RESEARCH IN FRONT CRAWL SWIMMING:** From a study of flutter kicking without body roll in swimmers at three levels of a learn to swim program and a group of skilled swimmers (Sanders, 2007) it is known that flutter kicking is characterised by a travelling body wave in the vertical plane aligned with the direction of swimming motion. In beginning swimmers the phase difference between hip and knee vertical undulations is too small, that is, the motions are too simultaneous. As a consequence the wave velocities from hip to knee are large and ineffective in producing propulsion (Table 2). Skilled swimmers have a larger difference in phase between the hip and knee so that the travelling wave is slower than all three levels of the swimmers in the learn to swim program. It is about the same speed, or slightly less than the wave travel from knee to ankle. From a hydrodynamic perspective this is more effective. It was also found that the skilled swimmers have smoother, more regular, and more symmetrical kicks across left and right sides than the learning swimmers.

**Table 2. Body Wave Velocities (Mean and SDs) of the Flutter Kick Obtained in the Sanders (2007) Study of Three Levels of Learners and a Group of Skilled Swimmers.**

	Hip-Knee Velocity	Knee-Ankle Velocity
L1	8.2 (7.1)	2.5 (0.5)
L2	8.3 (3.6)	4.1 (0.5)
L3	7.3 (4.3)	3.8 (0.8)
Skilled	2.8 (0.5)	3.2 (0.3)

To investigate the more complex situation of the whole stroke that incorporates a roll about the long axis 3D data were obtained from seven adult squad level swimmers simulating a 200m race (Sanders, submitted). Given that the downbeats and upbeats of right and left limbs are out of phase we can deal with the kick rhythms as being associated with rolling actions and investigate how 'torsional' waves travel along the body from shoulders to ankles.

The shoulders rolled approximately 50 degrees to each side and the variability was relatively small both within and between swimmers. The hip roll was less than half the shoulder roll and with slightly greater within subject variability than the shoulder roll. The amplitude of roll increased at the knees and reached about 70 degrees to each side in the ankles. The shoulder roll was very rhythmical with approx 98% of the power contained in a single sinusoid corresponding to the frequency of the stroke cycle. It is largely unaffected by the kicking action. The hip roll is also dominated by H1 but there is some of H3, presumably in reaction to the kicking pattern. The roll of the lines joining the knees is predominantly H3 due to the six beat kicking action but also contains considerable power in H1 indicating the influence of the upper body roll. The same applies, but to a lesser extent to the ankles. While the roll is dominated by H1 and H3, the power contained in H2 in the knee and ankle roll is not insignificant. It is due to the effect of the arm action occurring twice, that is two arm pulls, per cycle.

Only the H3 wave is transmitted in a consistent manner. The very high wave velocity for the H1 wave from shoulder to hip reflects that the shoulder and hip roll are almost exactly in phase. In contrast, the much slower wave velocity for H3 from hip to knee and from knee to ankle indicates a sequencing of the lower limb actions such that the H3 wave is transmitted in a cephalo-caudal direction. Small within and between subject variability in wave velocity indicated that this rhythmic pattern is characteristic of skilled front crawl swimmers using a six beat kicking pattern. The moderate H3 wave velocity, accelerating as it moves caudally, is consistent with the effective production of propulsion observed in marine animals.

**The travelling wave from an energetics perspective:** While the transmission of the H3 wave at a moderate velocity is desirable from a hydrodynamic perspective it is also of interest to determine whether the wave may transmit energy to yield advantages from a physiological economy perspective.

Figure 8 shows the energy typical of a skilled butterfly swimmer possessed by virtue of translation in the desired direction and the potential energy by virtue of raising the WBCM. Notice that both forms of energy fluctuate much less than that of the butterfly swimming. The increases in energy of translation occur predominantly during the underwater pull of each arm.

The potential energy increases are almost concurrent with the increase in energy of translation. This is due to the downward push of the other arm as it commences its pull. Thus, there is no evidence of energy exchange between potential energy and kinetic energy of translation that might lead to physiological energy savings.

Figure 9 shows the mechanical energy typically possessed by the trunk, lower limbs and upper limbs. The large peaks in the upper limb energy are associated with the rapid recovery of the arms and is not energy that would readily be transmitted along the body through the rolling action. Despite the large mass of the trunk, the energy possessed by the trunk is small relative to that of the upper and lower limbs. This is because the motion is mostly quite slow rolling motion of mass that is close to the axis of rotation. While there may be an influence on the motion of the lower limbs through the H1 wave, there is not a lot of energy to give up to the lower limbs relative to the energy generated by the six beat kicking action. Thus, unlike the situation in butterfly there is little evidence of 'free of charge' energy transmission from trunk to lower limbs in front crawl swimming.

