INTRODUCTION

Among the human movements, the sporting ones represent the most critical condition. In fact, either fast actions or perfect movements are required, from a technical point of view, in order to obtain the best performances and/or to avoid injuries during events or training. For these reasons sporting activities represent the boundary of body expression. The complexity of sport actions reflects upon quantitative analysis because of the necessity of very sophisticated and expensive instrumentation, signal processing algorithms, and laboratories, while often coaches would need simple instrumentation to continuously carry on field analysis to control the effect of training.

The purpose of this work is twofold: to point out the role of the analysed performances and to assess the frequency content (i.e., the bandwidth) of body landmark displacements during several (basic) track and field movements.

In biomechanics in order to study quantitatively human motion the modelling approach is widely used (see Hatze [1984] for a review. Zatsiorsky (1980)).
The body is modelled as a set of links connected by hinges. It is possible, provided the estimate of body parameters, such as segment weight and their center of mass position, to assess the dynamics of the movement. This analysis also requires the acquisition of ground reaction forces by means of force platforms.

Since the early studies, most of the equipment developed for kinematic studies are based on measurements of displacement data of a limited set of artificial landmarks fixed to the body segments (review in Woltring, 1984; Lanshammar, 1982b).

In order to correctly use such devices it is very important to point out the theoretical constraints to be respected during analysis in order to obtain meaningful data.

At first we must be sure that the sampling rate is sufficiently high to ensure that the signal recorded has negligible components outside the Nyquist bandwidth according to the Shannon sampling theorem. If this condition is not met, frequency components are folded back into the Nyquist bandwidth resulting in signal distortion: the well known phenomenon of aliasing in signal processing. For this reason it is very important to have some knowledge about the frequency content of the signal under analysis.

Moreover if velocities and accelerations are of interest and they are estimated by numerical differentiation of displacement data, it is necessary to include additional constraints. In fact whatever the devices used to acquire data, these will contain certain amounts of error (see Wood 1982 for a review of the sources of error and Lanshammar, 1982b for a short review on the errors in various kinematic equipments).

In general, such errors can be divided into two classes: those that can be described as "systematic" such as image distortion, inaccurate scales and placement of body markers, etc.; and those that can be described as "random" arising from digitization process and from algorithms like 3D reconstruction. Considering these random errors induced by multiple and independent causes it can be reasonably hypothesized that the additive result would be normally distributed (central limit theorem) and independent of the signal which is assumed to be some unknown deterministic process (Cappozzo et al., 1975; Lesh et al., 1979; Lanshammar, 1981). Now, while the first kind of error is often relatively harmless as far as differentiation is concerned, the second kind is much more serious, in fact differentiation acts as a high pass filter, and thus amplifies the random noise which extends at high frequencies (Lanshammar 1982-a). In this case Lanshammar (Lanshammar, 1981) demonstrated that the sampling theorem could be inadequate for the
determination of sampling rates when the collected data are subjected to
differentiation. In Gustaffson and Lanshammar (1977) a formula was
verified that relates the maximal precision, (minimal variance) in es-
timated derivations, to the measurement noise variance, the sampling in-
terval and the bandwidth of the measured signal. Assuming the measure-
ment data $y(t)$ as a sum of white noise $e(t)$ and useful signal $x(t)$:

$$y(t) = x(t) + e(t); \ t = o, T, 2T, ... \ (T \text{ is the sampling interval})$$

in which the signal $x(t)$ is strictly a band limited signal with the bandwidth $x \text{ (rad/sec)}$

The minimal variance formula is:

$$V_k \geq V_{k, \text{min}} = \frac{V_n^2 T \omega x^{(2k+1)}}{\Pi (2k+1)}$$  \hspace{1cm} (1)

where $V_{2k}$ is the variance of the estimated k-th order derivative, $V_{2k, \text{min}}$ is its minimal value and $V_n^2$ is the white noise variance. From this for-

1. At first we note that when measurement data are used for estimation of the signal itself the minimal noise transmission i.e. the ratio $V_{2k, \text{min}}/V_n^2$ is proportional to the signal bandwidth, but when first and second derivatives are estimated the minimal noise transmission grows as $\omega^3 x$ and $\omega^5 x$, respectively.

