MATHEMATICAL MODELLING AND OPTIMIZATION OF SNATCH LIFT TECHNIQUE

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Mathematical modelling and optimization of snatch technique based on dynamic synthesis is the main purpose of this study. The barbell trajectory was proposed as the main index which has been evaluated experimentally by several researchers who have introduced optimum trajectory according to the percentage of the athletes' success. We employ a five-link model to evaluate its behaviour and to obtain its optimum trajectory by minimizing a specific criterion. We solve motion equations together with an equation which represents the performance criterion by means of Pontryagin Maximum Principle (PMP) formulation simultaneously. The results of this model in comparison with other researchers' experimental observations show an improvement to introduce a good predictive model. This model can help the coaches to improve the performance of weightlifters.

KEY WORDS: sports biomechanics, weightlifting, motion analysis, optimization.

INTRODUCTION:

Barbell trajectory and other dynamic characteristics of motion, such as velocity of barbell during weightlifting, were the common subjects which have been investigated by several researchers like Baumann et al. (1988), Isaka et al. (1996), Gourgoulis et al. (2000), Byrd (2001), Garhammer (2001) and Schilling et al. (2002) over the years. The importance of optimizing the barbell trajectory is in agreement with most above researchers. Most of them have studied the differences between the elite weightlifters' characteristics of motion. They have categorized several of these lifting motion patterns as optimized one, such as the study coordinated by Baumann et al. (1988). These optimized patterns have been selected because of their owners' success, and none of mechanical parameters were considered. In recent years, several researchers like Park et al. (2005) have used mechanical parameters such as actuating torque, to introduce optimized patterns for lifting tasks. He has investigated the differences in motion patterns for goal-directed lifting activities and believes that the redundancy of degrees of freedom makes it possible to have an optimum motion pattern. But, there was no attempt to use this method for weightlifting which is more complicated than simple lifting task.

On the other hand, using the optimal control theory for optimizing the gait patterns by Rostami and Bessonet (2001) and the capability of this method for sport activities, encourage us to extend this method for weightlifting. We have formulated a mathematical model based on dynamic principles to predict the barbell trajectory which minimizes the specific criterion.

METHOD:

Equations of motion: The first step to build a Weightlifter Biomechanical Model is to translate the human's physical properties into the mathematical one. For this purpose, we employ the anthropometric models, which have been developed by several researchers. One of comprehensive models has been introduced by Chaffin and Anderson (1991). In this model, the body segments are converted to solid links and body joints to simple revolute joints. The second step is simplification of this model to a sagittal plane model for weightlifting or other general lifting activities. This is a common assumption, used by several researchers like Chang et al. (2001), Menegaldo et al. (2003), and Park et al. (2005). The third step is to define a kinematics model, represents the number of links and thus the number of degrees of freedom (DOF). Several researchers like Chang et al. (2001), Menegaldo et al. (2003) and Park et al. (2005) have used five-link model to analyze lifting
tasks, therefore we use the same model. Figure 1 shows the schematic diagram of this model at initial time which is made by five links represent shin, thigh, trunk, upper arm and forearm, respectively named L1 to L5. Also, five body joints: ankle, knee, hip, shoulder and elbow are represented O1 to O5 respectively.

Figure 1: Biomechanical Model of a Weightlifter at Initial Position

Numerical values of dimensional and physical parameters are calculated by using the formula, suggested by Chaffin and Anderson (1991). We employ the motion equations in the state space form. As indicated in Rostami and Bessonet (2001), the Hamiltonian dynamic model not only fulfills this requirement but strengthens the robustness of algorithm also. By reformulating the motion equations, we have the first order below equation:

\[ \dot{x}(t) = F(x(t)) + Bu(t) = F(x(t), u(t)) \]  

Where \( x = (x_1, \ldots, x_{2n})^T \) is the vector of state variables and \( u = (u_1, \ldots, u_n)^T \) is the vector of control inputs, joint actuating torques, and \( n \) is the number of degrees of freedom (DOF).

