

THE EFFECT OF THE STRAIN RATE ON THE SHORT TERM SHEAR RESPONSE OF ARTICULAR CARTILAGE

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LIST OF SYMBOLS:

a, b	Parameters of the elastic response	$T(e)$	Elastic response.
c	Constant, influencing relaxation rate.	λ	Deformation relative to AC thickness.
G	Reduced relaxation function.	τ	Auxiliary time-variable.
S	Relaxation time spectrum.	τ_1, τ_2	Lower and upper limit for non-zero $S(\tau)$.
t	Time.		
T	Measured or calculated force.		

INTRODUCTION

Articular cartilage (AC) consists mainly of water and a solid matrix, which contains a resilient collagen network and a water-imbibing proteoglycan (PG) gel containing a net surplus of fixed negative charges. The PG gel exerts a swelling pressure on the collagen network and keeps it rigid. It influences the flow resistivity and the equilibrium modulus of AC. On the other hand, the entrapment of the PG gel within the collagen network enhances the short term stiffness (Jurvelin et al., 1992). The effect of loading and deformation rate on AC stiffness has been studied by several investigators either in strain rates below 10^{-1} s^{-1} or at the domain of impact response, approx. 10^{+3} s^{-1} . Oloyede et al. (1992) has investigated compressive properties of AC both at strain rates from $5 \cdot 10^{-5}$ to $5 \cdot 10^{-2} \text{ s}^{-1}$ and at the rate of 10^{+3} s^{-1} .

The present work studies the way the apparent flow-independent stiffness of AC rises, when deformation rate increases. It is also investigated whether the strain rate-stiffness curve will at some time scale level to some "instant" rigidity. In the shear geometry, as it is applied in this work, fluid flow is assumed to be negligible, and time-dependent behavior is seen to reflect the intrinsic properties of the AC matrix.

METHODOLOGY

Fresh bovine knees were dissected in order to obtain cylindrical osteochondral plugs from the patellar groove. The diameter of the plugs ($N=9$) was 6 mm, and the thickness varied from 1.25 to 1.97 mm, as measured by a needle penetration technique (Hoch et al., 1983) from four sites around the center of the plug. An electrohydraulic universal materials testing apparatus (Figs 1, 2) was employed at the measurements. The device was controlled by PC/AT microcomputer via an DT2801-A 1/0 card (Data Translation). The specimen was fixed to the testing system using cyanoacrylate cement. Before testing, the specimens were preconditioned with cyclical shear loading for 5 minutes and after that were left to recover for 5 minutes. Approximately 0.1 mm shear ramp deformations were administered on each specimen at 11 different speeds. The deformation and the force signals were digitized at 1 kHz sampling frequency for 2

seconds, and there was a 5 minutes pause before each measurement.

For mathematical analysis of the time-dependent deformation and force data a quasilinear viscoelastic model (Fung, 1981) was used. In this formulation, the step response is seen as a product of the elastic response, which takes into account the nonlinearity, and the reduced relaxation function that relaxes from unity to equilibrium response (Figure 3). Intrinsically, this formulation assumes a finite instant response, but as it can be very high when compared to equilibrium response, this isn't problematic at finite deformation speeds, even if the real instant response is infinite.

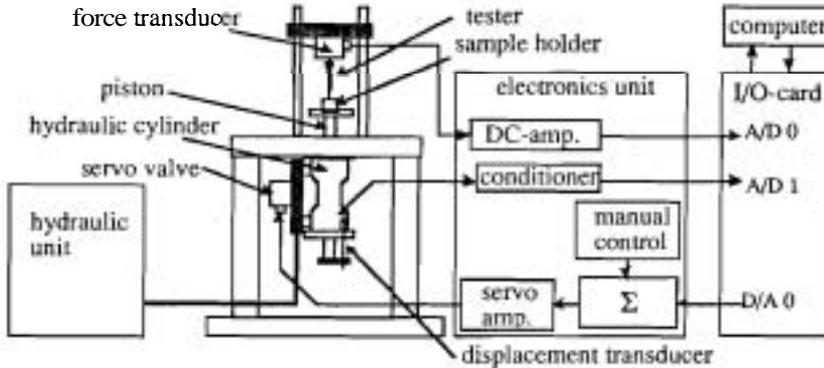


Figure 1. The electrohydraulic materials testing apparatus and its control system. The configuration of the figure is for indentation test.

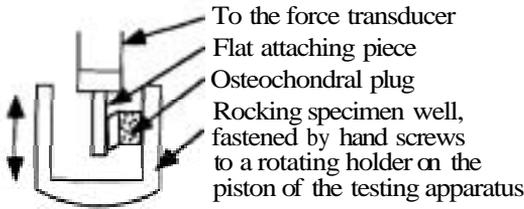


Figure 2. The shear testing geometry.

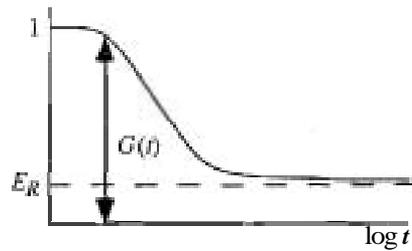


Figure 3. Unit step response in the quasilinear viscoelastic model by Fung (1981).

The measured force at a given moment can be seen as a convolution integral of the elastic response and the change rate of the reduced relaxation function:

$$(1a) \quad T(t) = T^{(e)}[\lambda(t)] + \int_0^t T^{(e)}[\lambda(t-\tau)] \frac{\partial G(\tau)}{\partial \tau} \cdot d\tau,$$

$$(1b) \quad T^{(e)}(\lambda) = a \cdot \lambda^2 + b \cdot \lambda.$$

Note that this makes it possible to calculate the force caused by an arbitrary deformation history. In this formulation, the reduced relaxation function is defined as:

$$(2) \quad G(t) = \frac{1 + \int_0^{\infty} S(\tau) \cdot e^{-t\tau} d\tau}{1 + \int_0^{\infty} S(t) d\tau}$$

Here, a following relaxation spectrum was employed:

$$(3) \quad S(\tau) = \frac{c}{\tau}, \quad \tau_1 \leq \tau \leq \tau_2, \quad 0 \text{ elsewhere.}$$

Thus, the material properties of the sample were described by five parameters: a , b , c , τ_1 and τ_2 . The data was analyzed in AT-MATLAB (The Mathworks Inc.) environment. The parameters were determined for each specimen separately by Nelder-Mead Simplex algorithm. The square sum of the differences between the measured force signal and that calculated by the displacement signal was formed for each measurement of different strain rate, and the square sum of these square sums was used as a function to be minimized. The shear moduli at 4% strain were calculated by using both the measured and the modelled force signals. The corresponding strain rates were assessed from the slope of the displacement signals at 4% strain.

RESULTS

The linearity of the elastic response varied strongly within the specimens. For one of the specimens, the shear moduli at different strain rates are shown in Figure 4. The data showed stiffening that was somewhat faster than logarithmic as a function of the strain rate.

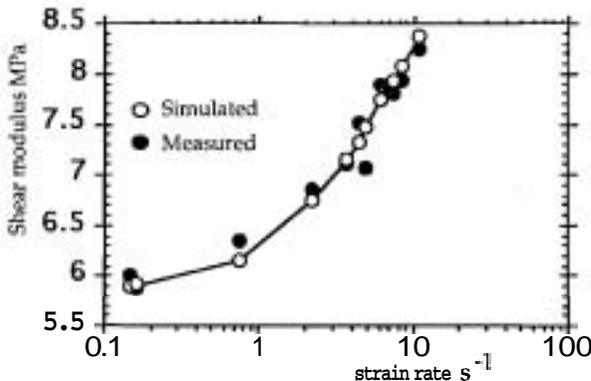


Figure 4. Simulated and measured shear moduli as a function of the strain rate.

The range of the strain rates is approximately from 0.06 s⁻¹ to 12 s⁻¹. The average shear

modulus at strain rate 10 s^{-1} was 4.85 MPa. From the strain rate 0.1 s^{-1} to rate 10 s^{-1} the shear modulus rose in average by a factor of 1.52.

DISCUSSION

As the strain rate advanced from about 0.1 to 10 s^{-1} , a significant rise of the shear modulus was found in this work. On the other hand, Oloyede et al. (1992) reported "a minimal" increase in stiffness while the strain rate in compressive tests for thawed specimens rose from $5 \cdot 10^{-2}$ to $1 \cdot 10^{-3} \text{ s}^{-1}$. In impact range of loading rate, Finlay and Repo (1979) found only negligible difference between compressive stress/strain data at strain rates 500 and 1000 s^{-1} . These results suggest an effective instant response that doesn't change, once the strain rate is high enough.

CONCLUSIONS

The linearity of the mechanical response varies strongly within the articular cartilage specimens. The employed mathematical model agrees reasonably well with the behavior of articular cartilage at the strain rates used in measurements. The rate at which the cartilage stiffens, when the strain rate is increased, is somewhat faster than logarithmic. A finite instant mechanical response of articular cartilage was not found in this experiment, although some other results may suggest it.

ACKNOWLEDGEMENT

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