Thus, for the estimation of the second derivative of measured time
series, a mistake in bandwidth selection could dramatically affect the
results, because the resulting precision is extremely sensitive to variations in the signal bandwidth.

For this reason algorithms that automatically select a signal bandwidth
based on the optimization of some objective parameters related to
measurement data (Woltring, 1985; Anderssen and Bloomfield; 1974,
D’Amico and Ferrigno, 1988; Wood and Jennings, 1978) must be preferred as cited in D’Amico and Ferrigno (1988), with respect to trial and error
methods (Reinsch 1967-71; Pezzack et al., 1977).

Another important fact can also be enlightened by formula (1): given the
precision of the measurements of data i.e. given the noise variance and
given the signal bandwidth, the only way to increase the K-th order deriva-
tion estimate precision is to increase sampling rate and from (1) the max-
imum sampling interval can be found in relationship with the required
precision on k-th derivative:
So the sampling rate suggested by the Shannon sampling theorem could not be suitable to obtain the required precision on derivatives. Moreover, if it possible to choose different acquisition data systems with different noise level added to the useful signal, is much better to prefer those featuring high precision even if not so fast, rather than fast but not highly accurate.

Moreover to confirm this fact it would be also emphasized that if the signal is varying slowly, then the assumption of noise whiteness is not valid if it is sampled too often (Lanshammar, 1982b) and this would be very dangerous because almost all differentiating algorithms work under the white noise hypothesis. In this frame the use of automatic motion analysers guarantees a known level of noise and its stationarity along the measurements.

Spectral estimation

The spectral estimation allows to compute an estimate of the power spectrum of a deterministic signal or of a stationary processes are addressed (Makhoul, 1975). The best known techniques span from the Periodogram (Schuster, 1898-99) based on Fourier analysis, to the Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA) models. Our approach has been oriented to the use of AR models which feature a wide variety of well proven algorithms.

**Autoregressive Algorithm**

The autoregressive model of a signal relies on the assumption that the signal could be modelled as a linear combination of its past values:

\[
y(k) = \sum_{i=1}^{P} a_i y(k-i)
\]  

where \( P \) is the model order and \( a_i \) is the \( i \)-th term of a vector of model parameters. It is possible to determine easily the parameters vector by minimizing the squared prediction error. This error is the difference between the model output \( y(k) \) and the actual value of the measurement \( y(k) \):

\[
T \leq T_{\text{max}} = \frac{V^2 k (2k+1)}{V_n^{2} \sin (2k+1)} \tag{2}
\]
\[ e = y(k) - y(k) - \sum_{i=1}^{P} y(k-i)a_i = \]
\[ = -\sum_{i=0}^{P-1} y(k-i)a_i \quad \text{with } a_0 = 1 \]
\[ E = \sum_{k=1}^{N-1} e_k^2 \]

where \( N \) is the total number of measured points.

Several algorithms can be used to obtain the model parameters; our choice has been oriented to the maximum sharpness in the details of the power spectrum density (PSD), with no assumptions on the signal outside the acquired record. We have used the forward-backward least squares algorithm (Ulrych and Clayton, 1976); (Nuttal, 1976) which gives very sharp PSD without showing Spectral line splitting (Kay and Marple, 1981; Marple 1987). The PSD estimate of the signal is easily computed by the model parameters vector as follows (Kay and Marple 1981):

\[ | S(z) |^2 = \frac{\nu^2 T^2}{1 + \sum_{k=1}^{P} a_k z^{-k}} \]

where \( S(z) \) represents the PSD, \( \nu^2 \) the standard deviation of the prediction error, \( T \) the reciprocal of the sampling rate and \( z \) is an arbitrary complex value.

The choice of the order has been discussed in D'Amico and Ferrigno (1988); for the present paper, \( P \) has been fixed to 9.

The AR model has been used also to extend the data before and after the acquired data in order to perform a filtering of the signal as described in D'Amico and Ferrigno (1988) and obtain the derivations of the data.

**Determined Frequencies**

For the purposes of this work three frequencies have been considered: the frequency at which the signal to noise ratio falls below 50 (also used for filtering), the frequency which bounds the 99.5% of the signal power and the frequency bounding the 99% of the signal power.

The first one (F1) is a measure of the compatibility of the measuring device with the data characteristics, it accounts for the maximum useful bandwidth in relation to the measurement noise. The second and third fre-
frequencies \((F_2, F_3)\) are more representative of the properties of the signal. The comparison between the first frequency \(F_1\) with respect to \(F_2\) and \(F_3\) allows an assessment of the validity of the use of a given instrumentation for a movement the power of which is bounded by \(F_2\) and \(F_3\). The reason we have used two frequencies \((F_2\ and\ F_3)\) instead of only one is that the difference between these two accounts for the consistency of the estimate and avoids methodical grossolane errors.

**Measuring Instrumentation**

The measurements of the kinematics of the movements that will be reported in the results have been performed with a fully automatic device: the ELITE system. This instrumentation, which measures the displacements of passive hemispheric lightweight landmarks applied to the subjects, does not interfere with either the athlete's movements or with the environment in which the tests were performed. This condition is mandatory when analysing sport movements in which the subject freedom must be guaranteed by the measuring devices. The ELITE system (Ferrigno and Pedotti, 1985) can guarantee such a non-invasivity in the measure because it recognizes the landmarks by their shape by means of a hardware implemented real time cross correlation algorithm. In fact this feature allows the use of very small unobtrusive landmarks (1 cm on a 2.8 meters of field of view), and make it possible to achieve a very high accuracy, up to one part on 2800 of the field of view, by computing the center of mass of the over threshold cross correlation markers for each landmark. The system used was a three dimensional one, equipped with two CCD TV cameras. In order to sharply sample the data, the cameras are electronically shuttered, i.e. they are sensitive to light for only 1 millisecond for each frame. The sampling rate is 50 images per second and the subjects were lit up by infrared flashes, mounted on the cameras, which work synchronously with the electronic shutter. The choice of the near infrared wavelength allows the use of a relatively high power flash (50 W for 1 millisecond) without giving any disturbance to the subject.

The 3D coordinates of the landmarks have been obtained by using the stereophotogrammetric parameters of the cameras which have been computed before the experiments by acquiring a control grid of landmarks of known geometry.

All the data were acquired and stored on a small personal computer IBM compatible with a 80286 INTEL microprocessor and mathematical coprocessor: an Olivetti PE28. All the subsequent processing for spectral estimation, \(F_1, F_2\ and \ F_3\) determination and for velocity computing have been carried out on a similar computer: an Olivetti M290.
The subjects of this study were two male athletes, trained for different track and field events: athlete A is a race walker, athlete B is 400 m hurdles runner. They were asked to perform the basic and technical movements as reported below.

The training background of the two subjects, ensured the correct execution of the movements with high technical content.

Athlete A:
- WAL. = walking at natural cadence;
- R.W. = race walking at competition speed;
- JOG. = running at warm up speed.

Athlete B:
- SPR. = sprinting;
- VJ. = vertical jump;
- D.J. = drop jump;
- BJ. = broad jump;
- STA. = starting from blocks, push off leg;
- HUR. = hurdle overcoming, leading leg;
- L.L. = take off exercises for long jumpers, leading leg.

Five reflective markers were fixed on the following anatomical points of the lower limb, chosen in order to permit the description of its kinematics through a four link model:
- marker 1 on the iliac crest;
- marker 2 on the head of the femur;
- marker 3 on the knee joint center of rotation;
- marker 4 on the external malleolus;
- marker 5 on the 5th metatarsal head.

RESULTS AND DISCUSSION

The exercise performed by the athletes describe a relatively large band of the human locomotion, as the peak values of speed amplitude measured on marker 1, the marker of the model nearest to the center of gravity, range from 2 m/s (walking) to 7.7 m/s (sprinting).

The results of the study are reported in the histograms of Figure 1, 2, 3, 4, and 5 where three cut off frequencies are shown for x and y coordinates of each marker. Black histograms represent the frequencies at which the
signal to noise ratio falls under 50, this is also the filter cut off frequency that neglects 1% and 0.5% of the signal power.

The results indicate numerous effects of the movement on the cut off frequency (frequency content) of each marker, moreover there are significant differences among the signals of the five anatomical points during the same movement, and between x and y coordinates of each marker during the same movement. The results will be discussed under respective sectional headings.

Figure 1

Figure 2
Figure 2

The cut off frequencies during each marker for each respective

X MARKER 2

Y MARKER 2

Figure 2
Figure 3
Figure 4
The means and standard deviation at 0.5%, reported in Table II, show that the lowest values are from 1% of the vertical movement. The standard deviation decreases as the percentage of power decreases from 1% to 0.5%. The trend in the standard deviation of power is from 0.5% to 1% to 0.1%. This result underlines the relationship between the control of the vertical movement and the loading of the vertical movement during vertical jump (y axis).

Another interesting point is the cut-off frequency of the vertical movement and its dependency on the marker during race walking. The head marker during race walking performs the filtering process with the kind of movement. The cut-off frequency at 1% of the signal power is lower, as a consequence of the filtering process. The difference between the muscle condition, and the cut-off frequency is high.

The analysis of the cut-off frequency preformed in order to observe the small differences of the consistency in the method. The filter cut-off frequency is shown in Table II, and the mean cut-off frequencies are computed by Rigano (1988).
Cut off frequency at 0.5% of signal power

The means and standard deviations of the five markers cut off frequency at 0.5%, reported in Table 1, demonstrate higher values in race walking (6.92 Hz) along the y axis and in vertical jump along x axis (4.84 Hz). The lowest values are from broad jump along both the axes (x = 2.88 Hz, y = 0.88 Hz).

All the movements have a lower cut off frequency along the principal direction of progression that is the x axis, and the same is true for marker coordinates of vertical jumping that evolves in the vertical direction.

The standard deviations range from 0.32 to 2.2 Hz for the x cut off frequencies and from 0.21 to 1.50 Hz for the y.

This result underlines differences of frequency content, varying from 0.6 to 5.2 Hz among the five markers of the lower limb during the same movement. The largest differences may be seen between markers 5 and 2 during vertical jump (x axis) and between markers 3 and 4 in race walking (y axis).

Another interesting observation concerns the differences between x and y cut off frequency of the same marker. They vary from marker to marker, and depend on the movement. The highest delta is shown by the knee marker during race walking (6 Hz) and the lowest by the fifth metatarsal head marker during jogging. All these observations emphasize the need to perform the filtering procedure with cut off frequency varying in relation with the kind of movement, the marker position and coordinates.

Cut off frequency at 1% signal power

Similar considerations may be drawn when the cut off frequencies at 1% of the signal power are analysed. Obviously, in this case, all the values are lower, as consequence of the larger amount of power neglected.

The difference between the frequencies ranges from 0.2 to 2 Hz, for the same condition, and there is the trend to have the higher differences, when the cut off value is high.

The analysis of the cut off frequency at 1% of signal power has been preformed in order to be sure that the cut off at 0.5% was correct. The small differences observed between these two values demonstrate a consistency in the method used.

Filter cut off frequency

Table II shows means and standard deviations of the five markers filter cut off frequencies, computed with the method proposed by D'Amico and Ferrigno (1988).
When the data are compared with those of Table I, it is possible to see that the filter always acts beyond the frequency which bound the 99.5% of the signal power.

This fact points out that the instrumentation used with its accuracy, although working at only 50 Hz of sampling rate, is appropriate. In fact, always more than 99.5% of the signal was considered for derivative computations, nevertheless a good signal to noise ratio was guaranteed.

### TABLE I
Mean values and standard deviations of the cut off frequencies binding the 99.5% of signal power.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>sd</th>
<th>Y</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAL.</td>
<td>0.98</td>
<td>0.32</td>
<td>3.56</td>
<td>0.95</td>
</tr>
<tr>
<td>R.W.</td>
<td>1.52</td>
<td>0.59</td>
<td>6.92</td>
<td>1.50</td>
</tr>
<tr>
<td>JOG.</td>
<td>1.08</td>
<td>0.33</td>
<td>3.40</td>
<td>0.94</td>
</tr>
<tr>
<td>SPR.</td>
<td>2.68</td>
<td>1.21</td>
<td>3.44</td>
<td>1.03</td>
</tr>
<tr>
<td>V.J.</td>
<td>4.84</td>
<td>2.20</td>
<td>2.80</td>
<td>1.47</td>
</tr>
<tr>
<td>D.J.</td>
<td>3.00</td>
<td>1.64</td>
<td>3.64</td>
<td>0.63</td>
</tr>
<tr>
<td>B.J.</td>
<td>0.88</td>
<td>0.41</td>
<td>2.88</td>
<td>0.86</td>
</tr>
<tr>
<td>STA.</td>
<td>1.88</td>
<td>1.23</td>
<td>4.04</td>
<td>0.72</td>
</tr>
<tr>
<td>HUR.</td>
<td>2.04</td>
<td>1.41</td>
<td>3.36</td>
<td>0.98</td>
</tr>
<tr>
<td>LL.</td>
<td>2.08</td>
<td>1.07</td>
<td>2.64</td>
<td>0.21</td>
</tr>
</tbody>
</table>

### TABLE II
Mean values and standard deviations of the filter cut off frequencies.

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>sd</th>
<th>Y</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAL.</td>
<td>5.08</td>
<td>1.26</td>
<td>5.72</td>
<td>1.41</td>
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<tr>
<td>R.W.</td>
<td>8.48</td>
<td>0.70</td>
<td>9.14</td>
<td>2.32</td>
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<tr>
<td>JOG.</td>
<td>5.76</td>
<td>1.53</td>
<td>6.72</td>
<td>2.02</td>
</tr>
<tr>
<td>SPR.</td>
<td>9.32</td>
<td>1.35</td>
<td>9.92</td>
<td>2.84</td>
</tr>
<tr>
<td>V.J.</td>
<td>7.24</td>
<td>1.76</td>
<td>6.64</td>
<td>0.76</td>
</tr>
<tr>
<td>D.J.</td>
<td>5.96</td>
<td>1.35</td>
<td>8.88</td>
<td>2.57</td>
</tr>
<tr>
<td>B.J.</td>
<td>6.72</td>
<td>0.95</td>
<td>6.80</td>
<td>1.93</td>
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<tr>
<td>STA.</td>
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<td>1.90</td>
<td>8.08</td>
<td>1.45</td>
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<tr>
<td>HUR.</td>
<td>9.16</td>
<td>2.11</td>
<td>10.12</td>
<td>1.37</td>
</tr>
<tr>
<td>LL.</td>
<td>8.56</td>
<td>1.84</td>
<td>8.96</td>
<td>1.16</td>
</tr>
</tbody>
</table>

REFERENCES


- (1899) The periodogram of magnetic declination as obtained from the records of the Greenwich Observatory during the years 1871-1895. Trans. Cambridge Philosophical Soc., 18,107-135.


