IMPROVING FEEDBACK ON SWIMMING TURNS AND STARTS EXPONENTIALLY

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The purpose of this study was to develop an objective method of quantifying velocity and estimating appropriate times of initiating the kick in turns and starts. A mathematical model based on an exponential function was fitted to the glide phase following push off from the wall in turns. The function was then extrapolated through the kicking phase and compared to the actual velocity to assess whether the timing of initiation of the kick was appropriate. It was concluded that this method was useful for analysis of swimmers and for feedback to swimmers and coaches on timing of the kick in turns. The method can also be applied to assess the timing of the kick following entry in a dive start.

KEY WORDS: swimming, resistive drag, starts, turns, video analysis, curve fitting

INTRODUCTION: Race analysis has shown convincingly that times spent in the turning and starting phases of swimming races are strongly related to swimming performance (e.g., Arellano et al., 1994; Mason & Cossor, 1999). Lyttle and Blanksby (2000) have shown that velocity of the swimmer following a turn or immediately after entry in the starting dive is greater than the velocity that can be achieved in underwater kicking. They found a maximal velocity kicking underwater of between 1.9 and 2.2 m\(s^{-1}\) in their group of experienced adult male swimmers. Kicking at higher velocities increases drag compared to that in a passive streamlined position. This has important implications. A swimmer can lose velocity unnecessarily if the kick is initiated prior to the maximum sustainable kicking velocity. Similarly, commencing the kick too late will result in a loss of velocity. Kicking too early has the added disadvantage of wasting energy.

The purpose of this study was to develop an objective method of quantifying velocity and estimating appropriate times of initiating the kick. This method can then be applied to advise competitive swimmers regardless of age, ability, and gender.

METHOD: The method requires a velocity-time record for the period immediately following push off to near the time of surfacing in the case of turns, or from entry to surfacing in the case of dive starts. Generally, this record would be obtained from digitized video of the swimmer from an underwater view. Ideally, the center of mass of the swimmer would be determined by digitizing joint centers of a full body model. However, for expediency in a testing and feedback application, a marker representing a fixed point on the body, such as the hip joint, can be used to represent whole body motion. This is deemed acceptable when the swimmer is maintaining a close-to-constant body posture, for example, following extension of the legs and arms to adopt the gliding posture. Hip and knee flexion associated with the kicking action introduces some error in estimates of whole body motion. However, this error is mostly in the vertical direction. Thus, estimating the horizontal velocity of a fixed point of the body may be regarded as reasonable for the purpose of providing useful feedback to swimmers and coaches while minimising the time between data collection and delivery of feedback.

Video data of the swimmer may be collected in various ways provided that the camera is viewing from below the water surface. Care must be taken to minimize or correct perspective errors and to calibrate accurately using a scale. We collected the data for the development of the method described here, using a panning technique. A JVC GR-DVL9800 handycam sampling at 60 Hz was set 12 m from the plane of motion of the swimmers. It was set on a tripod and panned so that its axis was horizontal. The camera axis was perpendicular to the plane of motion of the swimmer when the swimmer was 4 m from the wall. To correct for perspective error when the swimmer was not perpendicular to the camera axis, a FORTRAN
computer program was developed. This rescaled the data continuously using reference positions of markers on a scale line. The scale line comprised black markers positioned at 1 m intervals on a taut cable directly under the midline of the swimmer and aligned in the direction of travel of the swimmer. The subject wore a white body suit to maximize contrast of a black marker fixed in line with the hip joint. To maximize contrast of the scale line with the background, white plastic sheets were placed against the pool wall opposite the camera. These procedures enabled the scale line markers and the subject’s hip markers to be digitized automatically using an Ariel Performance Analysis System (APAS). After continuous scaling of the hip data and smoothing with a dual pass recursive 2nd order Butterworth filter at 6 Hz, velocity data were derived.

**MATHEMATICAL MODEL:** To determine the appropriate time to introduce kicking it is necessary to model the velocity that would occur in the absence of kicking, that is, when the swimmer is holding a passive glide position. When the velocity falls below the velocity that can be sustained by kicking, the swimmer should commence kicking. Because velocity reduces as a function of velocity, the glide phase is appropriately modeled as an exponential function of the form:

\[ v(i) = ae^{-kt(i)} \]

Where \( v(i) \) is the velocity at the ith data sample, \( a \) and \( k \) are constants to be determined. We determine \( a \) and \( k \) to minimise the sum of the squares ('least squares curve fitting', see, for example, Kreysig, 1983) of residuals between the data for the glide phase and the fitted exponential function:

\[ s = \sum_{i} (v(i) - ae^{-kt(i)})^2 \]

Where \( n \) is the number of samples from the start of the constant glide phase, that is, when a constant glide position is attained, to the time when kicking begins. This requires the partial derivatives:

\[ \frac{\partial s}{\partial a} = 0 \quad \text{and} \quad \frac{\partial s}{\partial k} = 0 \]

The equation \( \frac{\partial s}{\partial k} = 0 \) can be solved for \( a \) to get \( a = \frac{\sum_{i} v(i) t(i) e^{-kt(i)} - \sum_{i} v(i) e^{-kt(i)}}{\sum_{i} t(i) e^{-kt(i)}} \)

Then the equation \( \frac{\partial s}{\partial a} = 0 \) yields the following expression:

\[ z = (\sum_{i} 2v(i)t(i)e^{-kt(i)})(\sum_{i} e^{-2kt(i)}) + (\sum_{i} e^{-2kt(i)}t(i))(\sum_{i} v(i)e^{-kt(i)}) \]

where the range of all sums are from \( i = 1 \) to \( n \).

The value of \( k \) that yields \( z = 0 \) is solved numerically. Then \( a \) is determined by substitution into:

\[ a = \frac{\sum_{i} v(i)e^{-kt(i)}}{\sum_{i} e^{-2kt(i)}} \]

While a function can be found using this technique in some standard software packages, for example, Excel (Microsoft), coding the formulae into one’s own program affords some
advantages. For example, the values of $a$ and $k$ can be found for the glide phase and then the function extrapolated to predict the velocity for times beyond the glide phase. This can be plotted over the actual data obtained for the period after the commencement of kicking. In this manner it can be observed whether the kicking was effective in sustaining the velocity above the values that would have occurred by simply maintaining a passive glide, or whether the introduction of the kick was premature and resulted in a loss of velocity compared to a passive glide.

![Graph](image)

**Figure 1** - Actual velocity of the hip of a freestyle swimmer from 0.2 s after last contact with the wall (time zero) and the fitted velocity based on the exponential function with $a = 2.282$ and $k = 0.406$. The function was fitted for the section for 0.0 to 0.3 s corresponding to the glide phase of the turn and then extrapolated for the remainder of the period during which the swimmer was kicking.

**RESULTS:** Validation of the curve fitting program against commercial software (Microsoft Excel) showed that the mathematical model and numerical search routine had been correctly coded. The model has been applied to the turns of swimmers from the Scottish Institute of Sport and City of Edinburgh swim squads and has indicated that the timing of the kick is inappropriate in many cases. Most of these swimmers kick too early. Their performance would be improved by adopting and holding a streamlined glide position for a longer period before commencing the kick. It was apparent in most cases that, by rushing to start the kick, many swimmers were not in a streamlined position and lost speed rapidly after the turn. The preparation for the kick – in particular, flexion of the knees – slowed them down. Further, the kick itself failed to restore the velocity to the levels that they would have sustained by adopting a streamlined glide without kicking.
Figure 2 - Actual distance traveled by the hip of a freestyle swimmer from 0.2 s after last contact with the wall (time zero) and the fitted distance based on the exponential function.

Figure 1 shows an example of a fitted exponential curve and the actual velocity of the hip joint. This male swimmer timed the kick appropriately after having a streamlined glide for approximately 0.3 s. This was the section to which the exponential function was fitted ($a = 2.282; k = 0.406$). Time zero on the graph corresponds to attainment of the extended glide position at approximately 0.2 s after last contact with the wall. Notice that while the first kick did not produce an overall gain in velocity over the whole kick cycle, there was nothing lost by the first kick, except perhaps, some energy. Subsequent kicks clearly achieved an overall higher velocity than would have been maintained in a streamlined glide without a kick. The actual distance traveled by the swimmer and the distance traveled by the swimmer if he had held the streamlined glide are shown in Figure 2. Notice that the kick doesn't start to show any benefit until about 1.0 s (about 0.7 s after commencing kicking). At 1.90 s (corresponding to the start of the full freestyle swimming action as the surface is approached) the swimmer has gained about 0.1 m from the kicking actions. Given that drag increases as the swimmer gets closer to the surface, this is an underestimate of the advantage gained by kicking.

CONCLUSION: The development of a mathematical model and analysis program based on fitting the glide period of a push or dive with an exponential function has been found useful in assessing the timing of swimmers kicking in turns and starts.

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