COMPUTER MODELLING OF A SKI JUMP

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INTRODUCTION: Ski-jumping was qualified as a sporting event 100 years ago in Scandinavia and northern regions of Russia, and from the first years scientists helped athletes to sharpen their techniques. The first of them was R. Shtrauman, who created the "Norway" style of jump in 1924.

There have been several studies about ski-jumping published in the last two decades. Grozin (1971) made an integrated study of jump styles of the 1950s and 1960s. Aerodynamic coefficients are obtained from his experiment with figurines of ski-jumpers in an aerodynamic tube. Remizov (1973,1984) used these coefficients in the optimization problem solved by Pontryagin's maximum principle. Numerical methods were used for similar problems by Komi et al. (1974) and Tani et al. (1971). They all considered old techniques of jumping and, surprisingly, none considered the wind or used constraint on the landing velocity in the case of optimization problems. The most recent paper of Bagin et al. (1997) employs an original mathematical method based on the theory of complex variables and tries to add wind speed to movement equations, but they were written with an error.

The overview of the works above allows us to come to the conclusion that none of the following issues have been sufficiently studied: landing velocity, the effect of wind on a jump and the influence of the exact individual parameters of a fully-equipped skier. The main purpose of this study is to construct a mathematical model involving these parameters.

METHODS: This model takes into account the following factors: the wind, the push during break-off, mass, length, width of a ski-jumper in his equipment and the random nature of wind and position of a ski-jumper. We consider this problem in a 2D statement. Also, only the V-style jump is regarded, so that the angle of the skis with the horizontal is practically constant during the flight, and both the skier and skis are considered to be located in one plane.

In differential equations of motion the athlete is regarded as a moving material particle, affected by forces of gravity, air resistance and lift. The Reynolds number is about 105 for the average speeds of skiers, so the hydraulic estimation is valid and the aerodynamic coefficients can be used. The attack angle is a sum of two: the skis-horizon angle, which is constant, and the speed-horizon angle, which could be easily calculated at any point of the trajectory. We also need to know about the wind around and the aerodynamics of a fully equipped skier to find the flight path. The problem is solved by the 4-th order Runge-Kutta method.

Wind is measured only at one point during competition, but it may be different throughout the trajectory, so we need to forecast it at any place around the jumping hill, knowing it precisely at one point. Airflow around the jumping hill is found from the FEM solution of the border layer boundary problem with a no-slip condition on the jumping hill surface.

Aerodynamic coefficients are found from the FEM solution of the boundary problem of airflow around the ski-jumper with a no-slip boundary condition. The undisturbed flow velocity has been taken from the previous problem. The difference between these two problems is that airflow around a jumping hill is rather slow and laminar, while airflow around the skier is turbulent. The theory of quazilaminarity is used to describe turbulence.

Input factors of the model are: the jumping hill parameters; weight, height, and width of skier-on-skis; skis-horizon angle during flight (position of the skier); wind during the jump; starting velocity and push speed during break-off. To identify the model, a videorecording of a live broadcast from the "Four Hills" World Cup Contest held on January 4, 1998, on a K110 jumping hill in Innsbruck was used. One parameter – the air resistance coefficient in the beginning of the flight – was chosen from the problem of minimization of the difference between calculated and experimental distances with constraint on landing velocity.

Another optimization problem was solved to obtain the best position of the ski-jumper for maximizing the distance with constraint on landing velocity.

The very possibility of achieving maximum distance depends not only on the equipment, physical characteristics of the athlete and his jumping style, but also on the stability of holding an optimal position during flight. Random gusts of wind could influence flight distance but could not be considered by a jury. Thus, analysis of the stability of optimal flight trajectories with the statistical distribution of parameters is an inevitable part of flight investigation. Wind gusts and the skis-horizon angle are considered random continuous functions with normal distribution. We set the confidence level of distance, the skis' angle and wind speed at 0.95. The deviation interval of flight distance (for a given deviation interval of the skis' angle and wind speed) was found. The mathematical apparatus of statistical stability (Gitman et al., 1996) was used.

RESULTS AND DISCUSSION: The results identified by the model are the following: 1/6 of the computer model "jumpers" landed within 0.5 m of the actual distance, 50% – within 1-3 m and the rest – within 3-8 m.

The wind is blowing along the landing slope, which is known from the solution of the problem described above. The wind velocity is lower at the foot of the hill.

The optimal angle of skis to horizon was found for different masses and lengths of skier-on-skis to maximize the flight distance (Fig. 1). As is clearly seen, the optimal angle is near 0. Our model states that the heavier the skier and his equipment are and the shorter his skis, the larger is his optimal angle (i.e., the higher he should raise his ski tips). Lines of equal distance are pictured in Fig. 2. It shows that mass is more important than length in terms of "struggle for distance"; e.g., decrease of mass by 6.5 kg (7.7% of 84 kg) is equivalent to lengthening the skis by 25 cm (9.4% of 2.65 m).

Results of both statistical stability problems are presented in Table 1. Problem 1 finds the confidence interval of flight distance (A) for a given confidence interval of skis angle (A) and problem 2 finds the confidence

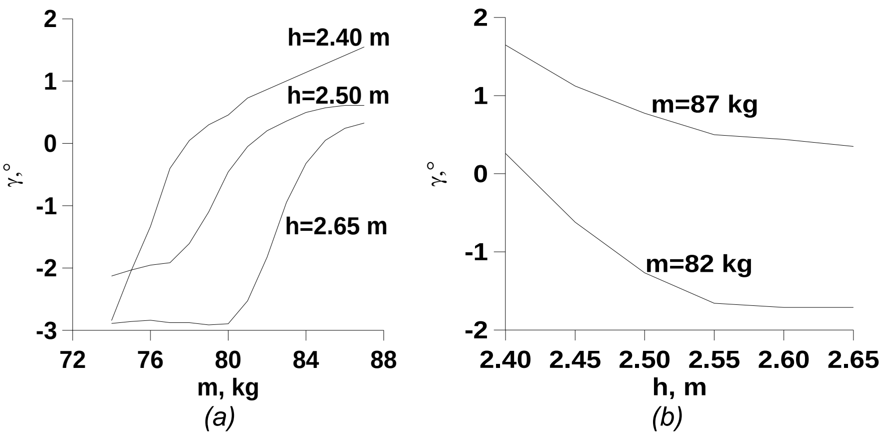

Fig. 1. Optimal angles for different lengths (a) and masses (b) of skier-on-skis.

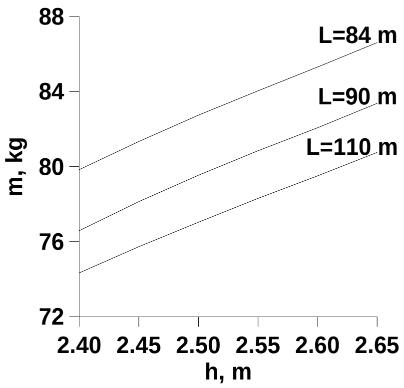

Fig.2. Isolines of flight distance in coordinates of mass-length of skier-on-skis system.

interval of flight distance (W) for a given confidence interval of wind speed (W). Note that optimal positions (and trajectories) of the skier are analyzed. Firstly, it is clearly seen that random deviations from the optimal position of about (5) during the flight do not noticeably decrease distance unless the skier is very tall or heavy. Secondly, random gusts of wind about 1 m/s can decrease flight distance as much as 2 m for light and tall jumpers. Bad weather is not as detrimental for heavy jumpers. It was noticed though that "almost-optimal" trajectories are generally more stable if there is a nasty wind or bad positioning.

Table 1. Results of statistical problems solution: h is the length of the skis; v0 is the break-off speed without push; m is the mass of the skier; (0 is the skis-horizon angle; results are obtained for the same push speed 3 m/s and no wind.

Initial Conditions	*Problem 1*		*Problem 2*	
	δ_A, °	ε_A, *m*	δ_W, *m/s*	ε_W, *m*
h=2.40 m, v_0=90 km/h, m=74 kg, γ_0=-2.6°	5.6	0.28	1.11	1.57
h=2.50 m, v_0=90 km/h, m=74 kg, γ_0=-2.1°	5.5	0.23	1.10	1.97
h=2.65 m, v_0=90 km/h, m=74 kg, γ_0= 0°	5.8	0.83	1.14	2.20
h=2.65 m, v_0=80 km/h, m=95 kg, γ_0= 2.3°	5.9	0.55	1.21	0.92

Fig. 3 shows that the aerodynamic coefficients used in this paper and known experimental results (Grozin, 1971) are similar in kind.

As is seen from Fig.1, the optimal flight angle is about 0, which correlates with experimental data: a video recording of the leading modern world athletes shows that skier and skis keep a practically horizontal orientation. Optimization and stability results were confirmed by the jumpers.

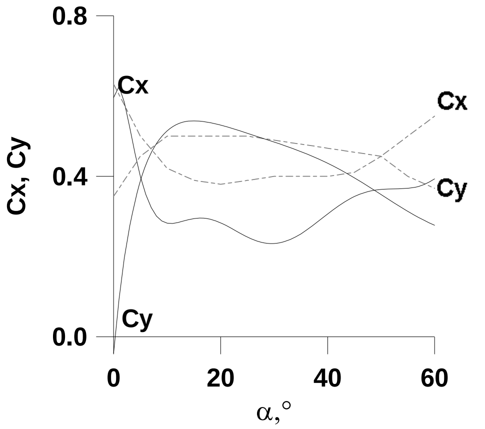

Fig. 3. Aerodynamic coefficients of forces of lift c_y and air resistance c_x vs. attack angle (solid line - model; dotted line - experiments).

CONCLUSION: The identification process included measuring, which has errors. For example, the error of angle determination is 1. The skier's mass and ski sizes were taken as average; only the most general corrections were considered. The permeability of ski clothing was not considered. Aerodynamic coefficients were obtained from the solution of the 2D airflow problem, while 2D and 3D turbulence have different laws. The theory of quazilaminarity is the simplest, and aerodynamic coefficients were found for a triangular plane not a figurine.
Considering everything, the model created has surprisingly high precision. It confirms a range of experimental facts which proves that the qualitative relations of parameters are described rather well even by such a simple model of a ski jump.

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