A MODELLING METHOD FOR DISCRETE LOW SAMPLING FREQUENCY
TEMPORAL SERIES ON THE EVALUATION OF INTRA-CYCLIC SWIMMING
SPEED FLUCTUATIONS

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INTRODUCTION

Profiles of intra-cycle swimming speed fluctuations has been widely used as a highly informative parameter on swimming biomechanics. Methods described in literature include: (i) free swimming and (ii) linked swimming approaches.

In a previous study (Vilas-Boas, 1992), we described a photo-optical method for the assessment of free swimming intra-cyclic velocity fluctuations. This method was characterized by a low sampling frequency and by a least adequate mathematical modelling method, since conventional polynomial regressions do not fully respect the cyclicity of the phenomena itself: both extremes of the model curve don't fit with each other, imposing-sudden velocity discontinuities that could only be explained through infinite accelerations and forces.

The purpose of this paper is to describe a modelling method for discrete intra cycle velocity fluctuation analysis with reduced sampling frequencies.

METHODOLOGY

Data acquisition: Intra-cyclic velocity (v) / time (t) pairs of values were obtained from a intermittent light-trace photographic method (Vilas-Boas, 1992). The method consists in the photographic registration, with prolonged exposure, of the trace produced by a pulse-light device attached to the waist of the swimmer, at a middle distance between the two hip joints. Photos (Canon T70, 35mm, Kodacolor 1000 ASA film) were digitized using a Calcamp digitizing table, the Sigma Scan software and a PC computer.

The modelling method: The first step consists in the superimposition of three consecutive breaststroke cycles, sampled with reduced frequency. This was performed subtracting, or adding, a estimated cycle period (T) to each t value of the extreme cycles sampled in each photograph. The initial T value was estimated from the time interval between two consecutive absolute v minimums. This first step was performed in order to: (i) increase the number of points to define the final model; and (ii) allow the individual model of the stroke cycle velocity / time curve to be calculated on more than one isolated stroke cycle. Once translation was accomplished, one first 8th degree polynomial regression was calculated:

\[ v = a + \sum_{i=1}^{8} b_i \cdot t \]

In order to allow the polynomial equation to respect the cyclical nature of the phenomena, two constrains were imposed to the regression, both on the initial (t = 0) and the final (t = T) instants of the stroke cycle model:

(i) \[ v(0) = v(T) \] (2)

and

(ii) \[ \frac{dv}{dt} \bigg|_{t=0} = \frac{dv}{dt} \bigg|_{t=T} \] (3)
This was accomplished as follows: 

Taking into account the equation (1),

\[ v(0) = a \] \hspace{1cm} (4)

and

\[ v(T) = a + \sum_{i=1}^{I} b_i \cdot T^i \] \hspace{1cm} (5)

Changing (4) and (5) into equation (2),

\[ a = a + \sum_{i=1}^{I} b_i \cdot T^i \] \hspace{1cm} (6)

then:

\[ \sum_{i=1}^{I} b_i \cdot T^i = \] \hspace{1cm} (7)

In the other hand, once:

\[ \frac{dv}{dt} = \sum_{i=1}^{I} i \cdot b_i \cdot T^{i-1} = b_1 + 2b_2 T + 3b_3 T^2 + \cdots + I \cdot b_I \cdot T^{I-1} \] \hspace{1cm} (8)

the first derivatives of the velocity in order of time for the first (t = 0) and last (t = T) points of the stroke cycle can be described as follows:

\[ \left. \frac{dv}{dt} \right|_{t=0} = b_1 \] \hspace{1cm} (9)

and

\[ \left. \frac{dv}{dt} \right|_{t=T} = b_1 + \sum_{i=2}^{I} i \cdot b_i \cdot T^{i-1} \] \hspace{1cm} (10)

Considering the second imposed constraint (3), is possible to note that:

\[ b_1 = b_1 + \sum_{i=2}^{I} i \cdot b_i \cdot T^{i-1} \] \hspace{1cm} (11)

and then:

\[ \sum_{i=2}^{I} i \cdot b_i \cdot T^{i-1} = \] \hspace{1cm} (12)

Starting from (12) is then possible to calculate the b1 coefficient of the regression equation, taking into account the optimized estimations of the coefficients b2 to bI-1, performed through the Marquardt (1963) algorithm.

If:

\[ 2b_2 T + 3b_3 T^2 + \cdots + I \cdot b_I \cdot T^{I-1} = \] \hspace{1cm} (13)

then:

\[ 2b_2 T + 3b_3 T^2 + \cdots + (I-1) \cdot b_{I-1} \cdot T^{I-2} + I \cdot \left( \frac{b_1}{T^{I-1}} \right) \cdot T^{I-1} = \] \hspace{1cm} (14)

It means:

\[ 2b_2 T + 3b_3 T^2 + \cdots + (I-1) \cdot b_{I-1} \cdot T^{I-2} = \left( \frac{b_1}{T^{I-1}} \right) \cdot T^{I-1} + \cdots \] \hspace{1cm} (15)

or:

\[ 2b_2 T + 3b_3 T^2 + \cdots + (I-1) \cdot b_{I-1} \cdot T^{I-2} = \left( \frac{b_1}{T^{I-1}} \right) \cdot T^{I-1} + \cdots \] \hspace{1cm} (16)

Working on, we obtain:

\[ -I b_1 \cdot (2-1) b_2 T \cdot (3-I) b_3 T^2 \cdot \cdots \cdot (-1) b_{I-1} \cdot T^{I-2} = \] \hspace{1cm} (17)

and:

\[ I b_1 \cdot (I-2) b_2 T \cdot (I-3) b_3 T^2 \cdot \cdots \cdot b_{I-1} \cdot T^{I-2} = \] \hspace{1cm} (18)
Developing in order to bl we have:

\[ b_1 = \frac{1}{T} [(I-2)b_2 T + (I-3)b_3 T^2 + \ldots + b_{I-1} T^{I-2}] \]  

or:

\[ b_1 = \frac{1}{T} \prod_{t=2}^{I} (I-1) b_{T^{-1}} \]  

Starting from (11) it is also possible to calculate the last coefficient of the regression equation \( (b_I) \).

If:

\[ b_1 T + b_2 T^2 + \ldots + b_{I-1} T^{I-1} = (21) \]

then:

\[ b_1 = \frac{1}{T} (b_1 T + b_2 T^2 + \ldots + b_{I-1} T^{I-1}) \]  

and:

\[ b_1 = \frac{1}{T} \prod_{t=1}^{I-1} \frac{b_{T^{-1}}}{} \]  

RESULTS AND DISCUSSION

Figure 1 presents velocity / time curves for three breaststroke techniques performed by 6 Portuguese male swimmers at 200 m race pace. It is possible to note the general coherence of the models.

Figure 1. Individual velocity / time curves obtained for 6 swimmers, each one performing three breaststroke techniques at 200 m race pace.

Using PC-Matlab (3.13) for integration and derivation of the special polynomial equations, it is possible to assess acceleration curves, and per phase resultant impulses (Vilas-Boas, 1994), as well as duration and horizontal distance covered per phase.

Results were highly compatible with previous reports (Vilas-Boas, 1993) and pointed out that: (i) mean minimum velocity associated with the recovery of the legs \((v1)\) was .40 (SD = .035) m.sec\(^{-1}\); (ii) mean maximal velocity associated with the leg kick \((v2)\)
was 1.43 (SD = .039) m.sec$^{-1}$; (iii) mean minimum intermediate velocity associated with the transition phase between leg and arm strokes ($v_3$) was 1.07 (SD = .027) m.sec$^{-1}$; (iii) mean peak velocity associated with the armstroke ($v_4$) was 1.26 (SD = .038) m.sec$^{-1}$; (iv) mean acceleration and resultant impulse between $v_1$ and $v_2$ were 3.03 (SD = .314) m.sec$^{-2}$ and 61.40 (SD = 3.726) Ns; (v) between $v_2$ and $v_3$ were -1.08 m.sec$^{-2}$ and -21.43 (SD = 3.478) Ns; (vi) between $v_3$ and $v_4$ were .69 (SD = .084) m.sec$^{-2}$ and 11.32 (SD = 1.853) Ns and (vii) between $v_4$ and $v_1'$ were -2.24 (SD = .026) m.sec$^{-2}$ and -51.13 (SD = .962) Ns.

CONCLUSIONS

Results of this study showed that the described mathematical method is feasible for modelling discrete low sampling frequency velocity / time curves of breaststroke swimmers. Nevertheless, further research on procedure fidelity should be conducted in the future.

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