**Constraints:** Initial and final constraints specify the conditions of the start position and the end of second pulling phase (or start of catching phase). In order to respect joint stops, prevent counter-flexion and moderate total joint coordinate variations, we have to prescribe bounds on the joint coordinates. Also we apply control-constraints that define limits on torques which act on the mechanical system and are produced by actuators (muscles).

**Optimization:** We want to generate an optimal motion that minimizes a performance criterion. This performance criterion is just like the actuating torques that have been used by Rostami and Bessonet (2001). Inequality constraints can be easily dealt with by using computing techniques like penalty method. The optimization problem may be summarized to finding a phase trajectory and a control vector, which minimize the integral cost of actuating torques and satisfy the state equation (1) together with the boundary conditions. Typically, we deal with a two-point boundary value problem. The two-point boundary value problem can be solved by using computing techniques such as finite difference algorithm or shooting method. We select the latter approach because of its efficiency and the simplicity of the implementation. Because of the strong non-linearity of equations, we employ the multiple-shooting method by means of solving the two-point boundary value problem, considering a short motion step. We continue the optimization algorithm iteratively by increasing the boundary values until the desired final values are reached. In the same way, any optimal solution can be used as a guess solution to solve swiftly a problem relating to the previous one.
RESULTS:

We solved a problem for a weightlifter with 55 kg mass and 1.6 m height who lifts a 90 kg barbell by snatch technique. We selected the final position, start of catch phase, so that the barbell could continue its motion and the weightlifter could move under the bar quickly. Figure 2 shows the barbell trajectory during the snatch lift from the time just prior to when the barbell left the floor (“lift-off”) until the bar reached to the start of catch phase. At this point the barbell continues to move as a “projectile” and lets the athlete to “move under the bar” to catch it. The experimental typical form of trajectory, described by Garhammer (2001) shows that when the barbell is lifted from the “lift-off” phase, it moves toward the athlete during the first pull, then away from the athlete and finally toward him again as it begins to descend during the catch phase. Figure 2 shows this typical form roughly. One can see the good agreement between optimized trajectory and experimental results, published by Garhammer (2001).

Figure 2: Optimized (solid line) and Experimental (dashed line) Barbell Trajectory during Snatch Lift

Figure 3 shows how actuating joint torques vary during the barbell motion. The role of each joint in making a complete snatch lifting motion can be realized by this diagram while considering the kinetics aspect. For instance, the importance of the role of the hip joint and also the ascending role of the ankle and knee joints during the snatch lift can be easily seen. Also these variables are good parameters for showing us the practical differences between an actual weightlifter’s snatch motion and the ideal optimized one which he/she can achieve. We can reduce these differences by advising a weightlifter about the strength training he/she should do to compensate for the weakness of a particular joint.

Figure 3: Optimized actuating joint torques until the start of catch phase
DISCUSSION:
Barbell trajectory which is produced by our optimized model shows the typical form, can be seen in experimental data. Since we obtain this optimized trajectory by using dynamic motion equations, we assure that this trajectory can be produced by a real weight lifter while other optimizing strategy like geometrical path optimization could not give us this assurance. On the other hand, we believe that the results show the relative success to predict the optimal motion based on our selective criterion. Therefore, we can conclude that the selected criterion is in agreement with the criterion is selected by a weight lifter. However selecting the best criteria for improving the performance of weightlifters requires more studies and it could consist of more than one criterion combined together during the full snatch. The results of this optimization can help us to train weightlifters to behave like the optimized kinematics parameters or to make their characteristics like resultant kinetic parameters. Actual parameters of our study in comparison with an optimized one, can guide us to achieve these results.
Introducing and modifying the proper criterion which is in agreement with human motion pattern is another advantage of this study and we believe that we are successful regarding to this matter.

CONCLUSION:
This dynamic model can provide an insight into control and improve the motions during the snatch lift. The determination of optimal motion during the motion of the snatch lift can help coaches to train weightlifters in a more systematic manner. This model can help them, not only to increase weightlifters’ success but to reduce their injury risks also. The good result that we obtained from the optimization problem shows that this method is very reliable. Therefore, the success achieved by our approach encourages us to improve our model by using techniques with more degrees of freedom.

REFERENCES: