The aim of this paper is to illustrate developments in inverse dynamics using selected examples. It will give a description of the method with emphasis on the critical parts. Results are discussed for several examples and the methodological difficulties are specified. It is shown how hidden parameters can be recovered with the help of inverse dynamics. The quantification of sports performance is demonstrated, and the applicability of inverse dynamics in the training process is illustrated.

KEY WORDS: Inverse dynamics, simulation, model creation, Fourier transformation, digital filters, trampolining, running, martial arts, judo

INTRODUCTION: At the 1998 ISBS Symposium, 27 presentations were devoted to the theme of 'Modeling, Simulation, Optimization in Sports Biomechanics'. These included the Geoffrey Dyson Memorial Lecture by Herbert Hatze (1998) from the University of Vienna and the Keynote Lecture by Maurice Yeadon (1998) from the University of Loughborough. Herbert Hatze explained the problem of simulating human movements. Besides examples of successful applications using simulations, he offered an overview of geometrico-mathematical models of the human skeletal subsystem and of neuromuscular models. Professor Hatze emphasized that, in a broad sense, the majority of all studies in sports biomechanics deal with the optimization of movements. Maurice Yeadon (1998) provided an inside view of various mathematical models with a wide range of complexity for various sport situations. He reviewed the successful approach of the simple two-segment model (Alexander, 1990) in jumping. For the simulation of the post-flight aerial phase of tumbling he presented an eleven-segment model (Yeadon, 1990), necessary to create a successful simulation. More than 50% of the presentations on the topic of 'Modeling, Simulation, Optimization in Sports Biomechanics' were concerned with simulation. Only two papers (Ambrósio et al., 1998 and Vieten et al., 1998) focused on inverse dynamics. This is rather surprising, since Hatze's above statement suggests that most sports biomechanists are trying to understand movement or even to develop concepts to describe optimal performance. Direct measurements cannot provide all the parameters necessary for a full description in biomechanical terms. Simulation can provide numbers. However, simulation has to be proven right to be of value. It is the author's opinion that inverse dynamics is a key to obtaining this proof. Furthermore, inverse dynamics helps us uncover hidden parameters (such as internal force, torque, or energy) and therefore provides initial data for developing simulations of higher complexity. In addition, temporary approaches in inverse dynamics can serve as monitoring tools during the athlete's workout sessions.

METHOD: The method of inverse dynamics consists of three distinct steps:
1. Implementing a subset of the laws of nature (on a computer).
2. Using a mathematical model that allows the calculation and display of the relevant entities (images of humans and the environment).
3. Provision of data for the animation of movements.

Taking the above into consideration and provided that gravity and none or one additional external force (what centre of action is located within a given area, e.g., a foot) are acting, the whole scenario is uniquely defined. In principle, all parameters can be calculated from the movement of the body, e.g., internal forces, joint moments and even muscle energies. If more external forces act on a model, then the forces and the locations they act on must be included in the simulation. In theory we can learn about movements in specific situations...
and about the general laws which govern them. Inverse dynamics can provide all the desired results. However, in practice inverse dynamics contains numerous imperfections! Modern simulation software packages provide the necessary subset of the laws of physics. For all our examples we used SDS simulation software, which was originally designed for industrial use, with rules based on developments in robotics (Walker et al., 1982). It includes the classical laws of mechanics and makes it possible to 'animate a virtual world' populated by entities with a defined topology and geometry. An additional component of the SDS system (SDS-Human) allows the easy construction of mathematical body models. Thirty-eight anthropometric measurements project an automatic construction of the individual subject's anatomy on the computer as a Hanavan model (Hanavan, 1964).

![Figure 1 - Human anatomy projected onto the Hanavan model](image)

Animation data were created in a process starting with the filming of the actual movement using a set of video cameras. In all the following examples we used three to five video cameras. Digitising was done using either the Peak 5 system in the manual mode or the WinAnalyze system in the automatic mode. When we used automatic tracking the human objects were tagged with up to 19 markers.

![Figure 2 - Marker positions (17 displayed)](image)

The result produced by the digitising process were either the three dimensional coordinates of the landmarks or the joint coordinates as functions of time. In the next step, these data had to be processed for use in inverse dynamics. The process is illustrated in the example
of a person changing from a sitting position to standing and back to sitting. The animation of
this first example contains three-dimensional data of 19 markers. The SDS software
requires the coordinates of the joints and the distal points at the ends of the kinematic links.
These coordinates must therefore be calculated from the marker coordinates, taking into
account the orientation of the segments. This is done by defining a vector pointing from
each marker to the respective joint. These vectors are fixed within their segment's
coordinate system. Once this is achieved, several matrix multiplications must be carried out
for each video frame and each marker. The process is automated within SDS. The result of
such a process is the joint coordinates as functions of time. These coordinate functions are
then converted to the coordinate functions of the base segment, the hip, the orientation of
the hip and all related segmental orientations. Figure 3 displays one frame of the sequence.
On the left the calculation was done using joint coordinates and on the right marker
coordinates.

Figure 3 - Posture calculated using joint (left) and marker (right) coordinates
The animation system requires further input to perform an analysis of inverse dynamics,
namely, the velocity and the acceleration of the base segment, the angular velocity, and the
angular acceleration of all segments. For each coordinate/ angular coordinate component a
spline of the 5th order is defined as:

\[ x(t) = c_0(n) + c_1(n) \cdot t + \frac{c_2(n)}{2} \cdot t^2 + \frac{c_3(n)}{6} \cdot t^3 + \frac{c_4(n)}{12} \cdot t^4 + \frac{c_5(n)}{20} \cdot t^5 \]

The respective velocity is

\[ v(t) = c_1(n) + c_2(n) \cdot t + \frac{c_3(n)}{2} \cdot t^2 + \frac{c_4(n)}{3} \cdot t^3 + \frac{c_5(n)}{4} \cdot t^4 \]

and

\[ a(t) = c_2(n) + c_3(n) \cdot t + c_4(n) \cdot t^2 + c_5(n) \cdot t^3 \]

represents the acceleration. The constants \( c_k(n) \) with \( k = 0, 1, \ldots, 5 \) are calculated for each
time interval from \( t = (n-1) \cdot \Delta t \) to \( t = n \cdot \Delta t \) with \( n \) running from 2 to \( N-2 \) (\( N \) = number of
frames). For the first and the last interval a spline of the 4th order was calculated. The
calculation is based on the neighbouring coordinates \( x_n \), the respective velocities \( v_n \),
the accelerations \( a_n \), and the jerks \( j_n \). The calculation of the velocity, acceleration, and jerk is
as follows:

\[
\begin{align*}
  v_n &= \frac{1}{2}(v_{n-1} + v_{n+1}) \\
  v_{n+\frac{1}{2}} &= \frac{x_{n+1} - x_n}{\Delta t}
\end{align*}
\]
These definitions allow the system to calculate parameters and the animation continuously between the first and the last data set and does not restrict the time to discrete values. In principle, all necessary data were supplied and a full calculation of the movement could be done. However, raw data from a digitising system are 'noisy' and need to be filtered. The amplitude of the Fourier transformation of the time signal indicates the noise level (see Figure 4).

![Scatterplot (1_31ROWI.STA5v*512c) not filtered](image)

**Figure 4** - Fourier transformation of the vertical component of the velocity of the center of gravity.

We used a FIR filter of the 8th order (Ifeachor & Jervis, 1993) combined with a 'Blackman Window' (see Figure 5 for the transfer characteristic) to reduce the noise level.
Figure 5 - On the left the transfer characteristic of the FIR filter combined with a Blackman Window. On the right is the Fourier transformation of the vertical component of the velocity of the center of gravity after a single filtering.

The normalised FIR filter reduces the amplitudes of frequencies mainly above 5 Hz. The lower frequencies in the interval of up to 2 Hz were amplified. The results of filtering of data done once and twice respectively for higher frequencies are shown in the next two figures.

Figure 6 - Fourier transformation for higher frequencies after filtering once and twice in the respective case. The vertical axes are truncated.
The filtering was performed on the original marker data before any other processing was done. Figure 7 shows the effects of the filtering. It depicts the vertical velocity component of the center of gravity for the above example. The dotted line represents the calculation using raw data and the solid lines for once and twice filtered input data.

![Figure 7 - Velocity of the centre of gravity calculated from unfiltered, once filtered, and twice (smoother line) filtered marker coordinates using an 8th order FIR low pass filter](image)

Results in inverse dynamics depend heavily on the quality of the input data. Therefore, filtering is a very important step throughout the entire process. Noise caused by stochastic processes during digitising has frequencies higher than 10% above the sampling frequency. A low pass filter can reduce the noise on the signal substantially if a sufficiently high sampling frequency is selected. In our studies we chose a sampling frequency ten times higher than the maximum frequency of the movement to be analysed.

RESULTS AND DISCUSSION: There are numerous papers on airborne movements in inverse dynamics as well as in computer simulation. The reason for this tendency is very simple. Only one external force, gravity, and the internal forces govern the whole movement. The next example falls into this category. It is a trampoline performance of three somersaults with a half twist. This is an excellent example to test the method of inverse dynamics. As known from classical physics, the trajectory of the centre of gravity of an airborne movement must be a parabola. The angular momentum of the body around the centre of gravity must be a constant. However, the results show deviations (Figure 8).
Figure 8 - Trajectory (left) and angular momentum of a trampoline performance (triple somersault with ½ twist)

These deviations are caused by the differences between the anatomy of the athlete and the mathematical model. In inverse dynamics the total movement of the model is controlled by the given animation. If, for example, the positions of the hip joints are incorrect, the centre of gravity is displaced by the same amount. By carefully adjusting the hip coordinates, the above curves will be closer to the real trajectories. A further step towards better results is to use more refined body models. The essential refinements will be discussed in the 'Conclusions'.

The following is an example of a combined inverse dynamics / simulation study of the above shown trampoline performance. Two identical anthropometrical models are used (see Figure 9). Body A (left entity) is animated as described above. Body B (right entity) is animated till it leaves the trampoline at $t=0.56$ sec. Thereafter, the segmental movement relative to each other is still guided, but the linear movement of the centre of gravity and the body's orientation are simulated.

Figure 9 - Trampoline performance – three somersaults with twist. Animation (model on the left) simulation (model on the right).

The simulation produced three somersaults almost identical to the animation, but instead of a half twist, as in the animation, the simulation produced 1 ½ twists. Figure 10 shows the comparison of the centre of gravity's trajectories on the left. The curve on the right displays the differences in the angular momentum between the animation and the simulation.
Inverse dynamics can give much insight regarding muscle energy and the power needed to perform specified movements. A method often discussed in the literature (Howley et al., 1974) which calculates muscle power from oxygen exchange during breathing is restricted to aerobic movements. In addition, only average muscle power can be calculated. There are no such restrictions with inverse dynamics. At the 1996 ISBS Symposium, Vieten et al. demonstrated that 80% of the average power exerted during running is created in the lower extremities (see Figure 11).

The main amount of muscle energy in slow running is provided during the support phase, while in fast running the support and flight phases contribute an almost equal amount. A comparison between literature data (oxygen exchange method) and the results of the study indicate a high level of agreement for slow running. For fast running the power increases exponentially (see Figure 12).
Figure 12 - Average Power/Mass as a function of the running speed

The high spin kick in martial arts is a technique in which the athlete spins on one leg while the other is extended to strike the target (see Figure 13).

Figure 13 - Generation of angular momentum in a martial arts High Spin Kick

The athlete's goal is to strike the target rapidly while maintaining high foot velocity. The angular momentum for the rotation axis, mainly the y-component in the above curves on the left, must be maximised when hitting the target (at \( t=0.9 \text{sec} \)). The initial angular momentum...
generated while two feet are on the ground is enhanced during the kick phase. The martial artist needs to go off-balance to generate the necessary angular momentum before impact with the target. Thereafter a counter-movement ensures a stable stance after the completion of the kick. Experienced martial artists reach the peak of their angular momentum curve while striking their target. The generation of angular momentum is also much more effective for experienced athletes. Inverse dynamics makes possible the quantification of the quality of martial arts techniques, as shown in the example of a high spin kick. Progress in the training process is directly displayable.

In another type of martial art, judo, linear momentum and momentum transfer is a major factor for a successful judo throw. For the example of a 'harai goshi' (Figure 14), data were taken from Walter (1997). The x-component – in this example the x-direction is the main movement direction - of the linear momentum (graph on the left) and the force on the athletes (graph on the right) – is shown.

**Figure 14 - The transfer of linear momentum in a 'harai goshi' judo throw**

The solid curves stand for the attacker and the dotted line for the person being thrown. During the pre-throwing phase (up to \( t=1 \)sec), which determines the success of the throw, the anti-cyclic behaviour of the curves clearly identifies the momentum transfer from the attacker to the thrown person. Many different judo throws can be judged based on a successful momentum transfer. Therefore, in this case as well the application of inverse dynamics during the training process can help monitor the athletes' development.

As shown in the previous two examples, inverse dynamics can provide training-relevant data without the need for defining the multiple external forces. The experimental problem is reduced to the creation of the body model and the generation of the animation data.
CONCLUSIONS: Inverse dynamics is a highly effective method for calculating hidden parameters such as energy, power, momentum, angular momentum, etc. Basic rules in human movement can be uncovered and quantified. The quality of an athlete’s performance can be judged based on stable parameters. Therefore, inverse dynamics can be used as a monitoring tool during a workout. The limiting factor in working with athletes during the training process is data collection. Otherwise, there is no reason not to use inverse dynamics during training, if the available digitising system can produce animation data at an adequate rate.

Mathematical body models consisting of rigid segments linked by simple joints are adequate for many movement situations. However, there are circumstances where such models are not satisfactory. As in the case of trampolining, extreme accelerations shift body masses away from their normal position. An improved mathematical model including a 'wobbling mass' (Gruber, 1987) for each segment can help us to achieve much more realistic results. In such a wobbling mass model a fraction of the mass of each segment could be coupled via a spring-damping system to the main mass of the segment. The model would apply both to shifted masses as well as to changes in inertia tensors and to a delay in the response to internal as well as external forces.

REFERENCES: