A MATHEMATICAL MODEL OF HUMAN DYNAMIC LOCOMOTION: THEORETICAL BASIS OF THE MODEL

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INTRODUCTION

Dynamic locomotion is characterized by a cycle of movement that includes a phase of support as well as a ballistic flight phase. The most current models of this type of locomotion involve the use of a simple or damped spring-mass system (McMahon and Green, 1979; Blickhan, 1989). Each of these models uses rather simple approximations (point-like mass, and massless spring) of the complex human anatomy. They use the dynamic variables but completely neglect the control process. Moreover, these models do not describe a realistic behavior of the system at some instant in time. In reality, a complicated process - depending on anatomy, posture, and muscle control - gives rise to a wide variation in system stiffness as the takeoff leg moves over the support foot. In previous models however, the system stiffness k is kept as a constant during the support phase. The problem encountered in developing an analytical approach for coaching is the unavailability of a mathematical model that accurately describes support phase mechanisms. The purpose of this study is to create a mathematical model that gives all the features influencing distance. This model will thus become a tool for coaches to design individual performance in a heuristic manner.

METHODOLOGY

We are dealing with the four important factors (dynamic variables, anatomy, posture, and controlling) which govern dynamic locomotion as far as translation is concerned. We use the dynamic variables - the linear momentum and the force as functions of time. The human anatomy is approximated by a point-like mass (the center of gravity of the individual human body), and a leg/foot system that provides for the right distance between the center of gravity and the center of pressure during the support phase. This point-like mass and leg/foot system is also what is necessary for describing the posture. A vital point of the dynamic locomotion of athletes is their ability to execute their movement by controlling the system stiffness during the support phase - a fact that most earlier models did not take into account.

In our model the system stiffness $k(t)$ is a function of time. Similar to the work of Blickhan (1989) we derive the equations:

$$y''(t) = y(t)\,\omega^2(t) \left[\frac{1}{\sqrt{y^2(t) + z^2(t)}} \right]^{-1} \tag{1}$$

$$z''(t) = z(t)\,\omega^2(t) \left[\frac{1}{\sqrt{y^2(t) + z^2(t)}} \right]^{-\varepsilon} \tag{2}$$

Here $y(t)$ is the horizontal jump direction, $z(t)$ the vertical jump direction, $y''(t)$ the acceleration in y-direction, $z''(t)$ the acceleration in z-direction, l is the distance betwe the center of gravity and the center of pressure at take-off, $k(t)$ is the system stiffness, g is the gravitational acceleration, and

$$\omega(t) = \sqrt{k(t) \setminus m} \tag{3}$$

is the frequency of the system which depends on the system stiffness and the mass m. However, unlike the work of previous authors, our use of the system stiffness makes it possible to describe the control process during the support phase, and allows damping and muscle work.

The system stiffness of an actual performance can be calculated from collected force data:

$$k(t) = \frac{F(t) \cdot \Delta l(t)}{\Delta l(t)^2} \tag{4}$$

Here, F is the force vector and l the vector between the center of pressure and the center of gravity whereas modifications of this measured curve are signs of a modified technique.

The necessary input parameters for our model are as follows: the system stiffness as a function of time during the support phase, the mass of the subject, the velocity vector of the center of gravity at touch-down, the distance between center of gravity and center of pressure at touch-down, distance between center of gravity and center of pressure at take-off, the touch-down angle at support phase, and the touch-down angle at the landing.

To solve the coupled differential equations (1) and (2) we use an iterative method on a PC and input the k(t) from a data file. The deviation caused by this PC calculation is smaller than 0.1%.

The first experimental proof of our model was done in the biomechanics laboratory at The University of Michigan, Ann Arbor. Here, we used a Bertec force plate, three video cameras (60 Hz), and a Motion Analysis System. A subject was marked with four reflective markers on the right side of the body: on the shoulder, the hip, the heel, and the toes. We used the Motion Analysis System to get the coordinates and the velocity of the four points during twenty jumps. The force plate gave the force with a frequency of 500 Hz.

The touch-down angle, the take-off angle and the distance between the center of gravity and the center of pressure had been calculated with the help of a computer program based on the HANAVAN MODEL, and using the individual anthropometric data and the posture of our subject.

RESULTS and DISCUSSION

Although simple, the model presents all the necessary parameters to calculate the coordinates of the center of gravity. This includes the derivations with respect to time - velocity and acceleration - as functions of time during support (see Figure 1) and flight phase. We calculate the jump distance by using the anatomical leg/foot lengths and the system stiffness as a function of time. Mechanical energy, the kinetic energy (translational energy but not rotational energy), the potential energy, and the elastic energy, are calculated for the entire jump (see Figure 2). Most important for coaches is the possibility to use this model to obtain the necessary input parameters for an optimal jump distance. This means, by designing the jump on the computer one can determine the best initial parameter for the individual athlete to excel.

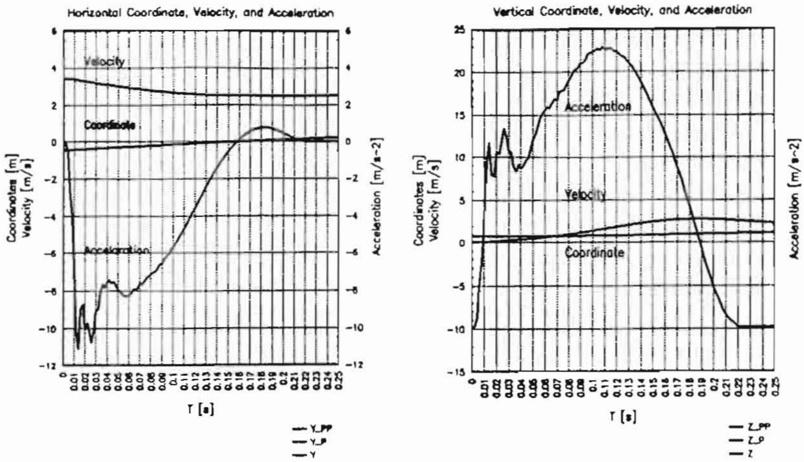

Figure 1. Support phase coordinates and derivatives.

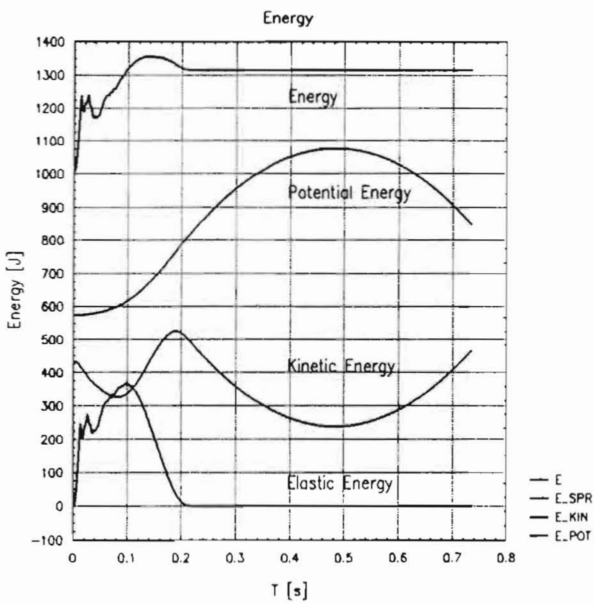

Figure 2. Mechanical energy.

In our first experimental proof we were able to demonstrate that our calculation consistently agreed with the results (Table 1). It is evident that this first proof is just a step in verifying our model experimentally. A more precise experiment has been planned using a different motion analysis equipment, appropriately chosen for this specific task, and sensitive enough to detect even minute differences in the performance.

Table 1. Results of jump based on the model.

Phase	Experimental Data	Mathematical Model
	Initial parameters	
Step		
horizontal velocity (m/s)	2.8 - 4.1	3.4
vertical velocity (m/s)	-0.5 - 0.5	0.15
touch down angle (°)	114 - 126	120
distance between CoG - CoP (m)	0.85 - 0.95	0.9
Landing		
touch down angle (°)	86 - 94	90
distance between CoG - CoP (m)	0.2 - 0.3	0.25
Results		
distance (m)	1.41 - 1.45	1.45
airborne time (s)	0.50 - 0.53	0.51
support time (s)	0.23	0.225

CONCLUSIONS

Our model exhibits all features necessary to design an optimal support phase for individual athletes. To achieve this, we calculate the optimal initial parameters by using a given and/or designed system stiffness. Our mathematical model is thus a tool for computer-aided training in horizontal jumps. A coach who is familiar with scientific methods can use it to guide athletes according to their individual ability.

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