Figure 10 shows the mechanical energy possessed by the thighs and shanks. Great energy is generated in the shanks. However, the fact that the thighs do not gain a lot of energy relative to that of the shanks indicates that much work must be done by the muscles to yield the large difference in energy of the shanks relative to the thighs. Although energy could be being absorbed from the trunk and thighs continuously by the negative work of muscles at

their origins and transmitted through the system by positive work at their insertions, there would be a physiological cost to this. There is no evidence for the possibility of 'free of charge' transmission through the transmission of a wave through the mechanical system. This differs from the situation in butterfly.

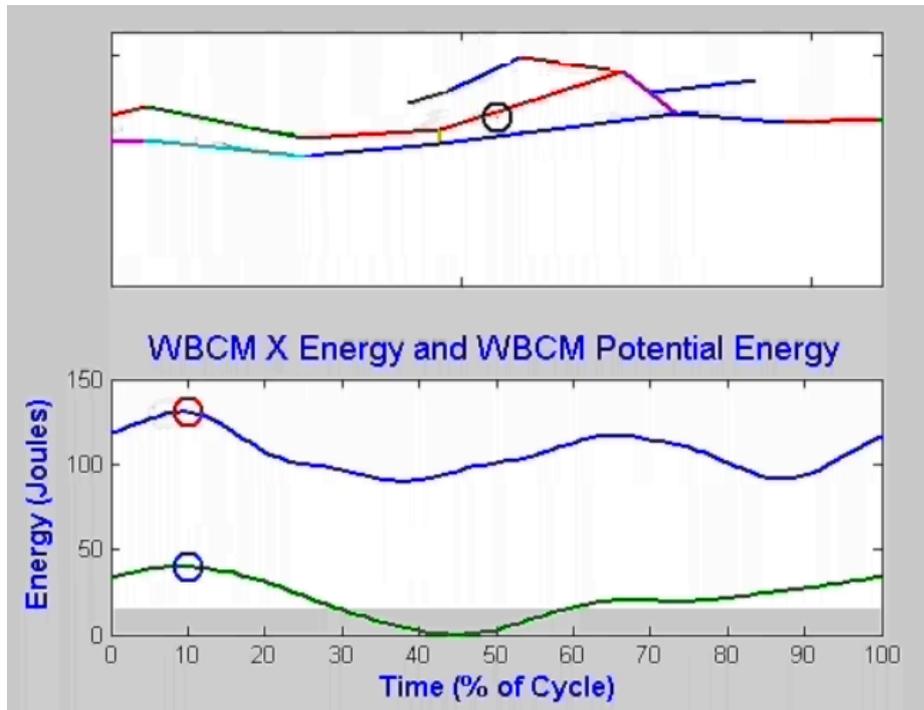


Figure 8: Typical energy possessed by virtue of translation in the desired direction (upper) and gravitational potential energy (lower) in front crawl swimming.

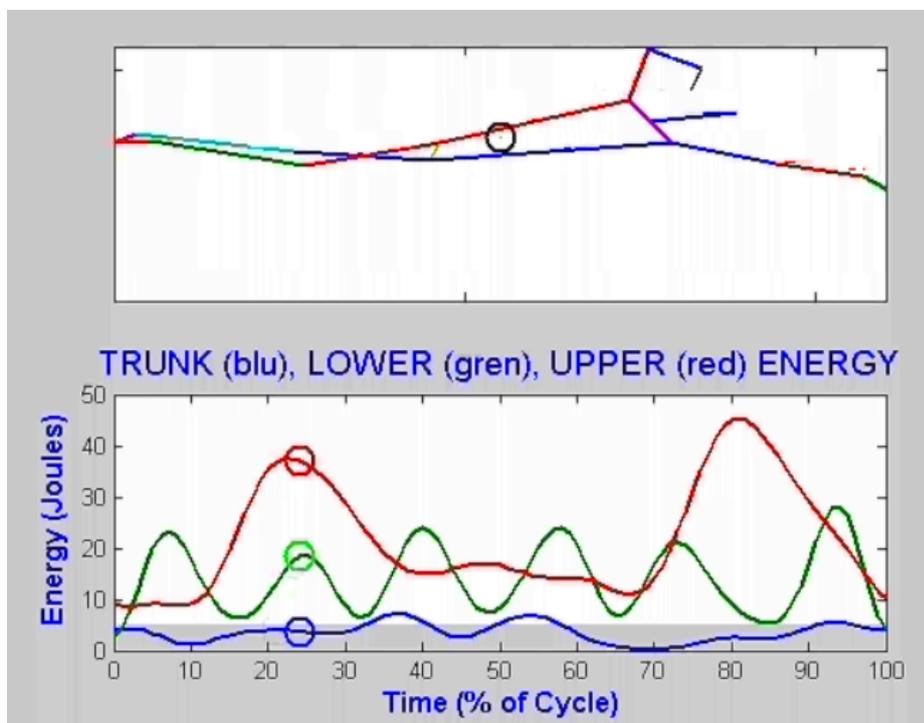


Figure 9: Typical trunk (blue), lower limb (green), and upper limb energy (red) in front crawl swimming.

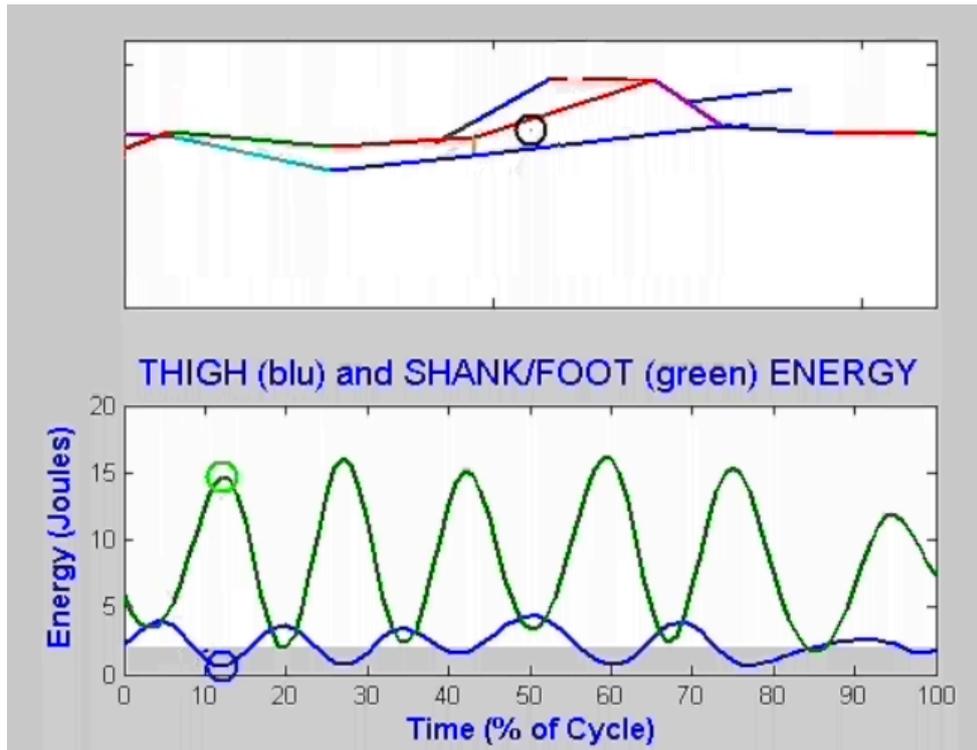


Figure 10: Typical thigh (blue), and shank (green) energy in front crawl swimming.

**Propulsion:** Figure 11 of WBCM acceleration shows that the propulsion of this typical front crawl swimmer is dominated by the action of the arms. However, the effect of the kick is apparent in the equispaced local maxima and minima. As in butterfly, the velocity vector of the centre of the feet indicated that there was no backward component of foot velocity at any stage during the cycle. This suggests that the mechanism of propulsion is not related to reaction forces from pushing undisturbed water backwards. Propulsion from vortex shedding remains possible but evidence of a more direct nature is required.

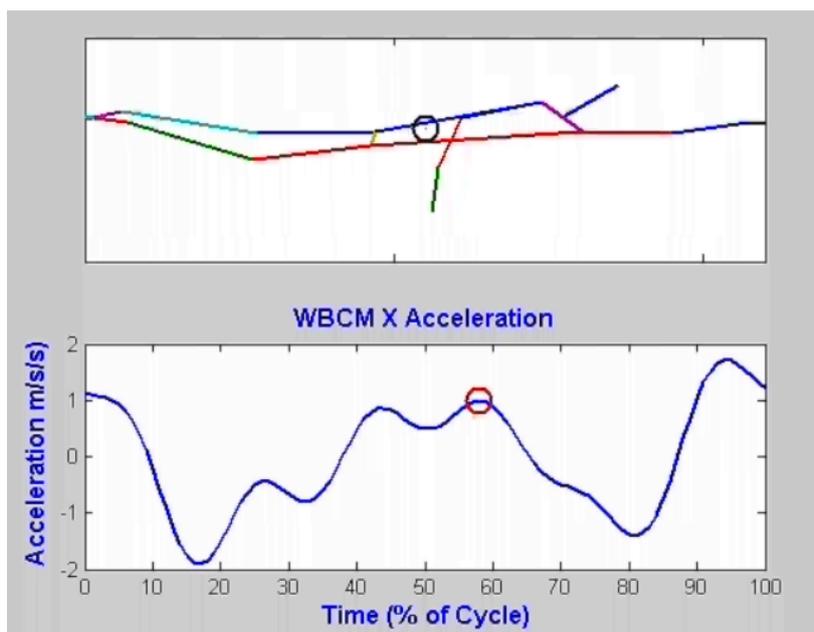


Figure 11: Typical acceleration of the WBCM in the desired direction in front crawl swimming.

**Roll kinematics and kinetics:** Figure 12 shows the roll angular momentum of the typical front crawl swimmer. The angular momentum possessed by the lower body exhibits the strong H3 rhythm. The roll angular momentum of the upper body reflects the H1 rhythm. In addition to the hydrodynamic forces from the upper limbs there are reaction torque effects from the actions of the lower limbs and, as indicated by the modelling work of Yanai (2004), there is a strong influence of the buoyancy force which is not in alignment with the WBCM when the arm is recovering. The rotation of the body is the result of the interaction of the torques from the hydrodynamic forces in reaction to the kick, the arm actions, and the torque from the buoyancy force.

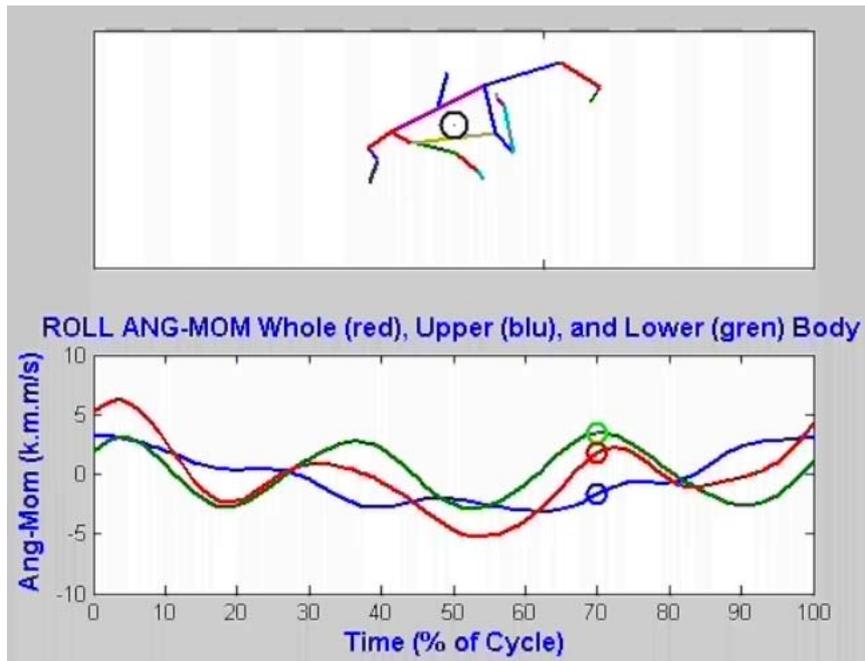


Figure 12. Typical angular momentum about the longitudinal principal axis (red) and contributions of upper (blue) and lower body (green) in front crawl swimming.

**CONCLUSIONS:** Having investigated the rhythms in butterfly and front crawl in combination with accurately derived kinematic and kinetic data some conclusions may be drawn.

1. Skilled butterfly swimming is characterised by 'rocking', that is, rotations about transverse axes, of body segments to yield wave-like undulations of shoulders, hips knees and ankles.
2. The undulations are coordinated to yield travelling 'body waves' to optimise performance from:
  - a. A hydrodynamic perspective
  - b. An energy transmission and re-use perspective.
3. The phase relationship between the one-beat H1 frequency and two-beat H2 is important to performance.
4. Skilled front crawl swimming using a 6 beat kick swimming is characterised by 'rolling' of body segments to yield wave-like rotations of shoulders, hips, knees and ankles.

5. The rolling actions are coordinated to yield a travelling 'torsional' wave from hip to ankles to optimise performance from a hydrodynamic perspective.

From a methodological perspective:

1. Analysis of rhythms, in combination with accurate kinematic and kinetic data provides additional insight into skill development and performance.
2. Rhythms can be regarded as part of the dynamic topology of the movement system and represented topographically.
3. Great potential to assist coaches in understanding, developing, and maintaining 'good rhythm'.

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