OPTIMIZATION OF GROUND REACTION FORCE DURING 100m-SPRINT

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The purpose of this study is to optimize the individual 100m-sprint performance by the optimal control of ground reaction force. Subject was a national level athlete, best performance 10.65s. Kistler force-plate and LOCAM Hi-speed camera was used to record the ground reaction force and the movement. Applying optimization theory and algorithms, the ground reaction force was altered reasonably, in order to increase the velocity of body c.m. during each support phase. It is found that the final velocity of body c.m. during support could be increased 0.05m/s through optimization of ground reaction force and the performance of subject for 100m could therefore be improved approximately 0.06-0.08s. This improvement is achieved only by changing the patterns of ground reaction force. In other words, it is obtained by rational force generation.

KEY WORDS: optimization, ground reaction force, sprint

INTRODUCTION: The horizontal running velocity of the athlete’s body center of mass (c.m.) is determined by the impulse of the horizontal ground reaction force during ground contact phase. Due to air resistance and horizontal braking impulse, the horizontal velocity of body c.m. is lost during the first part of the ground contact. The most critical issue in sprint running is the action of the leg before and while it is contact with the ground (Kunz and Kaufmann, 1981; Wood, 1987), since the leg action during these period is related to the amplitude and patterns of the ground reaction force. According to Tiupa et al (1978) and Baumann (1986), whether the horizontal braking impulse is reasonably produced or not, is an important criterion to evaluate the sprint technique. Therefore, the quality of sprint technique could be reflected by the patterns of the ground reaction force.

The purpose of this study is to simulate and optimize the individual sport performance during 100m-sprint by the optimization of ground reaction force.

METHODS: Subject was a national level athlete (body height: 189cm; body weight: 86kg), best performance of 100m sprint running was 10.65s. Kistler force platform (sample rate 600Hz) and LOCAM Hi-speed camera (150Hz) was used to synchronously record the ground reaction force of one ground contact and the movement of the subject during one stride in the maximal speed phase (40m to 50m from start). In addition, an infrared time measurement system (Biovision) was used to record the time between 40m and 50m from the start position.

According to the impulse-momentum equation, the velocity of body c.m. is determined by the body mass and the impulse of ground reaction force. In present research, the horizontal ground reaction force was chosen as a control variable. Applying optimization theory and algorithms the ground reaction force was altered reasonably, in order to maximize the final horizontal velocity of body c.m. at end of each support phase. Therefore, the object function of the optimization is to maximize the impulse of the horizontal ground reaction force \( F_x(t) \), subject to: i) unchanging the support time, the time of positive and negative impulse; ii) unchanging the maximal and minimal force values and iii) the rate of force development (RFD) must be less than or equal to the maximal value of the original rate of force development.

The object function of the optimization is written:

\[
\frac{1}{m} \int_{t_0}^{t_f} F_x(t) \, dt = \text{max}
\]
Subject to the constraints:

\[
\begin{align*}
F_{\text{min}}; & \quad t = t_1 \\
F'_x(t) = & \quad 0; \quad t = t_2 \\
F_{\text{max}}; & \quad t = t_3 \\
F_{\text{min}} \leq F'_x(t) \leq F_{\text{max}} \\
0 \leq \frac{dF'_x(t)}{dt} \leq D = \frac{dF'_x(t)}{dt} \bigg|_{\text{max}}
\end{align*}
\]

Here: 
- \( m \) – mass of the subject
- \( F_x(t) \) – original horizontal ground reaction force
- \( F'_x(t) \) – optimized horizontal ground reaction force
- \( F_{\text{min}} \) – minimal horizontal force values
- \( F_{\text{max}} \) – maximal horizontal force values
- \( D \) – maximal RFD of original horizontal force

The optimized horizontal force \( F'_x(t) \) was assumed to be as a polynomial \( \sum_{i=0}^{n} a_i t^i \) which pass through the points \((t_1, F_{\text{min}}), (t_2, 0), (t_3, F_{\text{max}})\) and its first derivatives at points \( t_1 \) and \( t_3 \) were equal to the original data (see Figure 1). One dimensional search method – golden section – introduced by Chen (1989) in optimization algorithms was used to find the best solution for the polynomial with fixed initial and final times, which meets the subject function under constraints condition. The aim of the optimization is that the impulse of horizontal force is maximized, i.e. the final velocity of body c.m. at end of support phase is maximized at the instant of takeoff, under the condition that the vertical ground reaction force and the strength conditioning of the subject keeps unchanged.

RESULTS AND DISCUSSION: See Figure 1, the horizontal force time curve \( F_x(t) \) displayed a short interval of positive value after the braking (negative) impulse phase and than reversed to the negative value before it turns into accelerating (positive) impulse phase. This phenomenon of the \( F_x(t) \) was also reported by other studies (Wood, 1987; Ae et al 1991). According to these studies, the reason for this phenomenon could be explained by: i) improper force generation; ii) concussion of soft tissue like muscle and viscus after impact; iii) elasticity of body limb. Based on the analysis of joint muscle torque, we found that the subject reduplicated flexion torque at knee joint at the first part of the ground contact phase (Liu, 1993) and hence, the above abnormal horizontal force curve was considered to be caused by improper force generation. For that reason, an optimal control of ground reaction force was conducted in order to optimize the individual 100m sprint running performance.

![Figure 1 - Horizontal ground reaction force time curve.](image-url)
In this study, the function of $F'_x(t)$ was to be expressed as a $5^{th}$ ($n=5$) polynomial and its coefficients were determined by optimization process. The aim of the optimization is that the impulse of horizontal force is to maximize the final horizontal velocity at the end of each contact phase under the condition that the strength conditioning of the subject keeps unchanged.

According to the experimental data, $F_{min}=1.15kN; F_{max}=0.76kN; t_1=12ms; t_2=29ms; t_3=80ms$. Based on these experimental data, the velocity of body c.m. was simulated by the optimal control of ground reaction force. The results reveal that the final velocity of body c.m. during ground contact at each step can be increased about 0.05m/s through optimization of ground reaction force. Figure 2 shows the original and optimized horizontal velocity time curve of body c.m. in one ground contact phase. The measured average horizontal velocity was 9.83m/s during ground contact phase and 10.07m/s during air phase. The distance and time passed through by one step were 1.06m and 0.001s during ground contact phase, 1.16m and 0.115s during air phase respectively. According to these data, assuming that the maximal speed phase includes 20 steps, the performance of the subject for 100m sprint running could therefore be improved approximately 0.06-0.08s. This improvement is achieved only by changing the patterns of ground reaction force, and the conditioning of the athlete and the movement time keep unchanged. In other words, it is obtained by rational force generation. Through analysis of the experimental data and the condition of the subject, we found that it is highly possible for the subject to achieve this objective. Hence, this simulated individual performance could also be called as optimal performance.

REFERENCES: